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Expression of Sin(x) and Cos(x) functions as cubic spline functions

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ABSTRACT

Cubic spline function is a useful technique to interpolate between known data points owing to its stable characteristics. In this study, cubic spline functions, used to investigate the relationships between dependent and independent variables, and its use in trigonometry fields were investigated. For this purpose, by giving various values for sin(x) and cos(x) functions in the range of $[0, 2\pi]$, the function values obtained as cubic spline functions were calculated. It was observed that the calculated values reached almost the same values with the observed values of the sin(x) and cos(x) functions. This result indicates that cubic spline functions are a very convenient method for calculating sin(x) and cos(x).

Key words: Cubic spline, sin(x), cos(x)

1. INTRODUCTION

Cubic spline functions allows investigation of the shape of the time dependence without requiring that the exact functional form first be specified. Cubic splines provide great flexibility in fitting data with relatively few parameters [1, 2].

The common method for data interpolation is a cubic spline function. There exist many types of spline basis function with respective degrees and its respective knots. But from literature, the most suitable spline for many applications is cubic splines interpolation. One of the principal reasons why cubic spline is the most utilizing basis function for data interpolation is that it is the lower degree splines that can obtain the C^2 continuity [3-5].

According to the literature review, studies with cubic spline functions were found. [6] developed a new spline method for solving two-point second order boundary value problems. [7], presented a numerical scheme for solving a finite difference approximation for discretizing spatial derivatives, and used the cubic spline collocation technique for the time integration of the resulting nonlinear system of ordinary differential equations. Analytical shaping method for low-thrust rendezvous trajectory using cubic spline functions was carried out by [8]. In other study, a modified cubic spline interpolation method was developed for chemical engineering application [9]. In other study, cubic spline function was used as an interpolation function for the simulation of groundwater flow [10].

In this study, it is aimed to write the functions sin (x) and cos (x), which are among the trigonometric functions, as cubic spline functions.

2. MATERIAL AND METHOD

If we draw a unit circle with the same center C, the are A'B' cut by the angle will have length θ, by the definition of radian measure [11]. Trigonometric ratios of an acute angle are presented in Figure 1 [12].

$$sin\theta = \frac{opposite}{hypotenuse} \qquad cos\theta = \frac{adjacent}{hypotenuse}$$

Sine: sinθ =y/r (a ratio of an opposite length to a hypotenuse).

Cosine: $cos\theta$ = x/r (a ratio of an adjacent length to a hypotenuse).

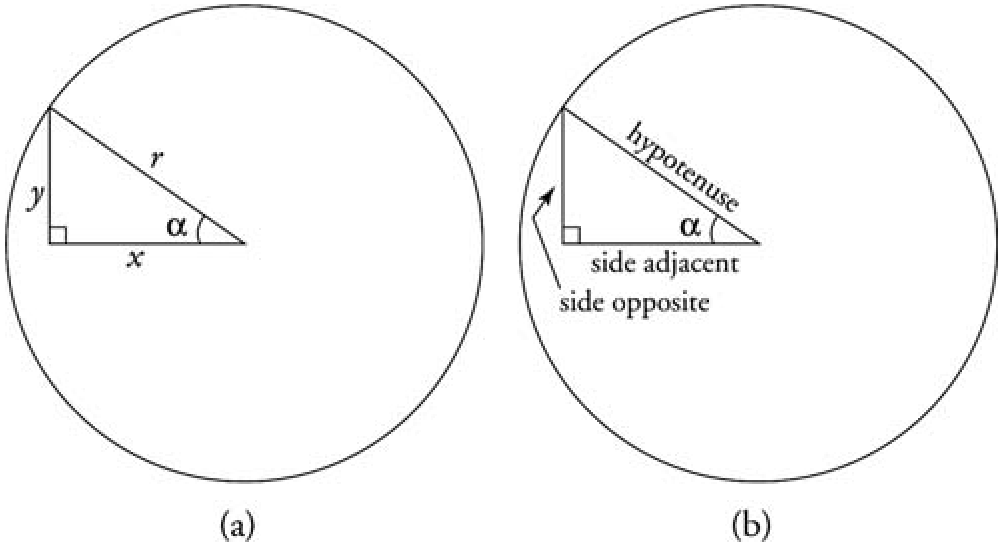

Fig. 1 Trigonometric ratios of an acute angle

$-1 \leq \sin x < 1$ and $-1 \leq \cos x < 1$.
$0 \leq x < 2\pi$ and $k \epsilon Z$,
$sinx = \sin(x + k.2\pi)$ and $cosx = \cos(x + k.2\pi)$ [13].
The spline function is to replace a single function, f, defined over the entire range of t with several low-order polynomials (splines) defined over subintervals of the range of t [1]. The points that divide the subintervals are called knots. These splines are continuous piecewise polynomials of degree m with continuous derivatives through order m-1. The values and first m-1 derivatives of the splines agree at either side of each knot, apart from the end knots. Thus, spline functions are smoothly joined piecewise polynomials. In general, if there are k knots at times t_i (i =1, . . . , k) it can be wrote a spline function of time as follow [14].

$$S(t) = \sum_{j=0}^{m} \beta_j t^j + \sum_{i=1}^{k} \theta_i (t - t_i)^m$$

There are k+m+1 regression coefficients, β and θ, for this mth-degree spline function.
Other commonly encountered special cases of spline functions include piecewise constants, or step functions, and piecewise linear functions. Cubic splines are often used to increase the flexibility of the spline function. Reviews of the theory and application of spline functions can be found in numerous references [1, 15, 16, 17].

3. RESULTS

The function (y=sin x) values corresponding to the various values of the sin x function in the range of 0 to 2π are given in Table 1, and cubic spline functions in different intervals are obtained according to these values.

Table -1 Sin x values corresponding to values in the range 0 to 2π

x	y=sin (x)
0	0
$\frac{\pi}{4} = 0.7854$	0.7071
$\frac{\pi}{2} = 1.5708$	1
$\frac{3\pi}{4} = 2.3562$	0.7071
π	0
$\frac{5\pi}{4} = 3.927$	-0.7071
$\frac{3\pi}{2} = 4.7124$	-1
$\frac{7\pi}{4} = 5.4978$	-0.7071
$2\pi = 6.2832$	0

Cubic spline functions are as follows created for sin(x) according to this information.

$$f(x_1) = -1.3416\,(x-0)^3 + 2.2\,(x-0)^2 \qquad 0 \le x < \pi/4$$

$$f(x_2) = 0.2509\left(x-\frac{\pi}{4}\right)^3 - 0.9611\left(x-\frac{\pi}{4}\right)^2 + 0.973\left(x-\frac{\pi}{4}\right) + 0.7071,\ \ \pi/4 \le x < \pi/2$$

$$f(x_3) = -0.0163\left(x-\frac{\pi}{2}\right)^3 - 0.3699\left(x-\frac{\pi}{2}\right)^2 - 0.0724\left(x-\frac{\pi}{2}\right) + 1,\ \ \pi/2 \le x < 3\pi/4$$

$$f(x_4) = 0.1685\left(x-\frac{3\pi}{4}\right)^3 - 0.4083\left(x-\frac{3\pi}{4}\right)^2 - 0.6835\left(x-\frac{3\pi}{4}\right) + 0.7071, \qquad 3\pi/4 \le x < \pi$$

$$f(x_5) = 0.1974\,(x-\pi)^3 - 0.0114\,(x-\pi)^2 - 1.0131\,(x-\pi), \qquad \pi \le x < 5\pi/4$$

$$f(x_6) = -0.1031\left(x-\frac{5\pi}{4}\right)^3 - 0.4538\left(x-\frac{5\pi}{4}\right)^2 - 0.6657\left(x-\frac{5\pi}{4}\right) - 0.7071, \qquad 5\pi/4 \le x < 3\pi/2$$

$$f(x_7) = 0.5694\left(x-\frac{3\pi}{2}\right)^3 + 0.2107\left(x-\frac{3\pi}{2}\right)^2 - 0.1438\left(x-\frac{3\pi}{2}\right) - 1,\ 3\pi/2 \le x < 7\pi/4$$

$$f(x_8) = -2.5285\left(x-\frac{7\pi}{4}\right)^3 + 1.5523\left(x-\frac{7\pi}{4}\right)^2 + 1.2409\left(x-\frac{7\pi}{4}\right) - 0.7071,\ 7\pi/4 \le x < 2\pi$$

For example, for $x = \frac{\pi}{3}$, the value of $\sin\frac{\pi}{3}$ can be calculated as follows.

$$f(x_2) = 0.2509\left(\frac{\pi}{3}-\frac{\pi}{4}\right)^3 - 0.9611\left(\frac{\pi}{3}-\frac{\pi}{4}\right)^2 + 0.973\left(\frac{\pi}{3}-\frac{\pi}{4}\right) + 0.7071$$

$$f(x_2) = 0.2509\left(\frac{\pi}{12}\right)^3 - 0.9611\left(\frac{\pi}{12}\right)^2 + 0.973\left(\frac{\pi}{12}\right) + 0.7071 = 0.8630$$

Likewise, sin (x) values can be calculated for other values. The actual and estimated values of the sin (x) function corresponding to the different values given to x are presented in Table 2.

Table -2 Estimation of y=sin (x) function values for different values given to x

X	$\pi/3$	$2\pi/3$	$5\pi/6$	$7\pi/6$	$4\pi/3$	$5\pi/3$	$11\pi/6$
Y=sin(x)	0.8660	0.866	0.5000	-0.5	-0.8660	-0.866	-0.5
$\hat{Y}$	0.8630	0.8655	0.4995	-0,4995	-0.8655	-0.863	-0.5052

Y: Actual values, $\hat{Y}$: Estimated value obtained with the cubic spline function (y prediction),

The graph of the values obtained by the sin (x) function and the cubic spline function is given in Figure 2. The cubic spline graph obtained as a result of the MATLAB program is also shown in Figure 3.

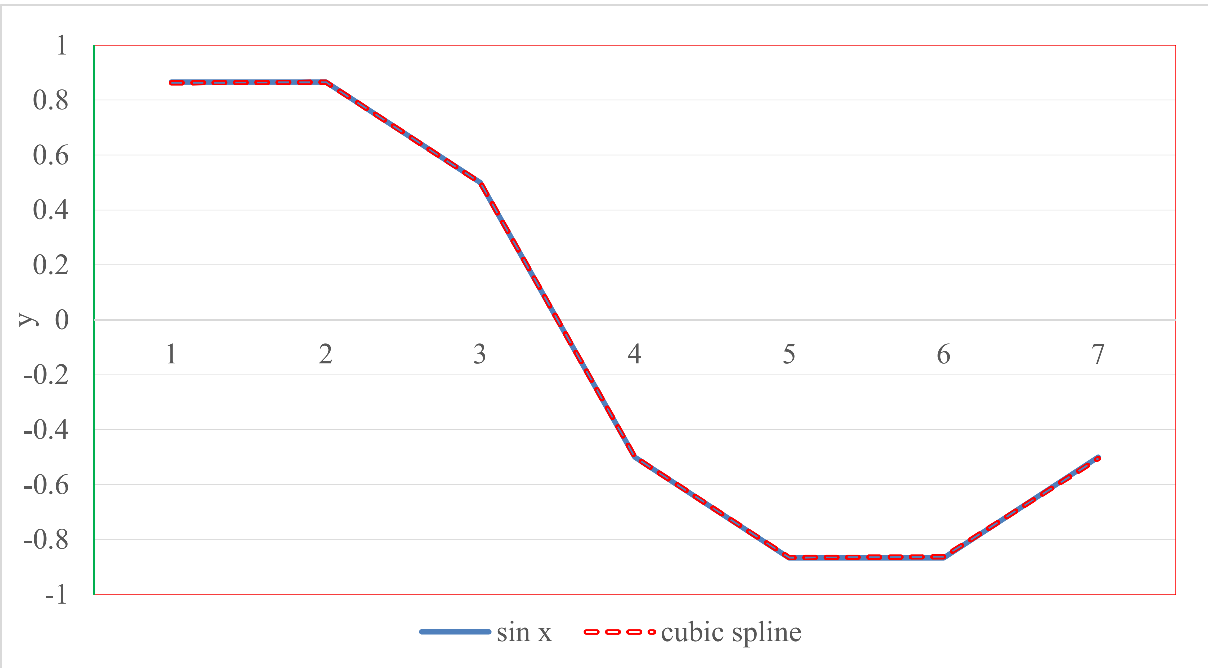

Fig. 2 sin (x) and cubic spline function values

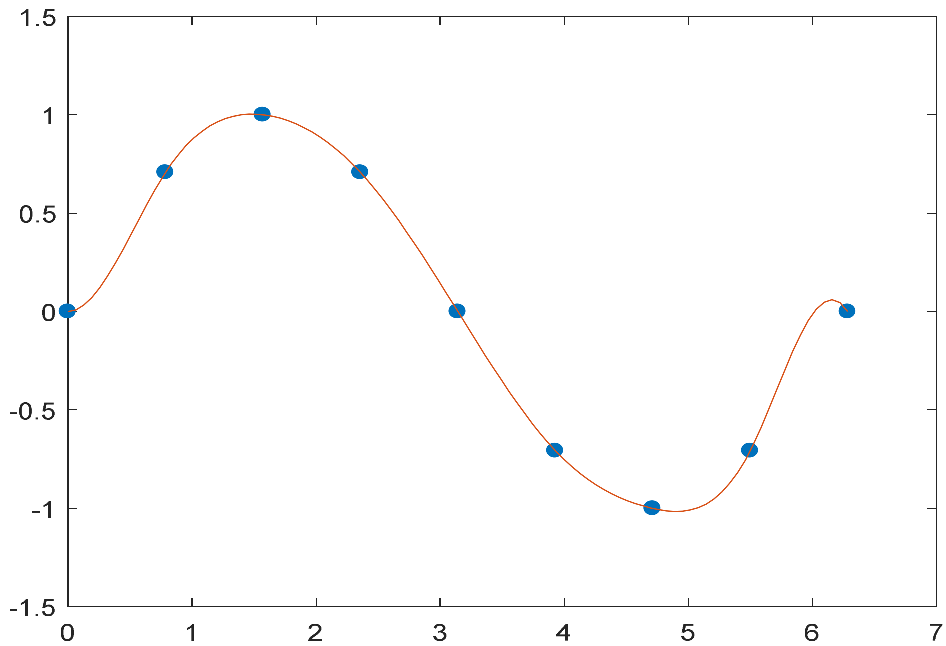

Fig. 3 Cubic spline graph for sin(x)

The function (y=cos x) values corresponding to the various values of the cos (x) function in the range of 0 to 2π are given in Table 3, and cubic spline functions in different intervals are obtained according to these values.

Table -3 Cos (x) values corresponding to values in the range 0 to 2π

x	cos x	Cubic spline
0	1	0
$\pi/4$	0.7071	0.7071
$\pi/2$	0	0
$3\pi/4$	-0.7071	-0.7071
π	-1	-1
$5\pi/4$	-0.7071	-0.7071
$3\pi/2$	0	0
$7\pi/4$	0.7071	0.7071
2π	1	1

Cubic spline functions are as follows created for cos (x) according to this information.

$$f(x_1) = -2.4034\,(x-0)^3 + 3.0399\,(x-0)^2 \qquad 0 \le x < \pi/4$$

$$f(x_2) = 1.3721\left(x-\frac{\pi}{4}\right)^3 - 2.6289\left(x-\frac{\pi}{4}\right)^2 + 0.3181\left(x-\frac{\pi}{4}\right) + 0.7071\,,\ \ \pi/4 \le x < \pi/2$$

$$f(x_3) = -0.1659\left(x-\frac{\pi}{2}\right)^3 + 0.6040\left(x-\frac{\pi}{2}\right)^2 - 1.2723\left(x-\frac{\pi}{2}\right),\ \ \pi/2 \le x < 3\pi/4$$

$$f(x_4) = 0.1465\left(x-\frac{3\pi}{4}\right)^3 + 0.213\left(x-\frac{3\pi}{4}\right)^2 - 0.6306\left(x-\frac{3\pi}{4}\right) - 0.7071\,, \qquad 3\pi/4 \le x < \pi$$

$$f(x_5) = -0.066\,(x-\pi)^3 + 0.5583\,(x-\pi)^2 - 0.0248\,(x-\pi) - 1\,, \qquad \pi \le x < 5\pi/4$$

$$f(x_6) = -0.2368\left(x-\frac{5\pi}{4}\right)^3 + 0.4028\left(x-\frac{5\pi}{4}\right)^2 + 0.73\left(x-\frac{5\pi}{4}\right) - 0.7071\,, \qquad 5\pi/4 \le x < 3\pi/2$$

$$f(x_7) = 0.1584\left(x-\frac{3\pi}{2}\right)^3 - 0.1552\left(x-\frac{3\pi}{2}\right)^2 + 0.9245\left(x-\frac{3\pi}{2}\right), \qquad 3\pi/2 \le x < 7\pi/4$$

$$f(x_8) = -1.2516\left(x-\frac{7\pi}{4}\right)^3 + 0.218\left(x-\frac{7\pi}{4}\right)^2 + 0.9738\left(x-\frac{7\pi}{4}\right) + 0.7071\,, 7\pi/4 \le x < 2\pi$$

The actual and estimated values of the cos(x) function corresponding to the different values given to x are presented in Table 4.

Table -4 Estimation of y=cos(x) function values for different values given to x

x	Cos (x)	Cubic spline
0	1	1
$\pi/4$	0.7071	0.7071
$\pi/3$	0.5	0.5461
$\pi/2$	0	0
$3\pi/4$	-0.7071	-0.7071
π	-1	-1
$5\pi/6$	-0.866	-0.8618
$7\pi/6$	-0.866	-0.8659
$5\pi/4$	-0.7071	-0.7071
$4\pi/3$	-0.5	-0.4991
$3\pi/2$	0	0
$5\pi/3$	0.5	0.4975
$7\pi/4$	0.7071	0.7071
$11\pi/6$	0.866	0.8707
2π	1	1

The graph of the values obtained by the cos (x) function and the cubic spline function is given in Figure 4. The cubic spline graph obtained as a result of the MATLAB program is also shown in Figure 5.

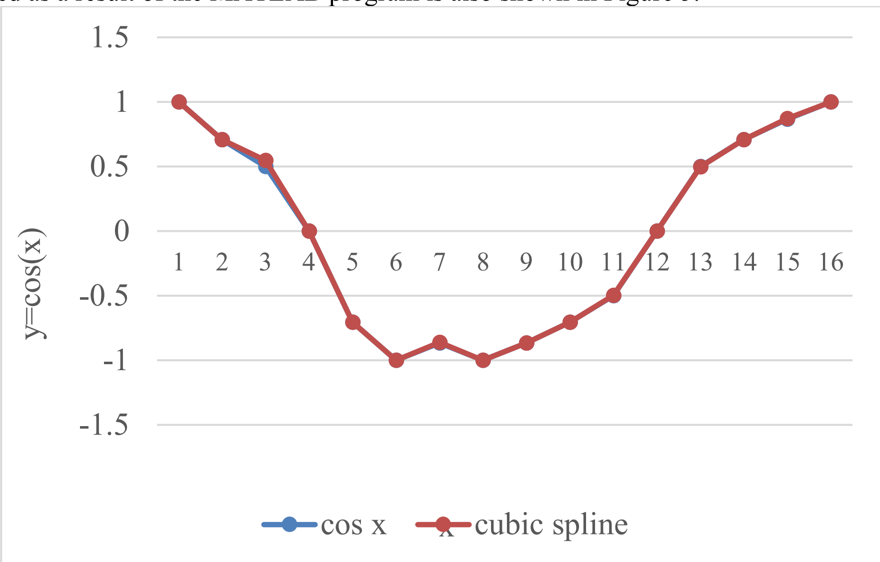

Fig. 4 Cos (x) and cubic spline function values

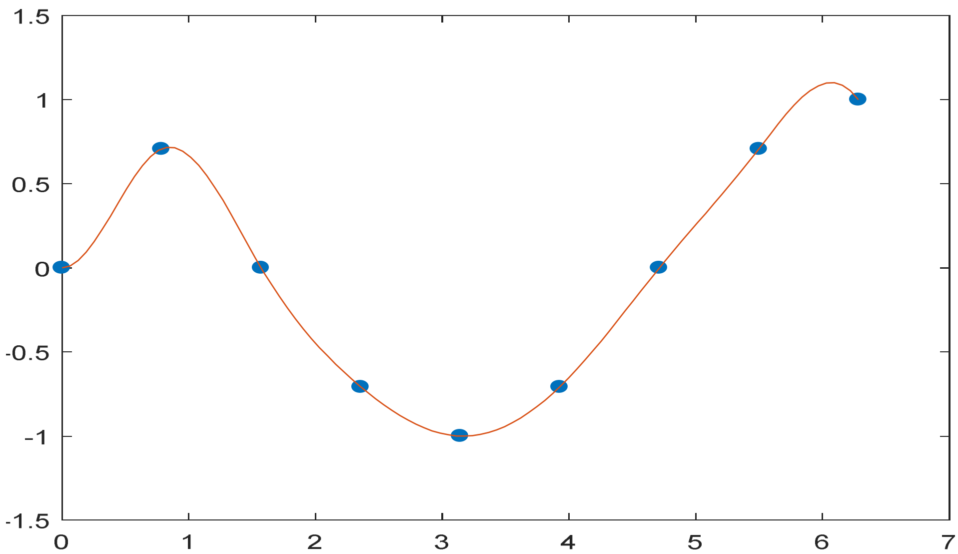

Fig. 5 Cubic spline graph for cos(x)

4. CONCLUSION

In this study, some trigonometric functions were expressed as cubic spline functions. The values of Sin(x) and Cos(x) functions according to the values in the interval [0, 2π] were also expressed with cubic spline functions. The values of sin(x) and cos(x) functions calculated with cubic spline functions were found very close to their true values. It has been seen that the graphs of the Sin(x) and Cos(x) functions and the cubic spline functions are in good agreement. It has been revealed that the cubic spline functions give very good results in calculating the values of sin(x) and cos(x) functions in the definition intervals.

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