Available online www.ejaet.com

European Journal of Advances in Engineering and Technology, 2022, 9(3):7-15

Research Article ISSN: 2394 - 658X

# Study in transient regime by analytical method of heat transfer through a two dimensional Kapok-plaster insulating material: influence of the heat exchange coefficient

**Seydou FAYE[1], Fatimata BA[1] and Mame Fadiamé THIAM[2]**

[1]Laboratory of Semiconductors and Solar Energy, Physics Department, Faculty of Science and Technology, University Cheikh Anta Diop, Dakar, Senegal
[2]The Higher Institute of Professional Education (ISEP) of Matam
E-mail: seydoufaye89@gmail.com

**ABSTRACT**

In this article, we present a study on the influence of the exchange coefficient through the propagation of heat in a material constitutes kapok-plaster.
The study in Cartesian coordinates of a material of a simple wall consisting of plaster-plaster of the following characteristics ($\lambda$=0, 1W.m$^{-1}$.$^{0}$C$^{2}$ and $\alpha$ = 4,73.10$^{-7}$m$^{2}$.s$^{-1}$).
This study is based on the description of the different temperature profiles and heat flow density depending on the depth and time for different values of the exchange coefficient.
The influence of the heat exchange coefficient has been studied in two-dimensional transitional dynamic regime.
This study made it possible to classify the Kapok as a satisfactory quality biodegradable natural insulation and the kapok-plaster has a good quality of thermal insulation.

**Key words:** Kapok-plaster-transient-temperature-temperature-density heat flow

## INTRODUCTION

Thermal insulation [1] plays an important role in thermal applications and cold production.
The aim of this study is to enhance and characterize materials of plant origin [2,3] such as kapok as a substitute for synthetic insulation [4, 5,6] (polyurethane, polystyrene, glass wool or rock wool).
Different heat transfer study models (finite medium, semi-infinite medium or finite medium) are proposed for the characterization of the kapok-plaster material. The measurement techniques thus make it possible to determine thermophysical parameters (thermal conductivity, thermal effusivity, thermal diffusivity coefficient, etc.) [7, 8,9].
In this work, we study the transient heat transfer [10, 11,12] in a material consisting of Kapok-plaster.
The resolution of the two-dimensional heat diffusion equation by the analytical method made it possible to determine the evolution of the temperature and the flux density [13] as a function of the depth and of the time under the influence of the coefficient of exchange on the front panel. The study showed that the Kapok-plaster material is a good thermal insulator [14,15].

## THEORETICAL STUDY

The fiber-plaster material is assumed to be homogeneous and of parallelepipedal shape. The depth of the material is $L = 0{,}05m$; the initial temperature of the material $Ti = 10^{0}C$ and that of the external ambient environments $Ta1 = Ta2 = 30^{0}C$. The heat exchange coefficients at the front and at the rear are respectively $h_1$ and $h_2$. The average thermal diffusivity is $\alpha = 2{,}07.10^{-7}m^{2}s^{-1}$ and the thermal conductivity is $\lambda = 0.15W.m^{-1}{}^{0}C^{-1}$.

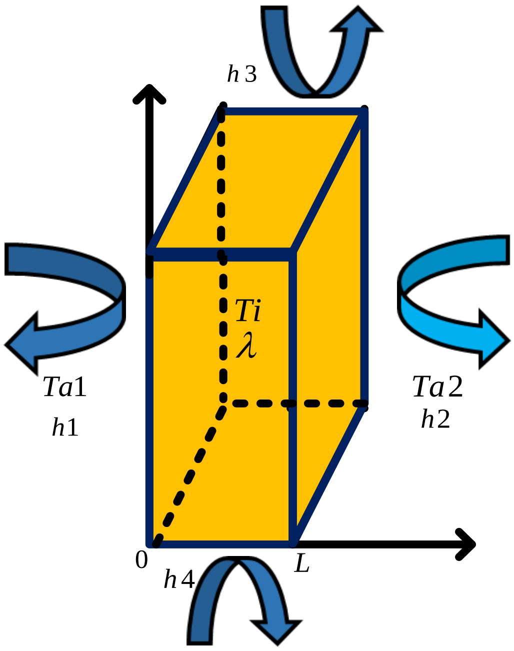


**Fig. 1** Sample Kapok-plaster

The unidirectional heat transfer in the yarn-plaster thermal insulation is governed by equation (1) below:

$$\frac{\partial^2 T(x,y,h_1,h_2,h_3,h_4,t)}{\partial x^2}+\frac{\partial^2 T(x,y,h_1,h_2,h_3,h_4,t)}{\partial y^2}-\frac{1}{\alpha}\frac{\partial T(x,y,h_1,h_2,h_3,h_4,t)}{\partial t}=0;(1)$$

$T=T(x,y,h_1,h_2,h_3,h_4,t)$ is the temperature inside the material; x the depth and t the time. Equation (2) gives the expression of the diffusivity α.

$$\alpha=\frac{\lambda}{\rho c};(2)$$

$\alpha$ is the coefficient of thermal diffusivity ( $m^2.s^{-1}$ )
$\lambda$ is the thermal conductivity ( $W.m^{-2}.c^{-1}$ )
$\rho$ is the density of the material ( $kg.m^{-2}$ )
Boundary conditions

$$\begin{cases}\lambda\left.\frac{\partial T(x,y,h_1,h_2,h_3,h_4,t)}{\partial x}\right|_{x\equiv 0}=h_1\left[T(0,y,t)-T_a\right];(3)\\ \lambda\left.\frac{\partial T(x,y,h_1,h_2,h_3,h_4,t)}{\partial x}\right|_{x\equiv L}=-h_2\left[T(L,y,t)-T_a\right];(4)\\ \lambda\left.\frac{\partial T(x,y,h_1,h_2,h_3,h_4,t)}{\partial y}\right|_{y\equiv 0}=h_3\left[T(x,0,t)-T_a\right];(5)\\ \lambda\left.\frac{\partial T(x,y,h_1,h_2,h_3,h_4,t)}{\partial y}\right|_{y\equiv L}=-h_4\left[T(x,L,t)-T_a\right];(6)\\ T(x,y,h_1,h_2,h_3,h_4,t=0)=T_i;(7)\end{cases}$$

Dimensionless heat equation

$$\theta(u,v,\tau)=\frac{T(x,y,t)-T_a}{T_i-T_a};(8)$$

with $\theta(u,v,\tau)$: reduced temperature;

$u=\frac{x}{L}$; is a space reduced variable

$v=\frac{y}{L}$ ; is a space reduced variable

and $\tau=\frac{\alpha t}{L^2}=F_0$

$F_0$ : Reduced time variable or Fourier number

The heat equation (1) becomes:

$$\frac{\partial^2\theta(u,v,\tau)}{\partial u^2}+\frac{\partial^2\theta(u,v,\tau)}{\partial v^2}=\frac{\partial\theta(u,v,\tau)}{\partial\tau};(9)$$

The boundary conditions (3), (4), (5) and (6) become (10), (11), (12) and (13):

$$\begin{cases}\left.\frac{\partial\theta(u,v)}{\partial\tau}\right|_{u=0}=\frac{h_{1x.}L}{\lambda}\theta(0,\tau);(10)\\ \left.\frac{\partial\theta(u,v)}{\partial\tau}\right|_{u=1}=-\frac{h_{2x.}L}{\lambda}\theta(1,\tau);(11)\\ \left.\frac{\partial\theta(u,v)}{\partial\tau}\right|_{u=0}=\frac{h_{1y.}L}{\lambda}\theta(0,\tau);(12)\\ \left.\frac{\partial\theta(u,v)}{\partial\tau}\right|_{u=1}=\frac{h_{2y.}L}{\lambda}\theta(1,\tau);(13)\end{cases}$$

Let us find the solution of equation (9) in the form of reduced variables separable in space and time given by relation (14):

$$\theta(u,v,\tau)=U(u)V(v)W(\tau);(14)$$

Using the relations (9) and (14) we obtain that of (15)

$$\frac{1}{U(u)}\frac{\partial^2U(u)}{\partial u}+\frac{1}{V(v)}\frac{\partial^2V(v)}{\partial v}+\frac{1}{W(\tau)}\frac{\partial W(\tau)}{\partial\tau}=-\gamma^2;(15)$$

$\gamma$ is a positive constant.

From relation (15) we obtain two differential equations:

- The differential equation in time is given by (16):

$$\frac{1}{W(\tau)}\frac{\partial W(\tau)}{\partial\tau}=-\gamma^2;(16)$$

- The differential equation in space (17) is written:

$$\frac{1}{U(u)}\frac{\partial^2U(u)}{\partial u}=-\beta^2;(17)$$

The boundary conditions space:

$$\begin{cases} \left.\dfrac{\partial\theta(0,\tau)}{\partial\tau}\right|_{u=0} = B_{i1x}\theta(0,\tau); (18) \\ \left.\dfrac{\partial\theta(1,\tau)}{\partial\tau}\right|_{u=1} = -B_{i2x}\theta(1,\tau); (19) \\ \left.\dfrac{\partial\theta(0,\tau)}{\partial\tau}\right|_{u=0} = B_{i1y}\theta(0,\tau); (20) \\ \left.\dfrac{\partial\theta(1,\tau)}{\partial\tau}\right|_{u=1} = -B_{i2y}\theta(1,\tau); (21) \end{cases}$$

Avec $B_{i1x} = \dfrac{h_{1x}.L}{\lambda}$; $B_{i2x} = \dfrac{h_{2x}.L}{\lambda}$ ; $B_{i1y} = \dfrac{h_{1y}.L}{\lambda}$ et $B_{i2y} = \dfrac{h_{2y}.L}{\lambda}$
respectively the Biot numbers on the front face and on the back face.
The general solution of the reduced temperature is in the form

$$\theta(u,v,\tau) = \sum_n \left[\left(a_n \cos(\beta_n u) + b_n \sin(\beta_n u)\right)\right]\left[c_n \cos(\mu_n v) + d_n \sin(\mu_n v)\right] e^{-\gamma^2\tau}; (22)$$

$$\begin{cases} \beta_n b_n = B_{i1x} a_n; (23) \\ -\beta_n a_n \sin(\beta_n L) + \beta_n b_n \cos(\beta_n L) = -B_{i2x}\left(a_n \cos(\beta_n L) + b_n \sin(\beta_n L)\right); (24) \end{cases}$$

$$\sin(\beta_n L)(a_n\beta_n - B_{i2x} b_n) = \cos(\beta_n L)(b_n\beta_n + B_{i2x} a_n); (25)$$

$$\tan(\beta_n L) = \frac{b_n\beta_n + B_{i2x} a_n}{a_n\beta_n - B_{i2x} b_n}; (26)$$

The following transcendent equation x:

$$\tan(\beta_n L) = \frac{\dfrac{h_{1x}L}{\lambda}\beta_n + \dfrac{h_{2x}L}{\lambda}\beta_n}{\beta_n^2 - \dfrac{h_{1x}h_{2x}L}{\lambda^2}}; (27)$$

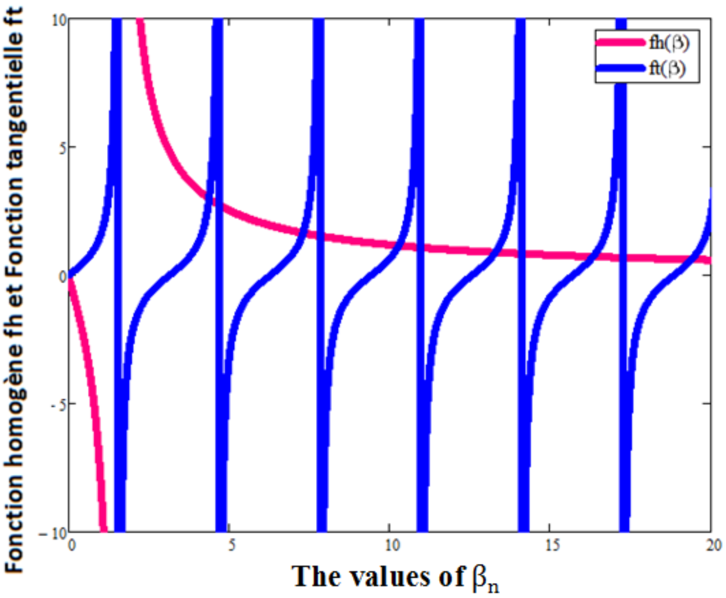


**Fig. 2** Curve of the following transcendent equation

The intersection of the two curves fh ($\beta_n$) and ft ($\beta_n$) corresponds to the solution.

Table 1 summarizes the eigenvalues found of $\beta_n$

**Table -1 The eigenvalues $\beta_n$ the equation**

| n | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\beta_n$ | 4,4 | 7,4 | 10,3 | 13,3 | 16,4 |

Transcendent equation following y

$$\begin{cases} \mu_n d_n = B_{i1y} c_n ; (28) \\ -\mu_n c_n \sin(\mu_n L) + \mu_n d_n \cos(\mu_n L) = -B_{i2y}\left(c_n \cos(\mu_n L) + d_n \sin(\mu_n L)\right); (29) \end{cases}$$

$$\sin(\mu_n L)(c_n \mu_n - B_{i2y} d_n) = \cos(\mu_n L)(c_n \mu_n + B_{i2y} c_n); (30)$$

$$\tan(\mu_n L) = \frac{d_n \mu_n + B_{i2y} c_n}{c_n \mu_n - B_{i2y} d_n}; (31)$$

$$\tan(\mu_n L) = \frac{\frac{h_{1y} L}{\lambda}\mu_n + \frac{h_{2y} L}{\lambda}\mu_n}{\mu_n^2 - \frac{h_{1y} h_{2y} L}{\lambda^2}}; (32)$$

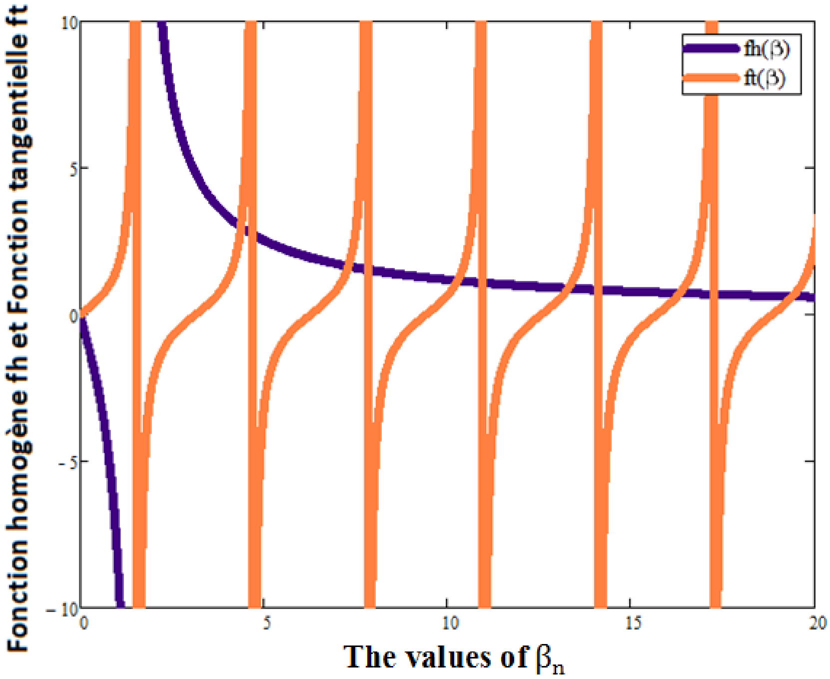


**Fig. 3** Curve of the following transcendent equation

The intersection of the two curves fh ($\mu_n$) and ft ($\mu_n$) corresponds to the solution.

Table 1 summarizes the eigenvalues found of $\mu_n$

**Table -2 The eigenvalues $\mu_n$ the equation**

| n | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\mu_n$ | 4,4 | 7,4 | 10,3 | 13,3 | 16,4 |

$$\theta(u; v, \tau) = \frac{T(x, y, h_{1x}, h_{2x}, h_{1y}, h_{2y}, t) - T_a}{T_i - T_a}; (33)$$

$$T(x, y, h_{1x}, h_{2x}, h_{1y}, h_{2y}, t) = T = T_a + (T_i - T_a)\theta(u; v, \tau); (34)$$

The general solution of temperature:

$$T = T_a + (T_i - T_a)\sum_n \left[ a_n \left( \cos\left(\beta_n \frac{x}{L}\right) + \frac{\frac{h_{1x}L}{\lambda}}{\beta_n}\sin\left(\beta_n \frac{x}{L}\right)\right) c_n \left( \cos\left(\mu_n \frac{y}{L}\right) + \frac{\frac{h_{1y}L}{\lambda}}{\mu_n}\sin\left(\mu_n \frac{y}{L}\right)\right)\right] e^{-\frac{\alpha t}{L^2}\gamma^2} \;;(35)$$

**Heat flux density:**

We get the expression for the density of the heat flow (or surface heat flow)
Which is the heat flux per unit area ($W.m^{-2}$) as follows:

$$\vec{\varphi}(x,y,h_{1x},h_{2x},h_{1y},h_{2y},t) = -\lambda\,\overrightarrow{grad}T(x,y,h_{1x},h_{2x},h_{1y},h_{2y},t);(36)$$

$$\vec{\varphi}(x,y,h_{1x},h_{2x},h_{1y},h_{2y},t) = \overrightarrow{\varphi_x}(x,y,h_{1x},h_{2x},h_{1y},h_{2y},t) + \overrightarrow{\varphi_y}(x,y,h_{1x},h_{2x},h_{1y},h_{2y},t);(37)$$

From these two expressions we get, the final expression of the heat flux density

$$\varphi(x,y,t) = \lambda(Ti - Ta)\left[\begin{array}{l}\left[\sum_n an\left(-\frac{\beta n}{L}\sin\left(\beta n\frac{x}{L}\right) + \frac{Bi1x}{L}\cos\left(\beta n\frac{x}{L}\right)\right)cn\left(\cos\left(\mu n\frac{y}{L}\right) + \frac{Bi1y}{L}\sin\left(\mu n\frac{y}{L}\right)\right)\right]^2 + \\ \left[\sum_n cn\left(-\frac{\mu n}{L}\sin\left(\mu n\frac{y}{L}\right) + \frac{Bi1y}{L}\cos\left(\mu n\frac{x}{L}\right)\right)an\left(\cos\left(\beta n\frac{x}{L}\right) + \frac{Bi1x}{L}\sin\left(\beta n\frac{x}{L}\right)\right)\right]^2\end{array}\right]^{\frac{1}{2}} e^{-\frac{\alpha t}{L^2}\gamma^2} \;;(38)$$

## RESULTS AND DISCUSSION

### Evolution of the temperature and the density of the heat flow as a function of the depth for different values of the exchange coefficient

The heat exchange coefficient on the rear face is assumed to be low and relatively high on the front face in Figures 3 and 4. The rear face being at a temperature close to the initial temperature of the material. The front face being hotter than the back face, the temperature decreases considerably in the medium and tends towards the initial temperature of the material for figure 3.

For Figure 4 the heat flux density gradually decreases before reaching the back side. Which is to say that the material has stored most of the heat. The observed phenomenon highlights the insulating character of the Kapok-plaster material. In this case the temperature and the density of heat flux transmitted from the front face to the rear face are completely absorbed before reaching the rear face (Figures 4 and 5). These observations show the thermal inertia of the material capable of storing heat but also of restoring it.

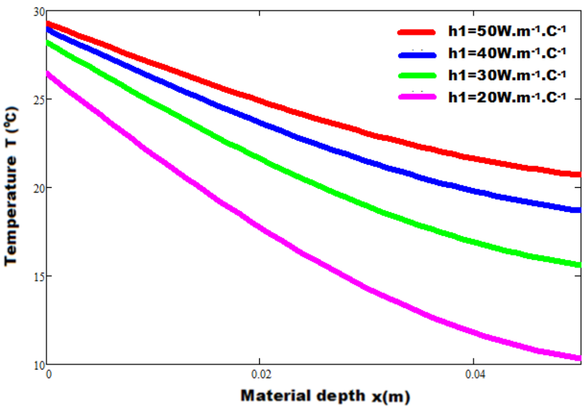


**Fig. 4** Temperature as a function of material depth. $h_2$=0.005$W.m^{-2}.C^{-1}$ ; t=100s

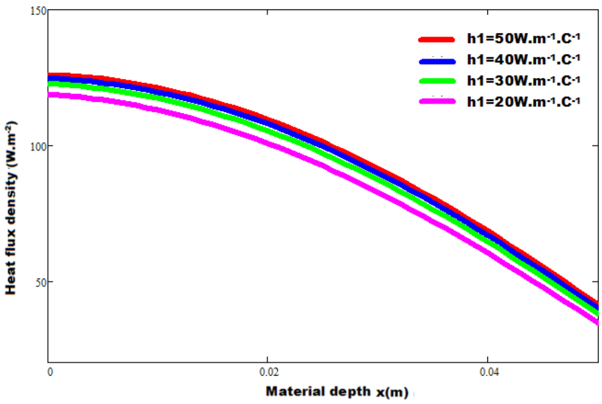


**Fig. 5** Heat flux density as a function of material depth.$h_2$=0.005$W.m^{-2}.C^{-1}$ ; t=100s

**Evolution of the temperature and the density of heat flow as a function of the exchange coefficient**

Figures 6 and 7 show the change in temperature and heat flux density in the material as a function of the heat exchange coefficient on the front face.

These figures show that the wall heats up as a function of the heat exchange coefficient reflecting a storage of thermal energy. The exchange of temperature and heat flux density for the different formulations is all the more important the higher the thermal coefficient.

This temperature change is all the more rapid. This corresponds to a strong period of excitement at the surface of the wall. The temperature changes until it reaches a limit value.

This is because the temperature tends towards the temperature of the fluid in contact with the surface of the material. By storing as much energy as possible, the material becomes independent of the change in the convective exchange coefficient when thermal equilibrium is reached.

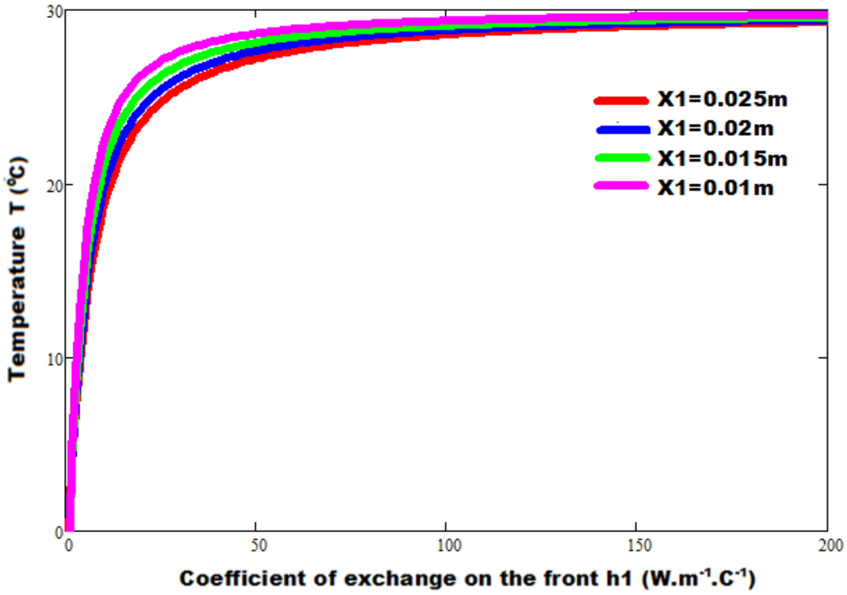


**Fig. 6** Temperature as a function to the heat exchange coefficient on the front face $h_1$ of the material. $h_2$=0.005W.m$^{-2}$.C$^{-1}$; t=100s

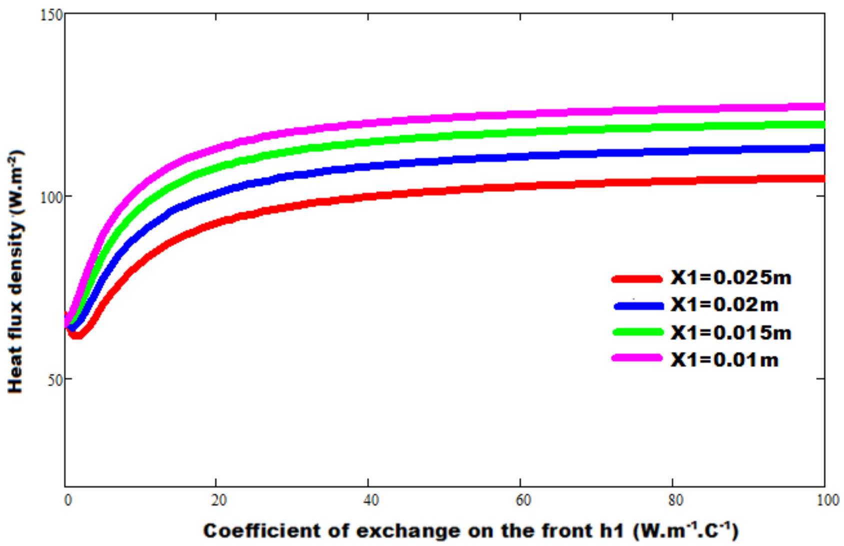


**Fig. 7** Heat flux density as a function of the exchange coefficient at the front face $h_1$material.$h_2$=0.005W.m$^{-2}$.C$^{-1}$; t=100s.

**Evolution of the Temperature and the Density of the heat flow as a function of time**

This change is made as a function of time for different values of the exchange coefficient of the front face in the material.

Figure 8 and 9 show the evolution of the temperature and the heat flux density as a function of time under the influence of h1. For the low values of the exchange coefficient at the front face, the material reacts then it keeps its initial temperature. This is the lag time. Beyond this time, the temperature increases when the exchange coefficient of the front face is high, then reaches a maximum, the material absorbs heat.

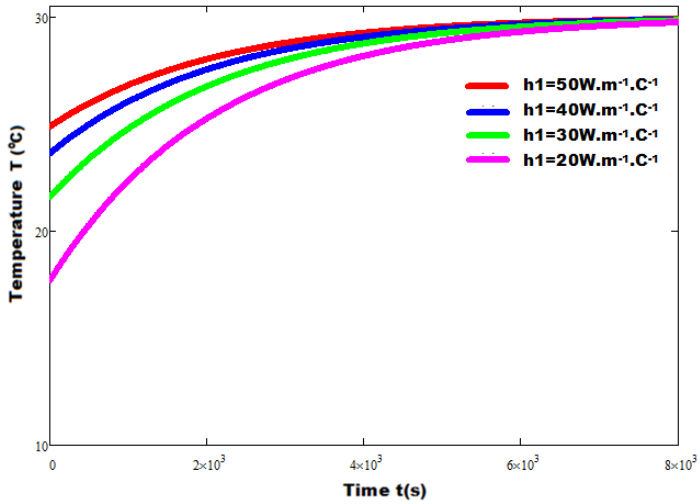


**Fig. 8** Evolution de la température en fonction du temps matériau ; x=0.01m ; h2=0.005W.m$^{-2}$.C$^{-1}$

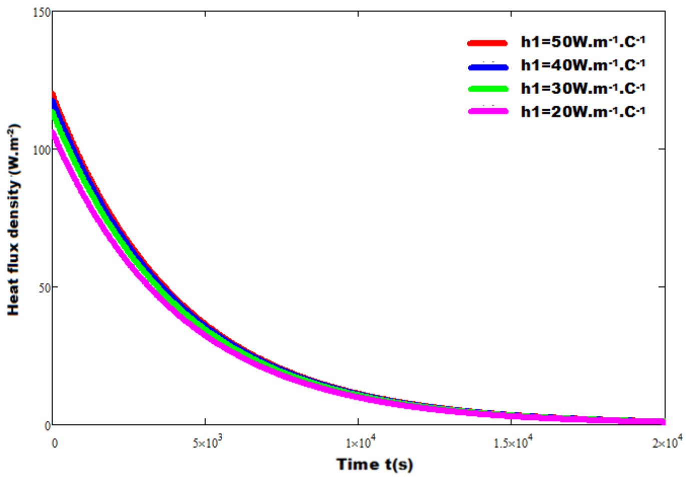


**Fig. 9** Evolution de la densité de flux de chaleur en fonction du temps; x=0.01m ; h2=0.005W.m$^{-2}$.C$^{-1}$

## CONCLUSION

In this article, the influences of heat exchange coefficients at the front face of a material on the diffusion of heat through a material and the flow of heat transferred from one face to another were presented. The influence of excitation time on diffusion and heat transfer has been shown. These studies show that the thermal diffusivity which is inversely proportional to the volume density leads to an increase in the optimum thickness and that the heat exchange coefficient at the front face still influences the optimum insulation thickness by disturbing the material. to achieve thermal equilibrium more quickly.

The modeling of the temperature and the heat flux density made it possible to highlight the quality of the kapok plaster material in thermal insulation.

## REFERENCES

[1]. I. Diagne, M. Dieng, M. L Sow, A. Wereme, F. Niang – G. Sissoko, « Estimation de la couche d'isolation thermique efficace d'un matériau Kapok-plâtre en régime dynamique fréquentiel », 2010, CIFEM2010, Edition Université de Rennes 1, pp 394-399.

[2]. Y Jannot., P Meukam " Simplified estimation method for the determination of the thermal effusivity using a low cost hot strip",., Measurement Science and Technology, vol.15, 2004.

[3]. J.C Marechal. & J.M. Devisme "Diffusivite thermique des materiaux de construction: Methode du signalperiodique". Anales I.T.B.T.P n° 357, Janvier 1978

[4]. Seydou Faye, Mohamed SidyaOuld Brahim, Youssou Traore, Aliou Diouf, Moussa Dieng, Abdoulaye Korka Diallo, Issa DIAGNE, Hawa Ly Diallo, Gregoire Sissoko « Study by Analytical Method of Transient Regime of Thermal Transfer Through a Insulation Material Tow-Plaster – Influence Coefficient of Thermal Exchange" IPASJ International Journal of Computer Science (IIJCS) Volume 4, Issue 7, pp 1-9, July 2016.

[5]. Jannot Y., Meukam P., Simplified estimation method for the determination of the thermal effusivity using a low cost hot strip, Measurement Science and Technology, vol.15, 2004.

[6]. Marechal J.C. and Devisme J.M., Metrology thermal insulation materials by measuring the flux in transient regime. Materials and constructions, (translated French), Anales I.T.B.T.P vol.7 n°41, 1974.

[7]. A. Bejan, The boundary layer regime in a porous layer with uniform heat flux from side, International Journal of Heat Mass Transfer, vol. 26, N° 9, pp. 1339-1346. 1983.

[8]. Hassan El Harfi1, Mohamed Naïmi1, Mohamed Lamsaadi, Abdelghani Raji1, Mohammed Hasnaoui, Mixed convection heat transfer for nanofluids in a lid-driven shallow rectangular cavity uniformly heated and cooled from the vertical sides: the opposing case. Journal of Electronics Cooling and Thermal Control JECTC, 3, 111-130. 2013.

[9]. Jannot Y., Meukam P. "Simplified estimation method for the determination of the thermal effusivity using a low cost hot strip", Measurement Science and Technology, vol.15, 2004.

[10]. P. Meukam, Y. Jannot, A. Noumowe and T.C Kofane" Thermo physical characteristics of economical building materials", Construction and Building Materrials, Volume 18, issue 6, July 2004.

[11]. TSIREL'MAN (N.M.)." Determining the convective heat transfer coefficient from the laws of constant-temperature front propagation."-J.Eng.Phys.vol.25, No2, August 1973.

[12]. M. F. Couturier, K. George, and M. H. Schneider, (1996). Thermophysical properties of wood-polymer composites. Wood Science and Technology, vol. 30, pp. 179-196.

[13]. Martin Rides, Junko Morikawa, Lars Halldahl, Bruno Hay, Hubert Lobo, Angela Dawson, and Crispin Allen, (2009). Intercomparison of thermal conductivity and thermal diffusivity methods for plastics. Polymer Testing, vol. 28, pp.480-489.

[14]. VyacheslavKremnev, Boris Basok, Andriy Timoshchenko, Sergei Tymchyshyn, (2018). Energy Saving in Construction by Wide Application of High-Quality Insulation Based on Basalt Fibers. Journal of Modern Physics, 9, 1724-1734 http://www.scirp.org/journal/jmp

[15]. A.Wereme, S. Tamba, M.Sarr, A. Diene, F. Niang, G. Sissoko, (2010). Caracterisation des isolants thermique locaux de types sciure de bois et kapok : messure de coefficient global d'echange thermique et de la conductivite thermique. Journal Des Sciences, Vol. 10, N°4 Pp39-46 (http://www.cadjds.org).