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# STABILITY OF QUADRATIC MAPPINGS IN A NON-ARCHIMEDEAN 2-BANACH SPACES AND RELATED TOPICS

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**ABSTRACT**
*In this paper, we investigate Hyers-Ulam-Rassias stability of the following quadratic functional equation:* $f(x+y+z)+f(x)+f(y)+f(z)=f(x+y)+f(y+z)+f(x+z)$ *in a non Archimedean 2-Banach spaces, and we prove the generalized Hyers-Ulam stability of functional quadratic functional equation in a non Archimedean 2-Banach spaces.*

**Key words:** Hyers-Ulam stability; a non Archimedean 2-Banach, Quadratic mapping, Stability

## 1. INTRODUCTION

In 1940, Ulam [1] suggested the stability problem of functional equations concerning the stability of group homeomorphism as follows:
Let $(G,\circ)$ be a group and let $(H,\star,d)$ be a metric group with the metric $d(.,.)$. Given $\epsilon>0$, does there exist a $\delta=\delta(\epsilon)>0$ such that if a mapping $F: G\rightarrow H$ satisfies the inequality:

$$d(f(x\circ y),f(x)*f(y))<\delta$$

for all x,y $\in G$. Then a homomorphism $F: G\rightarrow H$ exists $d(f(x),F(x))$ for all $x\in G$?
In 1941, Hyers [2] give a first (partial) affirmative answer to the question of Ulam for Banach spaces. Thereafter, we call type the Hyers - Ulam stability.
The result of Hyers was generalized by Aoki [6] for approximate additive mappings and by Th.m. Rassias [7] for a approximate linear mapping a following the difference Cauchy equation

$$\|f(x+y)-f(x)-f(y)\| \text{tobecontrolledby} \epsilon(\|x\|^p+\|y\|^p)$$

The functional equation

$$f(x+y)+f(x-y)=2f(x)+f(y) \quad (1)$$

is related to symmetric bi-additive function and called a quadratic functional and every solution of the quadratic (1) is said to be quadratic mapping. Skof [20] proved the Hyers - Ulam stability of quadratic functional equation (1).
The theory of linear 2-normed spaces was first developed by Gahler[8] in the mid 1960's, while That of 2- Banach spaces was studied later by Gahler [9] and White [10].
The functional equation

$$f(x+y+z)+f(x)+f(y)+f(z)=f(x+y)+f(y+z)+f(x+z) \quad (2)$$

Be another quadratic mapping, by Skof [20] proved the Hyers - Ulam stability of quadratic functional equation (2).
Jung [5] proved the Hyers - Ulam stability of quadratic functional equation (2) in the Banach space with respect to some conditions.
In 1978, Rassias[7] extended the Hyers- Ulam stability by considering variable. In 1994, it also has been generalized to function case by Gãvruta [18].
Hensel[21] has introduced a normed space which does not have the Archimedean property, Rassias[22] proved the generalized Hyers- Ulam stability of the additive functional equation and the quadratic functional equation in non-Archimedean spaces.
In this paper, we will prove the stability of quadratic functional equation in (2) in a non-Archimedean 2-Banach spaces under the approximately even (or odd) conditions and some asymptotic behaviours of quadratic and additive

mappings shall be investigated and generalized the stability of the same functional equation in a non-Archimedean 2-Banach spaces.

## 2. PRELIMINARIES

Now we introduce some notions for non-Archimedean spaces we be used in this paper.

**Definition 2.1** *[24] Let K be a field. A non-Archimedean absolute value (or valuation) on a filed K is a function* $|.|: K \longrightarrow R$ *such that for any* $a, b \in K$, *we have,*

- $|a| \geq 0$ and equality holds if and only if $a = 0$,
- $|ab| = |a||b|$
- $|a + b| \leq \max\{|a|, |b|\}$.

Condition (iii) is called the strong triangle inequality. By(ii), we have$|-1| = |1| = 1$.

Thus, by induction, it follows from (iii) that $|n| \leq 1$, for each integer n. we always assume in addition that $|.|$ is non-trivial, that is, there is an $a_0 \in K$ such tat $|a_0| \neq 0,1$

**Definition 2.2** *[24] Let X be a vector space over a non-Archimedean field K. A function* $\|.\|: X \rightarrow R$ *is called a non-Archimedean norm if it satisfies the following properties:*

- $\|x\| = 0$ if and only if $x = 0$,
- $\|rx\| = |r|\|x\|$
- $\|x + y\| = max\{\|x\|, \|y\|\}$

If $\|x\|$ is called a non-Archimedean norm on X and the pair (X, $\|.\|$) is called a non-Archimedean normed space

**Definition 2.3** *[22] Let* $(X, \|.\|)$ *a non-Archimedean normed space and* $\{x_n\}$ *a sequence in X. Then* $\{x_n\}$ *is said to be convergent in* $(X, \|.\|)$ *if there exists an* $x \in X$ *such that* $\lim_{n\to\infty} \|x_n - x\| = 0$. *In case, x is called the limit of the sequence* $\{x_n\}$*, and one denotes it by*
$\lim_{n\to\infty} x_n = x$ . A sequence $\{x_n\}$ is said to e Cauchy in $(X, \|.\|)$ if for all $p \in N$.

**Remark 2.4** *[22] By (c) in Definition (2.2),*

$$\|x_n - x_m\| \leq max\{\|x_{j+1} - x_j\| | m \leq j \leq n-1\} \qquad (n > m), \tag{3}$$

A sequence $\{x_n\}$ is Cauchy in $(X, \|.\|)$ if and only if $\{x_{n+1} - x_n\}$ converges to zero in a non-Archimedean normed space $(X, \|.\|)$. By a complete non-Archimedean normed space we mean one in which every Cauchy sequence is convergent.

**Definition 2.5** *[22] Let X be a real linear space over non-Archimedean field K with* $dimX > 1$ *and let* $\|x, y\|: X \times X \longrightarrow [0, \infty[$ *be a function satisfying the following properties:*

- $\|x, y\| = 0$ if and only if x and y are linearly dependent,
- $\|x, y\| = \|y, x\|$
- $\|\alpha x, y\| = |\alpha|\|x, y\|$
- $\|x, (y + z)\| \leq max\{\|x, y\|, \|x, z\|\}$

for all $x, y, z \in X$ and $\alpha \in R$. Then the function $(\|.,.\|)$ is called a non-Archimedean 2- norm on X and the pair $(X, \|.,.\|)$ is called a non-Archimedean 2-normed space.

**Lemma 2.6** *[22] Let* $(X, \|.,.\|)$ *be a non-Archimedean 2-normed space. If x* $\in$ *X and* $\|x, y\| = 0$ *for all y* $\in$*X, then x = 0.*

**Definition 2.7** *[22] A sequence* $\{x_n\}$ *in a non-Archimedean 2-normed space* $(X, \|.,.\|)$ *is called a Cauchy sequence if*

$$\|x_n - x_m, y\| = 0$$

for all $y \in X$

**Definition 2.8** *[23] A sequence* $\{x_n\}$ *in a non-Archimedean 2-normed space* $(X, \|.,.\|)$ *is called a Cauchy sequence if*

$$\|x_n - x, y\| = 0$$

for all $y \in X$ If $\{x_n\}$ converges to x, write $x_n \rightarrow x$ as $n \rightarrow \infty$ and call x the limit of $\{x_n\}$. In this case, we also write $\lim_{n\to\infty} x_n = x$ .

Let $\{x_n\}$ be a sequence in a non-Archimedean 2-normed space $(X, \|.,.\|)$.It follows from (N A4) that

$$\|x_n - x_m, y\| \leq max\{\|x_{j+1} - x_j, y\| | m \leq j \leq n-1\} (n > m)$$

for all $y \in X$ and so a sequence $\{x_n\}$ is a Cauchy sequence in $(X, \|.,.\|)$ if and only if $\{x_{n+1} - x_n\}$ converges to zero in $(X, \|.,.\|)$

A non-Archimedean 2-normed space $(X, \|.,.\|)$ is called a non-Archimedean 2-Banach space if every Cauchy sequence in $(X, \|.,.\|)$ is convergent.

**Lemma 2.9** *[23] For a convergent sequence $\{x_n\}$ a non-Archimedean 2-normed space $(X, \|.,.\|)$*

$$\lim_{n\to\infty} \|x_n, y\| = \left\| \lim_{n\to\infty} x_n, y \right\|$$

for all $y \in X$.

**Lemma 2.10** *[23] Let $(X, \|.,.\|)$ be a non-Archimedean 2-normed space. Then*

$$\|x, z\| - \|y, z\| \le \|x - y, z\|$$

for all $x, y, z \in X$.

**Definition 2.11** *[22] A non-Archimedean 2-Banach space X is called a normed non-Archimedean 2-Banach space if X is a 2-Banach space with norm.*

## 3 STABILITY OF QUADRATIC MAPPINGS IN A NON-ARCHIMEDEAN 2-BANACH SPACES

Throughout this section, let $X$ be a normed space and let $Y$ be a normed a non-Archimedean 2-Banach spaces.

**Lemma 3.1** *Let $f: X \to Y$ be mapping satisfies the following inequality:*

$$\|f(x+y+z) + f(x) + f(y) + f(z) - f(x+y) - f(y+z) - f(z+x), u\| \le \varphi(x, y, , z, u) \quad (4)$$

for all $x, y, z \in X$ and $u \in Y$, and f satisfies f(0)=0, where $\varphi: X^3 \times Y \to [0, \infty)$ is arbitrary mapping. Then,

$$\left\| f(x) - \tfrac{2^n+1}{2^{2n+1}} f(2^n x) + \tfrac{2^n-1}{2^{2n+1}} f(-2^n x), u \right\| \le \max\{\tfrac{|2^{i+1}-1|}{|2|^{2i+3}} \varphi(-2^i x, -2^i x, 2^i x, u), \tfrac{|2^{i+1}+1|}{|2|^{2i+3}} \varphi(2^i x, 2^i x, -2^i x, u): 0 \le i \le n-1\} \quad (5)$$

for all $x \in X$ , $u \in Y$ and $n \in N$

*Proof.* Let $f: X \to Y$ be mapping satisfies (4) and $f(0) = 0$,
by putting $x = y = -z$ in (4), we have,

$$\|3f(x) + f(-x) - f(2x), u\| \le \varphi(x, x, -x, u) \quad (6)$$

By substituting x by -x in (6), we obtain,

$$\|3f(-x) + f(x) - f(-2x), u\| \le \varphi(-x, -x, x, u) \quad (7)$$

Now we will use the induction on n to prove our lemma,

For n=1 on (5) we have for $x \in X$, $u \in Y$

$$\begin{aligned} &\left\| f(x) - \tfrac{3}{8} f(2x) + \tfrac{1}{8} f(-2x), u \right\| \\ &= \tfrac{1}{8} \|8f(x) - 3f(2x) + f(-2x), u\| \\ &= \left|\tfrac{1}{8}\right| \|9f(x) + 3f(-x) - 3f(2x) - 3f(-x) - f(x) + f(-2x), u\| \\ &\le \max\{\left|\tfrac{3}{8}\right| \|3f(x) + f(-x) - f(2x)\|, \left|\tfrac{1}{8}\right| \|-3f(-x) - f(x) + f(-2x), u\|\} \\ &\text{by(6)and(67)} \\ &\le \max\{\left|\tfrac{3}{8}\right| \varphi(x, x, -x, u), \left|\tfrac{1}{8}\right| \varphi(-x, -x, x, u)\} \end{aligned} \quad (8)$$

Which satisfies the inequality (5) for n=1.

its easy to show that

$$\begin{aligned} &f(x) - \tfrac{2^{n+1}+1}{2^{2n+3}} f(2^{n+1}x) + \tfrac{2^{n+1}-1}{2^{2n+3}} f(-2^{n+1}x) \\ &= f(x) - \tfrac{2^n+1}{2^{2n+1}} f(2^n x) + \tfrac{2^n-1}{2^{2n+1}} f(-2^n x) + \tfrac{2^{n+1}+1}{2^{2n+3}} (3f(2^n x) + f(-2^n x) \\ &-f(2^{n+1}x)) - \tfrac{2^{n+1}-1}{2^{2n+3}} (3f(-2^n x) + f(2^n x) - f(-2^{n+1}x) \end{aligned} \quad (9)$$

for all $x \in X$ and $n \in N$.

Now assume that the inequality (5) is true for n=m, we want to show that it's true for$n = m + 1$. So by (9) and Triangle inequality, we have

$$\left\| f(x) - \tfrac{2^{m+1}+1}{2^{2m+3}} f(2^{m+1}x) + \tfrac{2^{m+1}-1}{2^{2m+3}} f(-2^{m+1}x), u \right\|$$

$\leq \max\{\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(-2^ix,-2^ix,2^ix,u),\frac{|2^{i+1}+|}{|2|^{2i+3}}\varphi(2^ix,2^ix,-2^ix,u)$

$: 0\leq i\leq m-1\},\frac{|2^{m+1}+1|}{|2|^{2m+3}}\varphi(2^mx,2^mx,-2^mx,u),\ \frac{|2^{m+1}-1|}{|2|^{2m+3}}\varphi(-2^mx,-2^mx,2^mx,u)\}$

$\leq \{\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(-2^ix,-2^ix,2^ix,u),\frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^ix,2^ix,-2^ix,u): 0\leq i\leq m\}$

$\leq \{\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(-2^ix,-2^ix,2^ix,u),\frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^ix,2^ix,-2^ix,u): 0\leq i< m+1\}$

Therefore we verify the inequality (5) for m+1, Thus, we end the induction. Hence, the proof of lemma is holds for all $n\in\mathbb{N}$.

In the following theorem, Hyers-Ulam stability of equation(2) is proved under approximately even condition in a non-Archimedean 2-Banach spaces.

**Theorem 3.2** *Assume that* $\varphi: X^3\times Y\to[0,\infty)$ *and* $\psi: X\to[0,\infty)$ *are mappings such that for all* $x,y,z\in X$ *and* $u\in Y$*,*

$$\lim_{n\to\infty}\frac{\varphi(2^nx,2^ny,2^nz,u)}{|2|^{2n}}=0, \tag{10}$$

$$\lim_{n\to\infty}\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(2^ix,2^iy,2^iz,u): 0\leq i<n\}\quad \text{exists}, \tag{11}$$

$$\lim_{n\to\infty}\max\{\frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^ix,2^iy,2^iz,u): 0\leq i<n\}\quad \text{exists} \tag{12}$$

Also, let $f: X\to Y$ be a mappings satisfies f(0)=0 such that holds:
for all $x,y,z\in X$ and $\forall u\in Y$,

$$(i)\|f(x+y+z)+f(x)+f(y)+f(z)-f(x+y)-f(y+z)-f(z+x),u\|\leq\varphi(x,y,z,u) \tag{13}$$

$$(ii)\|f(x)-f(-x),u\|\leq\psi(x), \tag{14}$$

$$(iii)\lim_{n\to\infty}\frac{\psi(2^nx)}{|2|^{2n}}\quad \text{forallx}\in X \tag{15}$$

Then there exists a unique quadratic mapping $Q: X\to Y$ which satisfies (2) and the inequality:

$$\|f(x)-Q(x),u\|\leq\lim_{n\to\infty}\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(-2^ix,-2^ix,2^ix,u), \frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^ix,2^ix,-2^ix,u): 0\leq i<n\} \tag{16}$$

for all $x\in X$ and $u\in Y$.

Moreover if,

$$\lim_{n\to\infty}\lim_{k\to\infty}\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(2^ix,2^iy,2^iz,u): k\leq i<n+k\}=0, \tag{17}$$

and $$\lim_{n\to\infty}\lim_{k\to\infty}\max\{\frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^ix,2^iy,2^iz,u): k\leq i<n+k\}=0. \tag{18}$$

Then $Q$ is a unique quadratic mapping.

*Proof.* Let $f: X\to Y$ be a mapping satisfies the inequalities (13), then for all $x\in X$, and $u\in Y$, we have for $n\in\mathbb{N}$

$$\left\|\frac{2^n-1}{2^{2n+1}}f(2^nx)-\frac{2^n-1}{2^{2n+1}}f(-2^nx),u\right\|=\frac{|2^n-1|}{|2|^{2n+1}}\|f(2^nx)-f(-2^nx),u\|$$
$$\leq\frac{|2^n-1|}{|2|^{2n+1}}\psi(2^nx)\qquad \text{by(14)} \tag{19}$$

for all $x\in X$, $u\in$ Y and $n\in N$,

$$\left\|f(x)-\frac{1}{2^{2n}}f(2^nx),u\right\|=\left\|f(x)-\frac{1}{2^{2n}}f(2^nx)+\frac{2^n+1}{2^{2n+1}}f(2^nx)-\frac{2^n+1}{2^{2n+1}}f(2^nx)+\frac{2^n-1}{2^{2n+1}}f(-2^nx)-\frac{2^n-1}{2^{2n+1}}f(-2^nx),u\right\|$$
$$=\left\|f(x)-\frac{2^n+1}{2^{2n+1}}f(2^nx)+\frac{2^n-1}{2^{2n+1}}f(-2^nx)-\frac{1}{2^{2n}}f(2^nx)+\frac{2^n+1}{2^{2n+1}}f(2^nx)-\frac{2^n-1}{2^{2n+1}}f(-2^nx),u\right\|$$
$$=\left\|(f(x)-\frac{2^n+1}{2^{2n+1}}f(2^nx)+\frac{2^n-1}{2^{2n+1}}f(-2^nx)+(-\frac{1}{2^{2n}}f(2^nx)+\frac{2^n+1}{2^{2n+1}}f(2^nx)-\frac{2^n-1}{2^{2n+1}}f(-2^nx)),u\right\|$$
$$\leq\left\|f(x)-\frac{2^n+1}{2^{2n+1}}f(2^nx)+\frac{2^n-1}{2^{2n+1}}f(-2^nx),u\right\|+\left\|\frac{2^n-1}{2^{2n+1}}f(2^nx)-\frac{2^n-1}{2^{2n+1}}f(-2^nx),u\right\|$$

by(19) and lemma(3.1), the right side satisfies

$$\leq \max\{\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(-2^ix,-2^ix,2^ix,u),\frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^ix,2^ix,-2^ix,u) : 0\leq i\leq n-1\},\frac{|2^n-1|}{|2|^{2n+1}}\psi(2^nx)\} \quad (20)$$

for $n,m\in\mathbb{N}$ and $n\geq m$. This implies that,

$$\left\|f(x)-\frac{1}{2^{2(n-m)}}f(2^{n-m}x),u\right\|\leq \max\{\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(-2^ix,-2^ix,2^ix,u), \frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^ix,2^ix,-2^ix,u): 0\leq i\leq (n-m)-1\},\frac{|2^{n-m}-1|}{|2|^{2(n-m)+1}}\psi(2^{n-m}x)\} \quad (21)$$

for $n,m\in\mathbb{N}$ and $n\geq m$.

By replacing x by $2^mx$ in (21) we obtain

$$\left\|f(2^mx)-\frac{1}{2^{2(n-m)}}f(2^{n-m}2^mx),u\right\|\leq \max\{\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(-2^{i+m}x,-2^{i+m}x,2^{i+m}x,u), \frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^{i+m}x,2^{i+m}x,-2^{i+m}x,u): 0\leq i\leq (n-m)-1\},\frac{|2^{n-m}-1|}{|2|^{2(n-m)+1}}\psi(2^nx)\} \quad (22)$$

But $\|2^{-2n}f(2^nx)-2^{-2m}f(2^mx),u\|=|2|^{-2m}\left\|f(2^mx)-2^{-2(n-m)}f(2^{n-m}x),u\right\|$

So by (22) we have

$$\|2^{-2n}f(2^nx)-2^{-2m}f(2^mx),u\|\leq |2|^{-2m}\max\{\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(-2^{i+m}x,-2^{i+m}x,2^{i+m}x,u), \frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^{i+m}x,2^{i+m}x,-2^{i+m}x,u): 0\leq i\leq (n-m)-1\},\frac{|2^{n-m}-1|}{|2|^{2(n-m)+1}}\psi(2^nx)\} \quad (23)$$

By replacing $n$ by $n+1$ and $m$ by n in (32) we have,

$$\left\|2^{-2(n+1)}f(2^{n+1}x)-2^{-2n}f(2^nx),u\right\|\leq \max\{\max\{\frac{|1|}{|2|^3}\frac{\varphi(-2^nx,-2^nx,2^nx,u)}{|2|^{2n}}, \frac{|3|}{|2|^3}\frac{\varphi(2^nx,2^nx,-2^nx,u)}{|2|^{2n}}\},\frac{|1|}{|2||2|^{2(n+1)}}\psi(2^{n+1}x)\} \quad (24)$$

By (10) and (15), then for all $n\geq m$, the right side of the inequality (24) tends to 0 as $n$ tends to $\infty$.

This implies that the sequence $\{2^{-2n}f(2^nx)\}$ is Cauchy sequence in $Y$, by completeness of $Y$ this sequence is convergent. Define $Q:X\to Y$ by

$$Q(x)=\lim_{n\to\infty}2^{2n}f(2^nx) \qquad \forall x\in X. \quad (25)$$

$$\|Q(x+y+z)+Q(x)+Q(y)+Q(z)-Q(x+y)-Q(y+z)-Q(z+x),u\| = \lim_{n\to\infty}2^{-2n}|\ \ |f(2^n(x+y+z))+f(2^nx)+f(2^ny)+f(2^nz)-f(2^n(x+y)) -f(2^n(y+z))-f(2^n(z+x)),u|\ \ |$$

by (13), the right side

$$\leq \lim_{n\to\infty}\frac{\varphi(2^nx,2^ny,2^nz)}{|2|^{2n}}=0 \quad (26)$$

for all $x,y,z\in X$, $\forall u\in Y$ and $n\in N$. By lemma(2.6) we have,

$$Q(x+y+z)+Q(x)+Q(y)+Q(z)-Q(x+y)-Q(y+z)-Q(z+x)=0$$

Hence, $Q$ is a quadratic mapping.

Now let $Q(x)=\lim_{n\to\infty}2^{-2n}f(2^nx)$ and $Q(-x)=\lim_{n\to\infty}2^{-2n}f(-2^nx)$

$$\Rightarrow \|Q(x)-Q(-x),u\|=\lim_{n\to\infty}|2|^{-2n}\|f(2^nx)-f(-2^nx),u\|$$
$$\leq \lim_{n\to\infty}|2|^{-2n}\psi(2^nx)$$
$$=0 \quad \text{as} \quad n\to\infty$$

$\|Q(x)-Q(-x),u\|$=0 $\Rightarrow Q(x)=Q(-x)$, that is $Q$ is even.

By $|2|^n\leq 1$ for all $n\in\mathbb{N}$ we get,

$$\frac{|2^n-1|}{|2|^{2n+1}}\psi(2^nx)\leq \max\{\frac{|2|}{|2|^{2n+1}}\psi(2^nx),\frac{|1|}{|2|^{2n+1}}\psi(2^nx)\}=\frac{|1|}{|2|^{2n+1}}\psi(2^nx) \quad (27)$$

By (20) we have,

$$\|f(x)-\frac{1}{2^{2n}}f(2^nx),u\right\|\leq \max\{\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(-2^ix,-2^ix,2^ix,u),\frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^ix,2^ix,-2^ix,u): 0\leq i\leq n-1\},\frac{|2^n-1|}{|2|^{2n+1}}\psi(2^nx)\}$$

By (27)

$$\leq \max\{\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(-2^ix,-2^ix,2^ix,u), \quad \frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^ix,2^ix,-2^ix,u): 0\leq i\leq n-1\}, \frac{|1|}{|2|^{2n+1}}\psi(2^nx)\}, \quad (28)$$

so we have for all $x\in X$ and $\forall u\in Y$

$$\|f(x)-Q(x),u\| = \lim_{n\to\infty}\left\|f(x)-\frac{1}{2^{2n}}f(2^n x)\right\| \quad \text{bylemma}(2.9)$$
$$\le \lim_{n\to\infty}\max\{\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(-2^i x,-2^i x,2^i x,u),$$
$$\frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^i x,2^i x,-2^i x,u): 0\le i\le n-1\},$$
$$\frac{|1|}{|2|^{2n+1}}\psi(2^n x)\}$$
$$\le \lim_{n\to\infty}\max\{\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(-2^i x,-2^i x,2^i x,u),$$
$$\frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^i x,2^i x,-2^i x,u): 0\le i\le n-1\}\}$$

for all $x\in X$ and all $u\in Y$. This means that $Q$ is satisfies the inequality (16) be satisfied .

Let $T: X\longrightarrow Y$ be another quadratic mapping, which satisfies Equation (2) and inequality (16),
By [12], we have $Q(2^k x)=4^k Q(x)$ and $T(2^k x)=4^k T(x)$ for all $x\in X$ and $k\in N$
so for all $x\in X$, $u\in Y$ and $k\in\mathbb{N}$,

$$\|Q(x)-T(x),u\| = \frac{1}{|2|^{2k}}\|Q(2^k x)-T(2^k x),u\|$$
$$\le \frac{1}{|2|^{2k}}\max\{\|Q(2^k x)-f(2^k x),u\|,\|T(2^k x)-f(2^k x),u\|\}$$

By(??)

$$\le \max\frac{1}{|2|^{2k}}\max\{\lim_{n\to\infty}\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(-2^{i+k}x,-2^{i+k}x,2^{i+k}x,u),$$
$$\frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^{i+k}x,2^{i+k}x,-2^{i+k}x,u): 0\le i<n\}\}$$
$$\le \max\frac{1}{|2|^{2k}}\max\{\lim_{n\to\infty}\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(-2^i x,-2^i x,2^i x,u),$$
$$\frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^i x,2^i x,-2^i x,u): 0+k\le i<n+k\}\}$$

Now let $k\to\infty$ in both inequality, so by (17) and (18) the right side goes to 0, we have $\|Q(x)-T(x),u\|=0$ for all $x\in X$ and $u\in Y$. This implies that $Q(x)=T(x)$, hence $Q$ is a unique.

Therefore, the proof is complete.

Similarly, as the proof theorem (3.2), the Hyers-Ulam stability of equation(2) is proved under approximately odd condition in a non-Archimedean 2-Banach spaces.

**Theorem 3.3** *Assume that* $\varphi: X^3\times Y\to[0,\infty)$ *and* $\psi: X\to[0,\infty)$ *are mappings such that for all* $x,y,z\in X$ *and* $u\in Y$*,*

$$\lim_{n\to\infty}\frac{\varphi(2^n x,2^n y,2^n z,u)}{|2|^n}=0 \tag{29}$$

$$\lim_{n\to\infty}\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(2^i x,2^i y,2^i z,u): 0\le i<n\} \quad \text{exists,} \tag{30}$$

$$\lim_{n\to\infty}\max\{\frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^i x,2^i y,2^i z,u): 0\le i<n\} \quad \text{exists,} \tag{31}$$

also, let $f: X\to Y$ be a mapping satisfies f(0)=0 and such holds:
for all $x,y,z\in X$ and $\forall u\in Y$,

$$(i)\|f(x+y+z)+f(x)+f(y)+f(z)-f(x+y)-f(y+z)-f(z+x),u\|\le\varphi(x,y,z,u) \tag{32}$$
$$(ii)\|f(x)+f(-x),u\|\le\psi(x) \tag{33}$$
$$(iii)\lim_{n\to\infty}\frac{\psi(2^n x)}{|2|^n}=0 \quad \text{forallx}\in X \tag{34}$$

Then there exists a additive quadratic mapping $F: X\to Y$ which satisfies (2) and the inequality:

$$\|f(x)-F(x),u\|\le\lim_{n\to\infty}\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(-2^i x,-2^i x,2^i x,u)$$
$$\frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^i x,2^i x,-2^i x,u): 0\le i<n\} \tag{35}$$

forallx ∈ Xandu ∈ Y. Moreoverif,

$$\lim_{n\to\infty}\lim_{k\to\infty}\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(2^i x,2^i y,2^i z,u): k\le i<n+k\}=0, \tag{36}$$

$$\lim_{n\to\infty}\lim_{k\to\infty}\max\{\frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^i x,2^i y,2^i z,u): k\le i<n+k\}=0, \tag{37}$$

Then F is a unique quadratic mapping.

*Proof.* Let $f: X \to Y$ be a mapping satisfies the inequalities (32),

Now by (33) for all $x \in X$, and $u \in Y$, we have for $n \in \mathbb{N}$

$$\left\|\frac{2^n-1}{2^{2n+1}} f(2^n x)+\frac{2^n-1}{2^{2n+1}} f(-2^n x)\right\| \leq \frac{|2^n-1|}{|2|^{2n+1}} \psi(2^n x) \tag{38}$$

for all $x \in X$, $u \in$ Y and $n \in N$,

$$\begin{aligned}\left\|f(x)-\frac{1}{2^n} f(2^n x), u\right\| &= \left\|f(x)-\frac{2^n+1}{2^{2n+1}} f(2^n x)+\frac{2^n-1}{2^{2n+1}} f(-2^n x)\right. \\ &\quad \left.+\frac{-2^n-1}{2^{2n+1}} f(2^n x)-\frac{2^n-1}{2^{2n+1}} f(-2^n x)), u\right\| \\ &\leq \max\{\left\|f(x)-\frac{2^n+1}{2^{2n+1}} f(2^n x)+\frac{2^n-1}{2^{2n+1}} f(-2^n x), u\right\|, \\ &\quad \left\|\frac{2^n+1}{2^{2n+1}} f(2^n x)+\frac{2^n-1}{2^{2n+1}} f(-2^n x), u\right\|\}\end{aligned}$$

by(38) and lemma(3.1),

$$\begin{aligned}&\leq \max\{\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}} \varphi(-2^i x,-2^i x, 2^i x, u), \frac{|2^{i+1}+1|}{|2|^{2i+3}} \varphi(2^i x, 2^i x,-2^i x, u) \\ &: 0 \leq i \leq n-1\}, \frac{|2^n-1|}{|2|^{2n+1}} \psi(2^n x)\}\end{aligned} \tag{39}$$

This implies that for $n, m \in N$ and $n \geq m$.

$$\begin{aligned}&\left\|f(x)-\frac{1}{2^{(n-m)}} f(2^{n-m} x), u\right\| \leq \max\{\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}} \varphi(-2^i x,-2^i x, 2^i x, u), \\ &\frac{|2^{i+1}+1|}{|2|^{2i+3}} \varphi(2^i x, 2^i x,-2^i x, u): 0 \leq i \leq(n-m)-1\}, \frac{|2^{n-m}-1|}{|2|^{2(n-m)+1}} \psi(2^{n-m} x)\}\end{aligned} \tag{40}$$

By replacing x by $2^m x$ in (40)

$$\begin{aligned}&\left\|f(2^m x)-\frac{1}{2^{(n-m)}} f(2^{n-m} 2^m x), u\right\| \\ &\quad \leq \max\{\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}} \varphi(-2^{i+m} x,-2^{i+m} x, 2^{i+m} x, u), \\ &\quad \frac{|2^{i+1}+1|}{|2|^{2i+3}} \varphi(2^{i+m} x, 2^{i+m} x,-2^{i+m} x, u): 0 \leq i \leq(n-m)-1\}, \\ &\quad \frac{|2^{n-m}-1|}{|2|^{2(n-m)+1}} \psi(2^{n-m}. 2^m x)\}\end{aligned} \tag{41}$$

So by(41) we have

$$\begin{aligned}&\|2^{-n} f(2^n x)-2^{-m} f(2^m x), u\| \\ &\leq |2|^{-m} \max\{\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}} \varphi(-2^{i+m} x,-2^{i+m} x, 2^{i+m} x, u), \\ &\frac{|2^{i+1}+1|}{|2|^{2i+3}} \varphi(2^{i+m} x, 2^{i+m} x,-2^{i+m} x, u): 0 \leq i \leq(n-m)-1\}, \\ &\frac{|2^{n-m}-1|}{|2|^{2(n-m)+1}} \psi(2^{n-m}. 2^m x)\}\end{aligned} \tag{42}$$

Replacing $n$ by $n+1$ and $m$ by $n$ in (42) we have,

$$\begin{aligned}&\left\|2^{-(n+1)} f(2^{n+1} x)-2^{-n} f(2^n x), u\right\| \\ &\leq |2|^{-n} \max\{\max\{\frac{|2^{j+1}-1|}{|2|^{2i+3}} \varphi(-2^{i+n} x,-2^{i+n} x, 2^{i+n} x, u), \\ &\frac{|2^{j+1}+1|}{|2|^{2i+3}} \varphi(2^{i+n} x, 2^{i+n} x,-2^{i+n} x, u): 0 \leq i \leq(n+1-n)-1\}, \\ &\frac{|2^{n+1-n}-1|}{|2|^{2(n+1-n)+1}} \psi(2^{n+1} x)\}\end{aligned}$$

since $|4| \leq |2|$ since $|4|=|2+2| \leq \max\{|2|,|2|\}=|2|$ and $i=0$, we obtain,

$$\begin{aligned}&\left\|2^{-(n+1)} f(2^{n+1} x)-2^{-n} f(2^n x), u\right\| \\ &\leq \max\{\max\{\frac{|1|}{|2|^3} \frac{\varphi(-2^n x,-2^n x, 2^n x, u)}{|2|^n}, \frac{|3|}{|2|^3} \frac{\varphi(2^n x, 2^n x,-2^n x, u)}{|2|^n}\}, \\ &\quad \frac{|1|}{|2|^3|2|^n} \psi(2^{n+1} x)\} \\ &\leq \max\{\max\{\frac{|1|}{|2|^3} \frac{\varphi(-2^n x,-2^n x, 2^n x, u)}{|2|^n}, \frac{|3|}{|2|^3} \frac{\varphi(2^n x, 2^n x,-2^n x, u)}{|2|^n}\}, \\ &\quad \frac{|1|}{|2|^2|2|^{n+1}} \psi(2^{n+1} x)\}\end{aligned} \tag{43}$$

for all $n \geq m$, the right side of the inequality (43) tends to 0 as n tends to $\infty$).

This implies that the sequence $\{2^{-n} f(2^n x)\}$ is Cauchy sequence in $Y$, by completeness of $Y$ this sequence is convergent. Define $F: X \to Y$ by

$$F(x)=\lim_{n \to \infty} 2^{-n} f(2^n x) \qquad \forall \mathrm{x} \in \mathrm{X and n} \in \mathrm{N}. \tag{44}$$

For all $n \in N$, we know by a non Archimedean a absolute value $|2|^{2n+1} \leq |2|^{n+1}$ and $|2|^n \leq 1$, So we obtain,

$$\frac{|2^n-1|}{|2|^{2n+1}} \psi(2^n x) \leq \max\{\frac{|2|^n}{|2|^{2n+1}} \psi(2^n x), \frac{|1|}{|2|^{2n+1}} \psi(2^n x)\}=\frac{|1|}{|2|^{2n+1}} \psi(2^n x)$$

By (39) we have,

$$\left\|f(x)-\frac{1}{2^n}f(2^nx),u\right\| \le \max\{\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(-2^ix,-2^ix,2^ix,u),\frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^ix,2^ix,-2^ix,u)$$
$$:0\le i\le n-1\},\frac{|1|}{|2|^{n+1}}\psi(2^nx)\},$$

so we have for all $x\in X$ and $u\in Y$

$$\|f(x)-F(x),u\| = \lim_{n\to\infty}\left\|f(x)-\frac{1}{2^n}f(2^nx)\right\| \quad \text{bylemma}(2.9)$$
$$\le \lim_{n\to\infty}\max\{\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(-2^ix,-2^ix,2^ix,u),$$
$$\frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^ix,2^ix,-2^ix,u):0\le i\le n-1\}\}$$

This means the inequality (16) be satisfies.

The proof of uniqueness is the same way as that of theorem (3.2) by applying $F(2^{-n}x)=2^{-n}F(x)$ and $T(2^{-n}x)=2^{-n}T(x)$.

Similarly as proof of Theorem (3.2) due to (33) we see that mapping $F$ is odd.

By putting $z=-x$ in (2) and considering the oddness of $F$ and letting $u=x+y$, $v=x-y$, we get $2F(\frac{u+v}{2})=F(u)+F(v)$, since $F(0)=0$, the $F$ is additive. Hence the proof is complete.

Now, we will study the generalized stability of the following n-dimensional functional equation on a non-Archimedean 2-Banach spaces:

$$D_f(x_1,\ldots,x_k)=f(\sum_{i=1}^k x_i)+(k-2)\sum_{i=1}^k f(x_i)-\sum_{i=1}^k\sum_{j=1,j>i}^k f(x_i+x_j) \tag{45}$$

for any $k\ge 3$.

By Janfada[14] we can see that the quadratic function $f:X\to Y$ defined by $f(x)=x^2$ and any additive mapping not only satisfies the following equation functional:

$$f(x+y+z)+f(x)+f(y)+f(z)=f(x+y)+f(y+z)+f(x+z) \tag{46}$$

but also,

$$D_f(x_1,\ldots,x_k)=0 \tag{47}$$

For all $x_i\in X$, $i=1,2,..,k$.

In following we prove the generalized Hyers-Ulam-Rassias stability of (47) will be proved in non-Archimedean 2- normed spaces by using theorems (3.2) and (3.3).

Throughout this section, Let $X$ be a normed space and let $Y$ be normed non -Archimedean 2-Banach spaces.

**Theorem 3.4** *Let X and Y be common domain and range of the $f's$ in the functional equations (46) and (47). Then the functional equation (47) is equivalent to (46).*

*Proof.* See [14]

**Corollary 3.5** *Let $k\in\mathbb{N}$ and $k\ge 3$. Assume that mapping $f:X^k\times Y\to Y$ such that $f(0)=0$ and $f$ satisfies the following inequalities:*

$$\left\|f(\sum_{i=1}^k x_i)+(k-2)\sum_{i=1}^k f(x_i)-\sum_{i=1}^k\sum_{j=1,j>i}^k f(x_i+x_j),u\right\|\le\phi(x_1,x_2,\ldots,x_k,u) \tag{48}$$

$$\|f(x)-f(-x),u\|\le\psi(x) \tag{49}$$

Where $\phi:X^k\times Y\to[0,\infty)$ and $\psi:X\to[0,\infty)$ are mapping such that for all $x_1,x_2,x_3,\in X$ and $\forall u\in Y$,

$$\lim_{n\to\infty}\frac{\phi(2^nx_1,2^nx_2,2^nx_3,0,..,0,u)}{|2|^{2n}}=0 \tag{50}$$

$$\lim_{n\to\infty}\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(2^ix_1,2^ix_2,2^ix_3,0,0,\ldots,0,u):0\le i<n\}\quad\text{exists,} \tag{51}$$

$$\lim_{n\to\infty}\max\{\frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^ix_1,2^ix_2,-2^ix_3,0,0,\ldots,0,u):0\le i<n\}\quad\text{exists,} \tag{52}$$

$$\lim_{n\to\infty}\lim_{k\to\infty}\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(2^ix,2^ix,2^ix,0,0,\ldots,0,u):k\le i<n+k\}=0, \tag{53}$$

$$\lim_{n\to\infty}\lim_{k\to\infty}\max\{\frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^ix,2^ix,2^ix,0,0,\ldots,0,u):k\le i<n+k\}=0, \tag{54}$$

and,

$$\lim_{n\to\infty}\frac{\psi(2^nx)}{|2|^{2n}}=0\quad\text{forallx}\in X \tag{55}$$

Then there exists a unique quadratic mapping $Q:X\to Y$ which satisfies (47) and the inequality:

$$\|f(x)-Q(x),u\|\le\max\{\hat{\varphi}(-x,-x,x,u),\hat{\phi}(x,x,-x,u)\} \tag{56}$$

Where,

$$\varphi(x_1,x_2,x_3,u)=\{\phi(x_1,x_2,x_3,0,0,\ldots,0), \tag{57}$$

$$\hat{\varphi}(x_1,x_2,x_3,u) = \lim_{n\to\infty}\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(2^i x_1,2^i x_2,2^i x_3,u): 0 \le i < n\} \quad (58)$$

$$\hat{\phi}(x_1,x_2,x_3,u) = \lim_{n\to\infty}\max\{\frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^i x_1,2^i x_2,2^i x_3,u): 0 \le i < n\} \quad (59)$$

*Proof.* Let $f: X^k \times Y \to Y$ be mapping satisfies (48), $f(0) = 0$ and by setting $(x_1, x_2, x_3, \ldots, x_n, u) = (x_1, x_2, x_3, 0,0,\ldots,0, u)$ in (48), we obtain,

$$\|f(x_1+x_2+x_3) + f(x_1) + f(x_2) + f(x_3) - f(x_1+x_2) - f(x_2+x_3) - f(x_1+x_3), u\| \le \phi(x_1,x_2,x_3,0,..,0,u) \quad (60)$$

Now by Considering $\varphi(x_1,x_2,x_3,u) = \phi(x_1,x_2,x_3,0,0,\ldots,0,u)$ we see that $\varphi$ satisfies (10),(11),(12) and So using Theorem(3.2) we get (56).

**Corollary 3.6** *Let $k \in \mathbb{N}$ and $k \ge 3$. Assume that mapping $f: X^k \times Y \to Y$ such that $f(0) = 0$ and $f$ satisfies the following inequalities:*

$$\left\|f(\sum_{i=1}^{k} x_i) + (k-2)\sum_{i=1}^{k} f(x_i) - \sum_{i=1}^{k}\sum_{j=1,j>i}^{k} f(x_i+x_j), u\right\| \le \phi(x_1,x_2,\ldots,x_k,u) \quad (61)$$

$$\|f(x) + f(-x), u\| \le \psi(X) \quad (62)$$

Where $\phi: X^k \times Y \to [0,\infty)$ and $\psi: X \to [0,\infty)$ are mapping such that for all $x, y, z \in X$ and $\forall u \in Y$,

$$\lim_{n\to\infty}\frac{\phi(2^n x,2^n y,2^n z,0,..,0,u)}{|2|^n} = 0 \quad (63)$$

for all $x, y, z \in X$ and $\forall u \in Y$,

$$\lim_{n\to\infty}\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(2^i x,2^i y,2^i z,0,0,\ldots,0,u): 0 \le i < n\} \quad (64)$$

$$\lim_{n\to\infty}\max\{\frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^i x,2^i y,2^i z,0,0,\ldots,0,u): 0 \le i < n\} \quad (65)$$

are exists for all $x, y, z \in X$ and $\forall u \in Y$.

$$\lim_{n\to\infty}\lim_{k\to\infty}\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(2^i x,2^i x,2^i x,0,0,\ldots,0,u): k \le i < n+k\} = 0, \quad (66)$$

$$\lim_{n\to\infty}\lim_{k\to\infty}\max\{\frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^i x,2^i x,2^i x,0,0,\ldots,0,u): k \le i < n+k\} = 0, \quad (67)$$

$$\lim_{n\to\infty}\frac{\psi(2^n x)}{|2|^n} \quad \text{forallx} \in \text{X} \quad (68)$$

Then there exists a unique quadratic mapping $F: X \to Y$ which satisfies (2) and the inequality:

$$\|f(x) - F(x), u\| \le \max\{\hat{\varphi}(-x,-x,x,u), \hat{\phi}(x,x,-x,u)\} \quad (69)$$

Where,

$$\varphi(x,y,z,u) = \phi(x,y,z,0,0,\ldots,0,u) \quad (70)$$

$$\hat{\varphi}(x,y,z,u) = \lim_{n\to\infty}\max\{\frac{|2^{i+1}-1|}{|2|^{2i+3}}\varphi(2^i x,2^i y,2^i z,u): 0 \le i < n\} \quad (71)$$

$$\hat{\phi}(x,y,z,u) = \lim_{n\to\infty}\max\{\frac{|2^{i+1}+1|}{|2|^{2i+3}}\varphi(2^i x,2^i y,2^i z,u): 0 \le i < n\} \quad (72)$$

*Proof.* Let $f: X^k \times Y \to Y$ be mapping satisfies (61) and $f(0) = 0$.

Similarly as in proof of theorem(3.5) we can see that

$$\|f(x_1+x_2+x_3) + f(x_1) + f(x_2) + f(x_3) - f(x_1+x_2) - f(x_2+x_3) - f(x_1+x_3), u\| \le \phi(x_1,x_2,x_3,0,0,\ldots,0,u) \quad (73)$$

Consider $\varphi(x_1,x_2,x_3,u) = \phi(x_1,x_2,x_3,0,0,\ldots,0,u)$

This implies that

$$\lim_{n\to\infty}\frac{\varphi(2^n x_1,2^n x_2,2^n x_3,u)}{|2|^n} = \lim_{n\to\infty}\frac{\phi(2^n x_1,2^n x_2,2^n x_3,0,0,\ldots,0,u)}{|2|^n} = 0$$

Similarly, as proof of theorem (3.5) we see that $\varphi$ satisfies (29), (30) and (31), so we can use Theorem (3.3), we get (69) and by satisfy (36),(37) $F$ is unique.

## 4. CONCLUSION

A study of the stability properties of a type of quadratic equation in non-Archimedean 2-Banach spaces has been done. The stability quadratic functional equation with n-variable has been proved on the same space . It would be interesting also to study smilier properties for n-normed spaces.

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