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Method for Determining the Diffusivity and Thermal Resistance in Transient Regime from a Hollow Cylinder Wound with a Linen Insulator

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ABSTRACT

A new method of thermal characterization of materials is presented. It allows the determination of the thermal diffusivity (α) and the thermal resistance (Rth) between a solid surface of a hollow cylinder and its environment. This method is based on the analytical solution of the logarithm of the reduced temperature as a function of time, then the difference of temperature between the special reduced variable u = 0 and u = 1 as a function of heat flux. The use of these different parameters allows us to have the results. The characterization method is validated on a short hollow cylinder winding a linen insulator. This method constitutes a tool which lends itself well to the characterization of local materials which, for their implementation, must be the subject of a preliminary study.

Key words: Transient regime, Diffusivity, Thermal resistance

INTRODUCTION

The study of heat transfer by solving the Laplace equation is done in different ways in scientific work. These studies require data on the thermophysical parameters of these materials such as thermal diffusivity α, thermal conductivity λ, heat exchange coefficients h and thermal resistance RTH. Much work has been done on methods for determining the thermal diffusivity of materials. Most of these methods exploit time-series thermograms in the sample to trace the diffusivity value based on the exact analytical solutions of the transient temperature field.

These methods are classified into two groups: those using the frequency dynamic regime technique and those using the transient dynamic regime technique. Among the main methods of thermal diffusivity measurements using the dynamic frequency regime technique, we can cite the model of Marechal and Devism [1,2]. The main measurement methods below use the transient dynamic regime technique: the Parker and Jenkins model [3], the Degiovanni model [4], the hot wire method has been widely used by a large number of researchers including Blackwell [5], Foures [6] and Javelas (R) et al. [7], the regular regime method, developed by A. Vianou and A. Girardey [8] leads to the determination of thermal diffusivity by using the thermograms recorded in a cylindrical sample during its cooling.

In the work, we present a new method of characterization of construction materials [9]. It is a method which allows the determination of thermal diffusivity α and thermal resistance. The method is validated on a hollow cylinder in Lin.

STUDY MODEL

Study device

The change in temperature in a transient state in a short right cylinder is very difficult to determine analytically if a three-dimensional model is used. To facilitate its resolution, we will rely on the method of separation of variables and by applying the theorem of Von Neumann.

This theorem allows us to state that the dimensionless temperature of the short hollow cylinder is expressed as the product of the dimensionless temperatures of the infinite cylinder and the infinite plate. Indeed, the short right cylinder can be considered as the intersection of an infinite cylinder with an infinite plate, as illustrated in figure 1. Solving the two-dimensional problem is reduced to solving two one-dimensional problems.

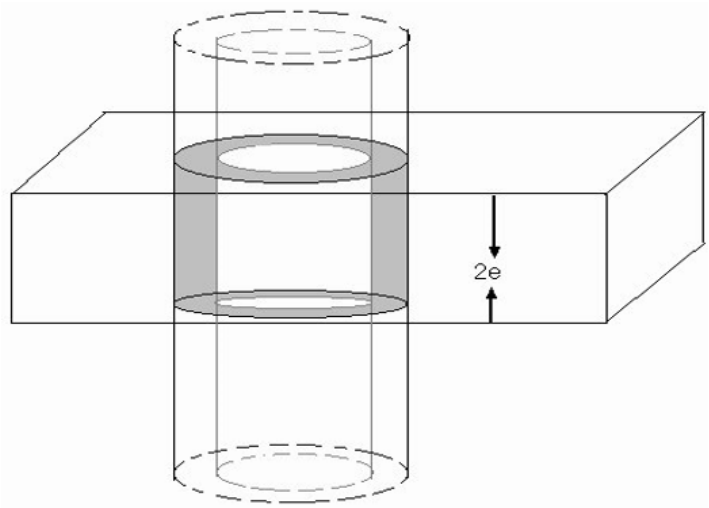

Fig. 1 Short cylinder: intersection of an infinite plate and an infinite cylinder

Theory

We consider a plate of thickness 2e initially at the uniform temperature T_0 in contact with a metal plate of temperature T_f at the upper surface of the plate and another made of plastic at temperature T_p at the lower surface, the two temperatures depend on the time. The surface of the plate being large compared to its thickness, the thermal field can be considered as one-dimensional. The contact of the plate with the two solids is characterized by the heat exchange coefficients h_f and h_p.

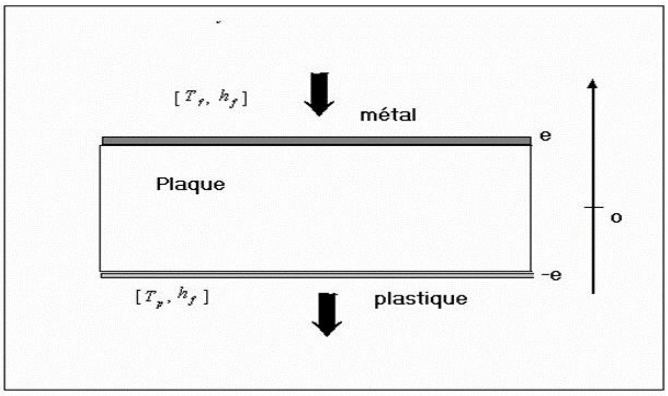

Fig. 2 Infinity plate

The heat equation is written as:

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T(x,t)}{\partial t} \tag{1}$$

With T (x, t) the temperature on the x- axis at the time t ; and a thermal diffusivity α

The conditions applied on the model of study are give :

- Initial Condition $T(x,0) = T_0$
- Boundary condition :

  At the surface metal-plate (x=e)

$$-\lambda \frac{\partial T(x,t)}{\partial x^2}\Big|_{x=e} = h_f\left[T(e,t) - T(t_1)\right] \tag{2}$$

At the surface plate- plastic (x=-e)

$$-\lambda \frac{\partial T(x,t)}{\partial x^2}\Big|_{x=-e} = h_p\left[T_p(t_1) - T(-e,t)\right] \tag{3}$$

λ Designed the thermal conductivity of the material.

We remark that the boundary conditions are functions of time. We can then apply DUAMEL's theorem for solving the equation

Application of DUAMEL's theorem [10]

Let $\Theta(x,t,t_1)$ the auxiliary solution of equation (1) be provided with the boundary conditions (2) and (3)

$$\frac{\partial\Theta(x,t,t_1)}{\partial x^2} = \frac{1}{\alpha}\frac{\partial\Theta(x,t,t_1)}{\partial t} \qquad (4)$$

$$-\lambda.\frac{\partial\Theta(x,t,t_1)}{\partial x^2}\Big|_{x=e} = h_f[T_p(t_1) - T(e,t,t_1)] \qquad (5)$$

$$-\lambda\frac{\partial\Theta(-e,t,t_1)}{\partial x}\Big|_{x=-e} = h_p[\Theta(-e,t,t_1) - T_p(t_1)] \qquad (6)$$

From the boundary conditions we have the transcendent equation below

$$\frac{w_i}{Bif} = \left(\frac{2.w_i.\cos(w_i) - Bif.\sin(2.w_i)}{w_i.\sin(2.w_i) + Bif.\cos(2.w_i) - Bip}\right) \qquad (7)$$

We present the graphical determination of the roots of the transcendent equation (8) which are the points of intersection of the curves of the two affine functions $f(w_i)$ and trigonometry $g(w_i)$

$$f(w) = \frac{w_i}{Bi_f} \quad \text{et} \quad g(w) = \frac{2.w_i.\cos^2(w_i) - Bi_p.\sin(2.w_i)}{w_i.\sin(2.w_i) + Bi_f.\cos(2.w_i) - Bi_p} \qquad (8)$$

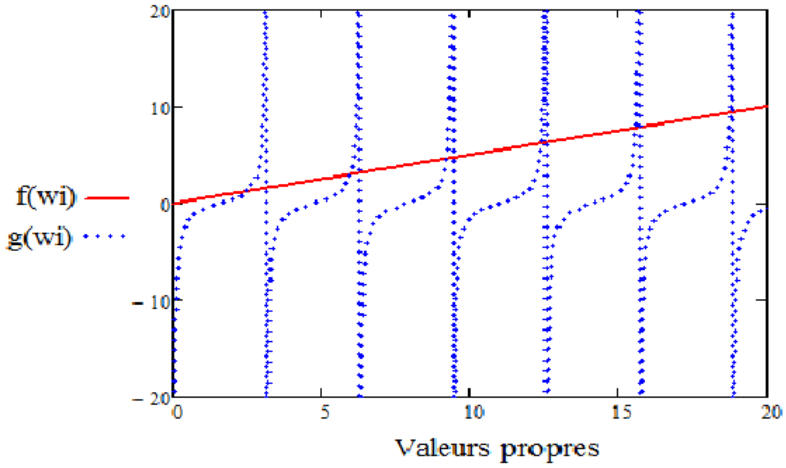

Fig. 3 Graphical determination of the roots of the transcendent equation for the plate

The table below groups together some values obtained thanks to the transcendent equation.

Table -1 Non-zero eigenvalues of the transcendent equation

n_i	0	1	2	3	4	5	6	7	8
w_i	0	2.14	3.17	6	6.36	9.41	12.46	15.66	18.78

From this graphic resolution, we realize the existence of a multitude of solutions w_i for a given couple.

Determination of constants

$$T_{1n} = \frac{Bi_p - 1}{2.Bi_p - 2}.\frac{2.\sin(w_{in})}{w_{in}} \qquad (9)$$

$$T_{2n} = \frac{Bi_p}{2.Bi_p - 1}\left(\frac{Bi_f - Bi_p}{2.w_{in} - (Bi_f - Bi_p).\tan(w_{in})}\right)\left(\frac{2.\sin(w_{in})}{{w_{in}}^2} - \frac{2.\cos(w_{in})}{w_{in}}\right) \qquad (10)$$

The equation (II.19) can be written of the form :

$$\delta\theta_n(u,\tau,\tau_1) = a_n.\delta\theta_0.U_{in}(u,\tau).(T_{1n} + T_{2n}).\exp(-{w_{in}}^2.\tau) \qquad (11)$$

$$\delta\theta(u,\tau_1,\tau) = \sum_n \delta\theta_n(u,\tau_1,\tau) \qquad (12)$$

Thermal field in transient regime in an infinite hollow cylinder

We consider an unlimited hollow homogeneous cylinder, of inner radius R_1 and outer radius R_2 and initial temperature distribution $T(o,r)=T_0$.

We consider an unlimited hollow homogeneous cylinder, of interior radius R_1 and exterior radius R_2 and initial temperature distribution. The contact of the cylinder with the two solids (metal, plastic) is characterized by the Heat exchange coefficients h_f and h_p.

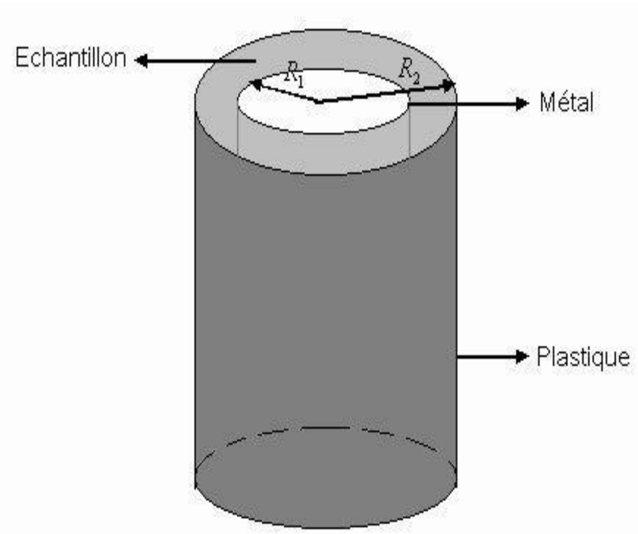

Fig. 4 Infinite hollow cylinder

The thermal field of the infinite hollow cylinder is governed by the following differential equation:

$$\frac{\partial^2 T(r,t)}{\partial r^2}+\frac{1}{r}\frac{\partial T(r,t)}{\partial r}=\frac{1}{\alpha}\frac{\partial T(r,t)}{\partial t} \tag{13}$$

The boundary conditions at the two surfaces are:

$$-\lambda\frac{\partial T(r,t)}{\partial r}\Big|_{r=R_1}=-h_f\left[T(t_1)-T_f(R_1,t)\right] \tag{14}$$

$$-\lambda\frac{\partial T(r,t)}{\partial r}\Big|_{r=R_2}=h_p\left[T(R_2)-T(t_1)\right] \tag{15}$$

We notice that the boundary conditions are functions of time. We can then apply DUHAMEL's theorem for the solution of the equation

Application of DUAMEL's theorem

Let $\Theta(r,t,t_1)$ be the auxiliary solution of the equation (16) provided with boundary cionsondit (14) et (15) we have :

$$\frac{\partial^2\Theta(r,t,t_1)}{\partial r^2}+\frac{1}{r}\frac{\partial\Theta(r,t,t_1)}{\partial r}=\frac{1}{\alpha}\frac{\partial\Theta(r,t,t_1)}{\partial \tau} \tag{16}$$

$$-\lambda\frac{\partial\Theta(r,t,t_1)}{\partial r}\Big|_{r=R_1}=h_f\left[T_f(t_1)-\Theta(R_1,t,t_1)\right] \tag{17}$$

$$-\lambda\frac{\partial\Theta(r,t,t_1)}{\partial r}\Big|_{r=R_2}=h_p\left[\Theta(R_2,t,t_1)-T_p(t_1)\right] \tag{18}$$

We note in the boundary conditions the existence of a constant term independent of the time variable 1. To simplify the resolution of equation (16), we will reduce it to a differential equation in the reduced form.

-Variable reduced space

$$u=\frac{r}{a} \tag{19}$$

With $a=R_1-R_2$ the thickness of the cylinder

-Variable reduced temperature

$$\theta(u,\tau,\tau_1)=\frac{\Theta(r,t,t_1)-T_0(t_1)}{T_0(t_1)} \quad (20)$$

Study of the infinite cylinder in transient regime:

The reduced heat equation of the infinite hollow cylinder in transient state is obtained by replacing the reduced temperature of the cylinder which is the sum of the reduced temperature in steady state and the reduced temperature in transient state, provided with the boundary conditions.

$$\frac{\partial\delta\theta(u,\tau,\tau_1)}{\partial u^2}+\frac{1}{u}\frac{\partial\delta\theta(u,\tau,\tau_1)}{\partial u}=\frac{\partial\delta\theta(u,\tau,\tau_1)}{\partial\tau} \quad (21)$$

With the boundary conditions:

$$\left.\frac{\partial\delta\theta(u,\tau,\tau_1)}{\partial u}\right|_{u=\frac{R_1}{a}}=Bif.\theta(\frac{R_1}{a},\tau,\tau_1) \quad (22)$$

$$\left.\frac{\partial\delta\theta(u,\tau,\tau_1)}{\partial u}\right|_{u=\frac{R_2}{a}}=-Bip.\theta(\frac{R_2}{a},\tau,\tau_1) \quad (23)$$

We will look for a solution of this equation in the form of separation of variables.
Then Equation can be put in the following form:

$$\frac{J_0(\mu_i.\frac{R_2}{a})}{Y_0(\mu_i.\frac{R_2}{a})}=\frac{J_0(\mu_i.\frac{R_1}{a})}{Y_0(\mu_i.\frac{R_1}{a})} \quad (24)$$

This is a transcendent equation in which the resolution can be done graphically

$$f(\mu)=\frac{J_0(\mu_i.\frac{R_2}{a})}{Y_0(\mu_i.\frac{R_2}{a})} \quad (25)$$

$$g(\mu)=\frac{J_0(\mu,\frac{R_1}{a})}{Y_0(\mu_i.\frac{R_1}{a})} \quad (26)$$

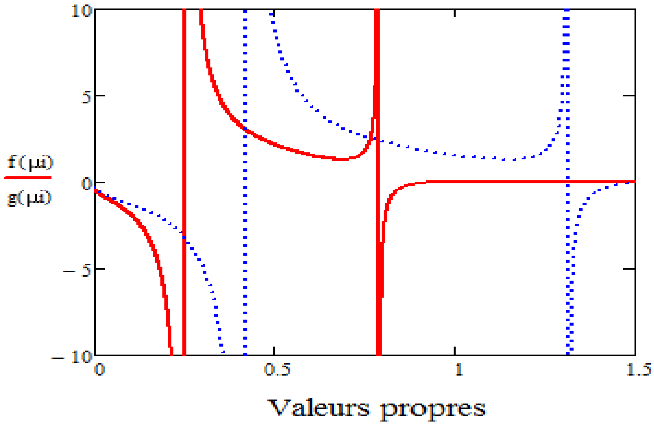

Fig. 5 Graphical determination of the roots of the transcendent equation for the cylinder

We present in the following table some solutions of the transcendent equation.

Table -2 Non-zero eigenvalues of the transcendent equation

n_i	0	1	2	3	4	5
μ_i	0	0.24	0.41	0.79	1.31	1.49

Determination of constants:

If we admit that $\delta\theta(u,\tau,\tau_1)$ then be represented by an expansion of the form.

$$\delta\theta(u,0,\tau) = \sum V_n(u,\tau_1).T_{in}(0) \quad (27)$$

It comes taking into account the orthogonality of the functions $V_{in}(u,\tau_1)$

$$T_{in}(0) = \frac{\int_{\frac{R_1}{a}}^{\frac{R_2}{a}} u.\delta\theta(u,0,\tau_1).V_{in}(u,\tau_1).du}{\int_{\frac{R_1}{a}}^{\frac{R_2}{a}} u.V_{in}^{\ 2}(u,\tau_1).du} \quad (28)$$

Solving this equation gives :

$$T_{ik}(0) = \delta\theta_0.\left[T_{1k}(o) + T_{2k}(0)\right] \quad (29)$$

$$T_{1k} = \pi.A.J_0(\mu_i.\frac{R_1}{a}).\frac{J_0(\mu_i.\frac{R_1}{a}).\ln(\frac{R_2}{a}) - J_0(\mu_i.\frac{R_2}{a}).\ln(\frac{R_2}{a})}{J_0^{\ 2}(\mu_i.\frac{R_1}{a}) - J_0^{\ 2}(\mu_i.\frac{R_2}{a})} \quad (30)$$

$$T_{2k} = \pi.B.\frac{J_0(\mu_i.\frac{R_1}{a})}{J_0(\mu_i.\frac{R_1}{a}) + J_0(\mu_i.\frac{R_2}{a})} \quad (31)$$

$$A = \left[1 - \frac{Bi_p.\ln(\frac{R_1}{R_2})}{Bi_p.\ln(\frac{R_1}{R_2}) - \frac{a}{R_2}}\right].\delta\theta_0 \quad (32)$$

$$B = \frac{Bi_p.\delta\theta_0}{Bi_p.\ln(\frac{R_1}{R_2}) - \frac{a}{R_1}} \quad (33)$$

And we have the final solution for the infinite cylinder

$$\delta\theta_n(u.\tau,\tau_1) = V(u,\tau_1).T(\tau).\exp(-\mu_i^{\ 2}.\tau) \quad (34)$$

Thermal fields in transient regime in a short hollow cylinder.

As we said, the general solution of the short hollow cylinder is the product of the solution of the infinite plate by the solution of the infinite cylinder.

$$\delta\theta(u,\tau,\tau_1) = U(u,\tau_1).V(u,\tau_1).T_{in}(0).T(0).\exp(-(\frac{w_i^2}{e^2} + \frac{\mu_i^{\ 2}}{a^2}).\alpha.t \quad (35)$$

Let $b_1 = U(u,\tau_1).V(u,\tau_1).T_{in}(0).T(0)$ and

$$b_2 = \left(\frac{w_1^{\ 2}}{e^2} + \frac{\mu_1^{\ 2}}{a^2}\right).\alpha$$

$$\delta\theta(u,\tau,\tau_1) = b_1.\exp(-b_2.t)$$

Determination of thermal diffusivity

Using the final result of the reduced temperature of the short hollow cylinder we will determine the thermal diffusivity of the material. Thus the plot of the logarithm of the reduced temperature as a function of time, gives us a straight line of slope b_2 which according to the following formula below, allows us to have the thermal diffusivity of the material with w_1 and μ_1 the eigenvalues which allows the convergence of the series whose respective values are 1.31 and 2.14.

$$\alpha = \frac{b_2}{(\frac{w_1}{e^2} + \frac{\mu_1}{a^2})}$$

$$\ln(\delta\theta(u,\tau,\tau_1)) = \ln(b_1) - b_2.t \quad (37)$$

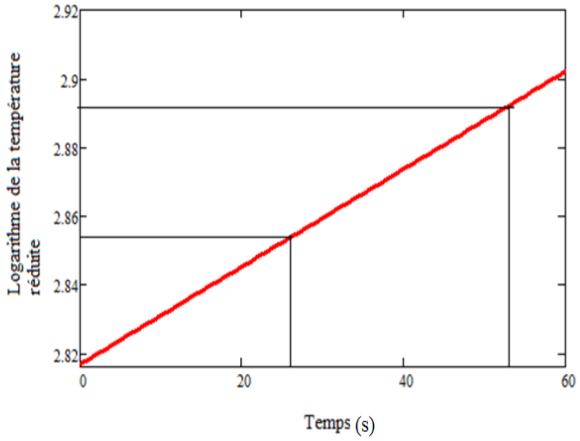

Fig. 6 Logarithmic evolution of the reduced temperature as a function of time

The table below gives the values of the slopes of the lines obtained by plotting the evolution of the logarithm of the reduced temperature, in three (3) points

Table -3 Values of the slopes of the lines obtained and of the diffusivity at each point

Δln (δθ) (°K)	3.83	5.83	1.461	3.71
Δt (s)	27	41	10	26
Slopes b_2 10^{-4} s^{-1}	1.4222	1.4219	1.4610	1.4350
Diffusivity ($x10^{-7}m^2s^{-1}$)	7.140	7.14	7.14	7.136

Table 3 gives the values extracted from the exploitation of the theoretical results obtained graphically. The thermal diffusivity of flax experimentally is equal to **8.10^{-7} $m^2.S^{-1}$** for a thickness of 200 mm [4]. We have found an equal value **7, 1361 10^{-7} $m^2.S^{-1}$** for a thickness also of 200mm and we have a very small deviation of the order of **10^{-7}**. We can deduce that the method we used is reliable and that the error is very small.

Determination of thermal resistance

To determine the thermal resistance of the material, we will try to plot the difference in the reduced temperature between the values u = 0 and u = 1 of the reduced space variables as a function of the density of the heat flow and we have a straight line. The calculates slopes for different gives us the thermal resistance of the materials by thermal electrical analogy. We give the correspondence between the electrical and thermal quantities in the table below:

Table -4 The electrical and thermal quantities of the materials

Electrical Quantities	Thermal Quantities
Electrical current : $I = \frac{dq}{dt}$	Flux density $\varphi = -grad T$
Potential V	Temperature en K°
Electrical impedance : $Z_E = \frac{\Delta V}{I}$	$Z_T = \frac{\Delta T}{\varphi}$

The expression of the flux density is:

$$\varphi(u,\tau) = -\lambda \frac{\partial\delta\theta(u,\tau,\tau_1)}{\partial u} \quad (38)$$

After deriving the expression for the reduced temperature we have:

$$\varphi(u,t) = -\lambda[(U_d.V(u) + U(u).V_d(u)).T_{i1}(0).T_{k1}(0)\exp(-(w_1^2 + \mu_1^2).\alpha.\tau \quad (39)$$

$$U_d(u) = -J_1(u).Y_0 + J_0.Y_1(u) \quad (40)$$

$$V_d = -a_n.\sin(w_{in}.u) + b_n \cos(w_{in}.u) \quad (41)$$

The difference of temperature is given by :

$$\Delta\delta\theta(u,\tau,\tau_1) = \delta\theta(0,\tau,\tau_1) - \delta\theta(u,\tau,\tau_1) \quad (42)$$

The profile of the difference temperature as a function of the heat flux density is:

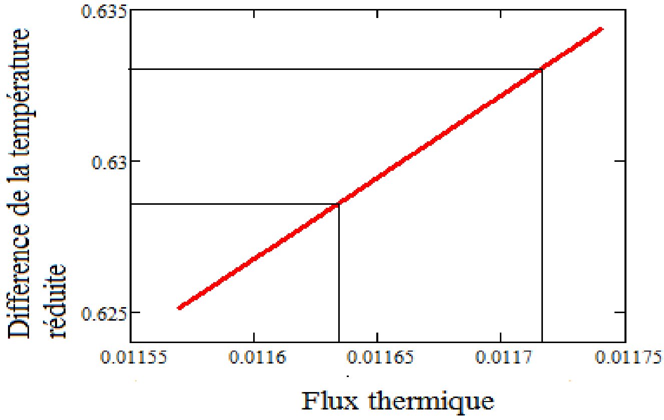

Fig. 7 Variation of the temperature difference between u = 0 and u = 1 depending on the heat flow for Bip = 100 and Bif = 1000

$$\Delta\delta\theta = R_{th}.\Delta\varphi \qquad R_{th} = \frac{\Delta\delta\theta}{\Delta\varphi}$$

Table -5 Different resistance values at each point

$\Delta\delta\theta$ (°K)	9.20	3.07	3.08	3.05
$\Delta\varphi$ (W)	1.70	0.5678	0.5706	0.5670
R_{th} (°KW^{-1})	5.411	5.406	5.397	5.362

CONCLUSION

This work was mainly focused on the characterization of the thermophysical parameters which are the determination of the thermal diffusivity and the thermal resistance of flax. In general, we have found that we need to know a certain number of thermal fields involving techniques for solving the heat equation in cylindrical coordinates

At the end of the results obtained graphically by our lines and our curves, we can draw certain conclusions:

The values of diffusivity and thermal resistance found are in agreement with those found in the literature. Linen is an excellent ecological thermal insulator. The various curves studied show that better heat transfer in transient conditions is obtained for a high reduced temperature and a low exchange coefficient on the rear face.

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