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A Survey of the Noise Terms Phenomenon in Adomian Method: A Special Case

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ABSTRACT

In this work, we apply an iterative method (Adomian Decomposition Method*) for solving a special case of nonlinear PDEs.* The efficiency *of this method is illustrated by investigating the* convergence results *for this equation. We show that the noise terms are conditional for nonhomogeneous equations and the numerical results show the reliability and accuracy of the ADM.*

Key words: Adomian method, noise terms, semi-numerical method, iterative method

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INTRODUCTION

Adomian Decomposition method (ADM) is a semi-numerical method for solving ODE and PDE. Adomian first introduced the concept of ADM and this technique constructs an analytical solution in the form of a polynomial. This technique is an alternative procedure for obtaining analytical series solution of the differential equations. The series often coincides with the Taylor expansion of the true solution at the point $x_0 = 0$, although the series can be rapidly convergent in a small region.

The noise term phenomenon provides major advantages by a fast convergence of the solution and may appear only or in homogenous PDEs. These are equal but different expressions, which appear in the various components u_k of the series. It can be verified that if the terms in u_0 are cancelled in iteration u_1, then it can be seen that those that were not cancelled in the first iteration constitute the solution of the equation.

DESCRIPTION OF ADM

The Adomian Decomposition Method is applied to a nonlinear equation

$$Lu + Ru + Nu - g = 0, \tag{1}$$

Where, the linear terms are decomposed into $L+R$ and the nonlinear terms are represented by Nu. Here, L is the operator of the highest-ordered derivatives with respect to t and R is the remainder of the operator. Thus, we get

$$Lu = -Ru - Nu + g. \tag{2}$$

Now, there is an inverse operator L^{-1} of L defined by

$$L^{-1}(\cdot) = \int_0^t \int_0^t (\cdot)\ dt\, dt \tag{3}$$

Now, if L is a second order operator, then L^{-1} is defined by a two-fold indefinite integral

$$L^{-1}Lu = u(x,t) - u(x,0) - t\frac{\partial u(x,0)}{\partial t} \tag{4}$$

Now, operating on both sides of Eq. (1) by using L^{-1} we obtain

$$L^{-1}Lu = L^{-1}g - L^{-1}Ru - L^{-1}Nu \tag{5}$$

Therefore, we have

$$u(x,t) = u(x,0) + t\frac{\partial u(x,0)}{\partial t} + L^{-1}g - L^{-1}Ru - L^{-1}Nu \tag{6}$$

The ADM represents the solution of Eq. (6) as a series

$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)$ (7)
Now, the operator Nu (nonlinear) is decomposed as
$Nu = \sum_{n=0}^{\infty} A_n$ (8)
Therefore, substituting (7) and (8) into (6) we obtain
$\sum_{n=0}^{\infty} u_n(x,t) = u_0 - L^{-1}R\sum_{n=0}^{\infty} u_n(x,t) - L^{-1}\sum_{n=0}^{\infty} A_n$ (9)
Where
$u_0 = u(x,0) + t\frac{\partial u(x,0)}{\partial t} + L^{-1}g$ (10)
Then, we can get

$$u_1 = -L^{-1}Ru_0 - L^{-1}A_0$$
$$u_2 = -L^{-1}Ru_1 - L^{-1}A_1 \qquad (11)$$
$$\vdots$$
$$u_{n+1} = -L^{-1}Ru_n - L^{-1}A_n$$

Here, $u_n(x,t)$ will be determined recurrently and A_n are the polynomials (Adomian) of $u_0, \dots, u_n$ defined by
$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}[F(\sum_{n=0}^{\infty}\lambda_i u_i)],\ \ n = 0,1,2,\dots$ (12)

In this case, we get

$$A_0 = f(u_0)$$
$$A_1 = u_1 f'(u_0)$$
$$A_2 = u_2 f''(u_0) + \frac{1}{2!}u_1^2 f''(u_0) \qquad (13)$$
$$\vdots$$

Therefore, if we introduce the parameter λ, we can obtain that
$u(\lambda) = \sum_{n=0}^{\infty} \lambda^n u_n$ (14)
Where we can write
$N(u(\lambda)) = \sum_{n=0}^{\infty} \lambda^n A_n$ (15)
Finally, expanding by Taylor's series at $\lambda = 0$ we have

$$N(u(\lambda)) = \sum_{n=0}^{\infty}\frac{1}{n!}\left[\frac{d^n}{d\lambda^n}N(u(\lambda))\right]\lambda^n$$
$$N(u(\lambda)) = \sum_{n=0}^{\infty}\frac{1}{n!}\left[\frac{d^n}{d\lambda^n}N(\sum_{n=0}^{\infty}\lambda^i u_n)\right]\lambda^n \qquad (16)$$

The Adomian's polynomials A_n can be calculated using the recurrence equation
$A_n = \sum_{n=0}^{\infty}\frac{1}{n!}\left[\frac{d^n}{d\lambda^n}N(\sum_{n=0}^{\infty}\lambda^i u_n)\right]$ (17)
at $\lambda = 0$.
If we are working with systems of differential equations (or equally of an algebraic type), the non-linear terms *N* can be of the form

$$N = N(u_1, \dots, u_k, \dots)$$

Where

$$u_k = \sum_{n=0}^{\infty} u_{ki}.$$

Similarly, Adomian's polynomials can be obtained by using the recurrence equation

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}N\left(\sum_{n=0}^{\infty}u_{1i}, \sum_{n=0}^{\infty}u_{2i}, \dots, \sum_{n=0}^{\infty}u_{ki}, \dots\right)\right]$$

The level of precision for the approximation of u will be much better the more components are calculated, i.e,

$$u = \lim_{n\to\infty}\varphi_n$$

Where

$$\varphi_n = \sum_{k=0}^{n-1} u_k$$

THE NOISE TERMS

In applications using ADM, the appearance of noise terms (sometimes) makes it necessary to calculate more terms of these polynomials. We can see that the cancellations vanishes in the limit and then the noise terms are the identical terms with opposite sign that appear within the components u_0 and u_1. It is important to note that these terms do not appear between u_0 and u_1, then it is necessary to calculate more components of the solution $u(t)$. Another important element is that not all non-homogeneous equations have the noise terms phenomenon.
Therefore, it is necessary to verify that the non-cancelled terms of u_0 satisfy the PDE. A necessary condition for the generation of the noise terms for inhomogeneous PDEs is that the zeroth component u_0 must contain the exact solution

$u(t)$ among other terms. To give a clear overview of the content of this work, an example of partial differential equation have been selected to demonstrate the efficiency of the method and to confirm the necessary condition needed for the generation of the noise terms.

RESULTS AND DISCUSSION

Numerical example.

We consider the problem

$$\begin{cases} u_{xx} = u_x u_{tt} - x + u \\ u(0,t) = \sin t \\ u_x(0,t) = 1 \end{cases} \quad (18)$$

Now, in terms of the operator, the equation associated with the problem (18) takes the form

$L_{xx}u = L_x u L_{tt} u - x + u$

Then, applying the inverse operatorL_{xx}^{-1} to this expression we have

$$L_{xx}^{-1}(L_{xx}u) = L_{xx}^{-1}(L_x u L_{tt} u) - L_{xx}^{-1}(x) + L_{xx}^{-1}(u) \quad (19)$$

As in the previous section, we proceed to evaluate the left side of (19) and perform the respective operations concerning the double integral of the middle end together with the initial conditions obtaining

$$u(x,t) = L_{xx}^{-1}(u) + L_{xx}^{-1}(L_x u L_{tt} u) + \sin t + x - \frac{x^3}{6}$$

where we finally get the recurrence equation

$$\sum_{n=0}^{\infty} u_n(x,t) = \sin t + x - \frac{x^3}{6} + L_{xx}^{-1}\left(\sum_{n=0}^{\infty} u_n\right) + L_{xx}^{-1}\left[L_x\left(\sum_{n=0}^{\infty} u_n\right)L_{tt}\left(\sum_{n=0}^{\infty} u_n\right)\right]$$

With $u(x,t) = \sum_{n=0}^{\infty} u_n$ and non-linear term$u_x u_{tt} = N(u)$which is subsequently written

$$N(u) = u_x u_{tt} = \sum_{n=0}^{\infty} A_n$$

Now, assuming that $u_0 = \sin t + x - \frac{x^3}{6}$ we have

$u_1 = L_{xx}^{-1}(u_0) + L_{xx}(A_0)$

$u_2 = L_{xx}^{-1}(u_1) + L_{xx}(A_1)$

$\vdots$

$u_{n+1} = L_{xx}^{-1}(u_n) + L_{xx}(A_n)$

Now, having chosen u_0, we calculate the first polynomial of Adomian as follows

$$A_0 = N(u_0) = (u_0)_x (u_0)_{tt}$$
$$= \left(\sin t + x - \frac{x^3}{6}\right)_x \left(\sin t + x - \frac{x^3}{6}\right)_{tt}$$
$$= \left(1 - \frac{x^2}{2}\right)(-\sin t) = -\sin t + \frac{x^2}{2}\sin t$$

Therefore, the first iteration has the form

$$u_1 = L_{xx}^{-1}\left(\sin t + x - \frac{x^3}{6}\right) + L_{xx}^{-1}(A_0)$$
$$= \frac{x^3}{6} - \frac{x^5}{120} + \frac{x^4}{24}\sin t$$

If we look closely, the sum of u_0 with u_1, would result in cancellation of similar terms thus giving rise to the phenomenon described in this section, i.e., the noisy terms. Therefore, no further iterations would be necessary and then we will get the solution $u(x,t) = x + \sin t$.

It is important to emphasize that the modified method is slower in convergence than the method presented in this example, which leads us to conclude that not always using modifications accelerates convergence in solution.

CONCLUSION

The appearance of noise terms using ADM plays a major role in accelerating the convergence of the solution of PDE and minimizing the size of calculations if an exact solution exists. If the case that a closed form solution cannot be obtained, then solution determine in approximation form. In this paper, we demonstrated that this iterative method is quite efficient to determine solution in closed form. The new scheme obtained using ADM yields an analytical solution in the form of a rapidly convergent series and therefore ADM makes the solution procedure much more attractive.

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