Available online www.ejaet.com

European Journal of Advances in Engineering and Technology, 2020, 7(7):19-28

Research Article **ISSN: 2394 - 658X**

Dynamic Response of Rayleigh Beam with Axial Force to Partially Distributed Moving Loads Resting on Linear and Nonlinear Viscoelastic Foundation

Akinpelu F. O.

Department of Pure and Applied Mathematics, LadokeAkintola University (LAUTECH) Ogbomoso, Oyo State, Nigeria

Current Address: Department of Mathematics, University of Lagos, Akoka Lagos, Nigeria

Email address: foakinpelu@yahoo.co.uk/ fakinpelu@unilag.edu.ng

ABSTRACT

The response of Rayleigh beam with axial force resting on a nonlinear Pasternak foundation with viscous damping subject to a partially distributed moving loads was investigated. The nonlinear Pasternak foundation is assumed to be a cubic. The governing equation of fourth order partial differential equation was reduced to second order nonlinear ordinary differential equation by assuming a solution in terms of series form and then solve numerically using the Maple software to obtain the result for the moving loads and moving force. The effects of rotary inertia, speed, viscosity, axial force (tensile and compressive forces) were discussed. The numerical investigation shows that the dynamics of Rayleigh beams supported by elastic foundation need super higher modes: furthermore, the comparison between moving load and moving force shows that the moving load is higher than moving force for different parameters. It was also observed that as the axial force increases the displacement decreases for the moving force and the moving mass for the tensile force while in the case of the compressive force, as the axial force increases the displacement is equally increases.

Key words: Rayleigh beam, nonlinear elastic foundation, moving load, viscous damping and axial force

1. INTRODUCTION

The response of elastic beam traversed by a moving load at a specific speed has attracted the attention of many researchers since 19th century. Such problems have been extensively studied in connection with machine process, guide way systems and design of Railway bridge. However, with the increase of road traffic and vehicle loads, earlier damage of asphalt pavement on high ways has become more and more serious which has greatly shortened the service life of pavement. Many researchers have shown that one of the most important reasons for road damages is the vehicle load. These pavement vehicle systems can be theoretically model as beams supported by foundations subject to moving forces. Therefore, the studies of the dynamic behaviour of structures under moving loads have received enormous attention [1]. Winkler foundations is mostly considered to represent the elastic foundation for mathematically simplicity [2]. This kind of model is considered as a system of mutually independent; linear springs and it is assumed the deflection of foundation at any point on the surface is directly proportional to stress. This does not accurately represent the continuous characteristics of practical foundations because the interaction between lateral spring is not taken into account. To find a foundation model that is both closer to physical reality and mathematically simple, researchers have developed various two parameter [3-8]. Uzzal, Bhat and Ahmed [9] presented the dynamic response of Euler-Bernoulli, beam, lying on the Pasternak foundation under a load. Cao and Zhong [10] investigated the dynamic response of Euler-Bernoulli beam on the Pasternak foundation subject of moving loads. They found that the maximum deflection of a beam resting on the two parameter foundation is much smaller than that of a beam on the Winkler foundation. Gbadeyan and Agboola [6] studied the dynamic behaviour of a double Rayleigh beam-system due to uniform partially distributed moving load. They observed that the maximum amplitude of deflection of the upper beam increase while the lower beam decrease as a result of the rotarory inertial [11]. Sapountzakis and Kampitsis [12] studied the nonlinear response of the beams resting on a nonlinear three- parameter foundation. They found the discrepancy between the results from the linear and non-linear analyses is remarkable. Kargarnovin, Younesian, Thompson *et al* [13] investigated the nonlinearity effects on the beams response and compared the results for a nonlinear and equivalent linear model. Yan, Hu Ding and Li-Qun [14] studied Dynamic response to a moving load of a Timoshenko beam restingon a nonlinear viscoelastic foundation, the Galerkin method and its convergence were studied for the response of a Timoshenko beam supported by a nonlinear foundation. In this paper, the rotatory inertia, the viscoelasticity and the nonlinearity of foundation are considered, as well as their effects, the dynamic response of Rayleigh beam on a nonlinear cubic Pasternak foundation with viscous damping are numerically determined via Maple software

2. GOVERNING EQUATION

Consider a Rayleigh beam resting on nonlinear viscoelastic foundation to partially distributed moving loads a shown in fig (1). F and V represent the magnitude of load and speed. V is assumed to be constant X and W are spatial coordinate along the x-axis of the beam and displacement function. The foundation is taken as a nonlinear Pasternak foundation with linear plus cubic stiffness and viscous damping as follows

$$P(x,t) = K_1 W(x,t) + K_3 W^3(x,t) - 2wb\frac{\partial W}{\partial t} \mp N\frac{\partial^2 W}{\partial x^2} \qquad (1)$$

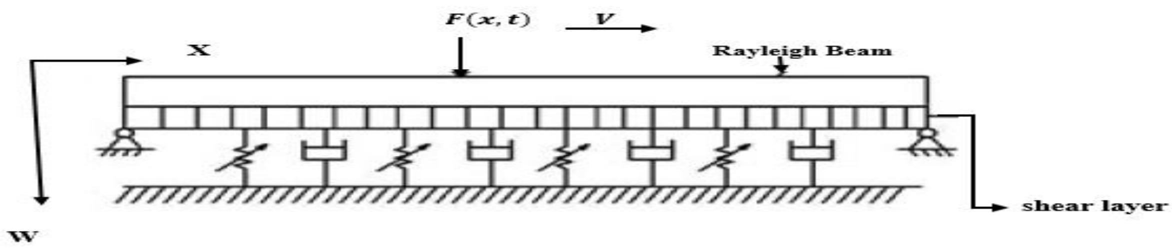


fig 1. The model of a finite Rayleigh beam on a nonlinear viscoelastic Elastic foundation

Fig. 1 The model of a finite Rayleigh beam on a nonlinear viscoelastic foundation

The governing equation of motion for the beam is given as

$$EI\frac{\partial^4 W}{\partial x^4} + \mu\frac{\partial^2 W}{\partial t^2} - R_0^2\frac{\partial^4 W}{\partial x^2 \partial t^2} + K_1 W + K_3 W^3 + 2\omega_b\frac{\partial W}{\partial t} \pm N\frac{\partial^2 W}{\partial x^2}$$
$$= Mg - M\left[\frac{\partial^2 W}{\partial t^2} - 2V\frac{\partial^2 W}{\partial x \partial t} + V^2\frac{\partial^2 W}{\partial x^2}\right]\left[H\left(x-\xi+\frac{\in}{2}\right) - H\left(x-\xi-\frac{\in}{2}\right)\right] \qquad (2)$$

Where E = modulus of elasticity, I= moment of inertial, μ =mass per unit length, R_O = rotary inertia, K_1 and K_3 are linear and nonlinear foundation parameters, ω_b and μ are damping coefficient, shear deformation and $\xi = vt + \frac{\in}{2}$

Subject to following boundary conditions

$$W(0,t) = W'' = (0,t) = 0, \quad W(L,t) = W'' = (L,t) = 0 \qquad (3)$$

and initial conditions is assumed as

$$W(x,0) = W(x,0) = 0 \qquad (4)$$

Solution Technique

(3) Normal mode.

The harmonic equation is assumed in the form.

$$W = (x,t) = \sum \psi_k e^{i\omega_\kappa t} \qquad (5)$$

To obtain a Rayleigh beam equation for free vibration

$$\frac{\partial^4 \psi}{\partial x^4} - a\frac{\partial^2 \psi}{\partial x^2} + b\psi = 0 \qquad (6)$$

Where $a = \frac{R_0^2 \omega_k^2 \mp \mathrm{N}}{EI}$, $b = \frac{-\mu\omega_k^2 + k_1 + 2i\omega_\kappa \omega_b}{EI}$

On solving equation (6) two possible solutions can be obtained for b$<$ 0 and b>0

$$\psi(x) = C_1 \cosh \lambda_1 x + C_2 \sinh \lambda_2 x + C_3 \cos \lambda_3 x + C_4 \sin \lambda_4 x \qquad (7)$$

By applying the boundary conditions in (3) to (7)gives

$$\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\left[1 \pm \frac{N}{EI}\left(\frac{L}{i\pi}\right)^2 + \frac{K_1 L^4}{EIi^4\pi^4}\right]^{1/4} \frac{i\pi x}{L}\right) \qquad (8)$$

To reduce the fourth order partial differential equation in (2) to second order ordinary differential equation, the solution is then assumed as

$$W(x,t) = \sum_{i=1}^{n} W_i(x) T_i(t) \qquad (9)$$

Substitute equation (9) together with the orthogonality property into equation (2) to get;

$$EI(x)\sum_{i=1}^{n} W_i^{iv}(x)T_i(t) \pm N\sum_{i=1}^{n} W_i^{''}(x)T_i(t) + m(x)\sum_{i=1}^{n} W_i(x)\ddot{T}_i(t) - R_0^2\sum_{i=1}^{n} W_i^{''}(x)\ddot{T}_i(t)$$

$$+K_3\left(\sum_{i=1}^{n} W_i(x)T_i(t)\right)^3 + K_3\left(\sum_{i=1}^{n} W_i(x)T_i(t)\right)^3 + 2\omega_b\sum_{i=1}^{n} W_i(x)\dot{T}_i(t)$$

$$= \left(Mg - M\sum_{i=1}^{n} W_i(x)\ddot{T}_i(t) + 2MvW_i^{'}(x)\dot{T}_i(t) - mv^2 W_i^{''}(x)T_i(t)\right)\left(H(x-\xi+\frac{\varepsilon}{2}) - H(x-\xi-\frac{\varepsilon}{2})\right) \qquad (10)$$

Multiply (10) by $W_j(x)$ and integrate along the length of the beam and apply the orthogonality condition then, equation (10) becomes

$$EI\sum_{i=1}^{n}\int_0^L W_i^{iv}(x)T_i(t)W_j(x)dx \pm N\sum_{i=1}^{n}\int_0^L W_i^{''}(x)T_i(t)W_j(x)dx + \mu\sum_{i=0}^{n}\int_\circ^L W_i(x)\ddot{T}_i(t)W_j(x)dx$$

$$-R_0^2\sum_{i=0}^{n}\int_\circ^L W_i^{''}(x)\ddot{T}(t)W_j(x)dx + K_1\sum_{i=0}^{n}\int_\circ^L W_i(x)T_i(t)W_j(x)dx + K_3\int_0^L\left(\sum_{i=0}^{n} W_i(x)T_i(t)\right)^3 W_j(x)dx$$

$$+2\omega_b\sum_{i=1}^{n}\int_0^L W_i(x)\dot{T}_i(t)W_j dx = Mg\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}} W_j(x)dx - M\sum_{i=1}^{n}\ddot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}} W_i(x)W_j(x)dx$$

$$-2MV\sum_{i=1}^{n}\dot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}} W_i^{'}(x)W_j(x)dx - MV^2\sum_{i=1}^{n} T_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}} W_i^{''}(x)W_j(x)dx. \qquad ((11))$$

For free vibration, $EIW^{iv} = \mu\omega^2 W$ so that equation (10) becomes

$$\mu\omega^2 T_i(t) \pm NT_i(t) + \mu\ddot{T}_i(t) + K_1 T_i(t) - R_0^2\ddot{T}_i(t) + 2\omega_b\dot{T}_i + K_3\int_0^L\left(\sum_{i=0}^{n} W_i(x)T_i(t)\right)^3 W_j(x)dx = Mg\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}} W_j(x)dx$$

$$-M\sum_{i=1}^{n}\ddot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}} W_i(x)W_j(x)dx - 2Mv\sum_{i=1}^{n}\dot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}} W_i^{'}(x)W_j(x)dx - Mv^2\sum_{i=1}^{n} T_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}} W_i^{''}(x)W_j(x)dx \qquad (12)$$

For simply supported with respect to elastic foundation, the displacement is give

$$W_i(x)=\sqrt{\frac{2}{L}}\sin\left(\left[1\pm\frac{N}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{K_1L^4}{EIi^4\pi^4}\right]^{\frac{1}{4}}\frac{i\pi x}{L}\right) \tag{13}$$

Differentiate (12) and substitute in (11) and applying trigonometry identity to have

$$\begin{aligned}
&\left(m-R_0^2\right)\ddot{T}_i+2\omega_b\dot{T}+\left(m\omega^2\pm N+K_1\right)T_i+K_3\int_0^L\left(\sum_0^n\sin\left(P\frac{i\pi x}{L}\right)T(t)\right)^3\sin\left(Q\frac{j\pi x}{L}\right)dx\\
&=\left[\frac{2Mg\sqrt{2L}}{Qmj\pi}\left(\left(\frac{L}{Qj\pi}\right)\cos\left(Qj\frac{\pi}{L}(\xi+\frac{\varepsilon}{2})\right)-\cos\left(Qj\frac{\pi}{L}(\xi-\frac{\varepsilon}{2})\right)\right)\right]\\
&+\left(\frac{M}{Pm\pi}\right)\sum_{i=1}^n\ddot{T}_i(t)\left[\begin{array}{l}\left(\frac{1}{(i+j)}\sin P\frac{\pi}{L}(i+j)(\xi+\frac{\varepsilon}{2})-\frac{1}{(i-j)}\sin Q\frac{\pi}{L}(i-j)(\xi+\frac{\varepsilon}{2})\right)\\-\left(\frac{1}{(i+j)}\sin\left(P\frac{\pi}{L}(i+j)(\xi-\frac{\varepsilon}{2})\right)-\frac{1}{(i-j)}\sin\left(Q\frac{\pi}{L}(i-j)(\xi-\frac{\varepsilon}{2})\right)\right)\end{array}\right]\\
&+\frac{2MiV}{mL}\sum_{i=1}^n\dot{T}_i(t)\left[\begin{array}{l}\left(\frac{1}{(i+j)}\cos\left(\frac{p\pi}{L}(i+j)(\xi+\frac{\varepsilon}{2})\right)-\frac{1}{(i-j)}\cos\left(Q\frac{\pi}{L}(i-j)(\xi+\frac{\varepsilon}{2})\right)\right)\\-\left(\frac{1}{(i+j)}\cos\left(\frac{p\pi}{L}(i+j)(\xi-\frac{\varepsilon}{2})\right)-\frac{1}{(i-j)}\cos\left(Q\frac{\pi}{L}(i-j)(\xi-\frac{\varepsilon}{2})\right)\right)\end{array}\right]\\
&-\frac{MV^2Pi^2}{mL^2}\sum_{i=1}^n T_i(t)\left[\begin{array}{l}\left(\frac{1}{(i+j)}\sin\left(\frac{p\pi}{L}(i+j)(\xi+\frac{\varepsilon}{2})\right)-\frac{1}{(i-j)}\sin\left(Q\frac{\pi}{L}(i-j)(\xi+\frac{\varepsilon}{2})\right)\right)\\-\left(\frac{1}{(i+j)}\sin\left(\frac{p\pi}{L}(i+j)(\xi-\frac{\varepsilon}{2})\right)-\frac{1}{(i-j)}\sin\left(Q\frac{\pi}{L}(i-j)(\xi-\frac{\varepsilon}{2})\right)\right)\end{array}\right]
\end{aligned} \tag{14}$$

where $P=\left[1\pm\frac{N}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{K_1L^4}{EIi^4\pi^4}\right]^{\frac{1}{4}}$ $Q=\left[1\pm\frac{N}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{K_1L^4}{EIi^4\pi^4}\right]^{\frac{1}{4}}$

further simplification of (14)resulted into

$$\begin{aligned}
&\left(m-R_0^2\right)\ddot{T}_i+2\omega_b\dot{T}+\left(m\omega^2\pm N+K_1\right)T_i+K_3\int_0^L\left(\sum_0^n\sin\left(P\frac{i\pi x}{L}\right)T(t)\right)^3\sin\left(Q\frac{j\pi x}{L}\right)dx\\
&=\frac{Mg\sqrt{8L}}{Qmj\pi\varepsilon}\left(\sin\left(Qj\frac{\pi\xi}{L}\right)\sin\left(Qj\frac{\pi\varepsilon}{2L}\right)\right)\\
&+\frac{M}{Pm\varepsilon\pi}\sum_{i=1}^n\ddot{T}_i(t)\left[\left(\frac{1}{(i-j)}\cos Q\frac{\pi}{L}(i-j)\xi\sin P\frac{\pi}{L}(i-j)\frac{\varepsilon}{2}\right)-\left(\frac{1}{(i+j)}\cos Q\frac{\pi}{L}(i+j)\xi\sin P\frac{\pi}{L}(i+j)\frac{\varepsilon}{2}\right)\right]\\
&+\frac{2MiV}{\varepsilon mL}\sum_{i=1}^n\dot{T}_i(t)\left[\left(\frac{1}{(i-j)}\cos Q\frac{\pi}{L}(i-j)\xi\sin P\frac{\pi}{L}(i-j)\frac{\varepsilon}{2}\right)-\left(\frac{1}{(i+j)}\cos P\frac{\pi}{L}(i+j)\xi\sin Q\frac{\pi}{L}(i+j)\frac{\varepsilon}{2}\right)\right]\\
&-\frac{MV^2Pi^2}{mL^2\varepsilon}\sum_{i=1}^n T_i(t)\left[\left(\frac{1}{(i-j)}\cos Q\frac{\pi}{L}(i-j)\xi\sin P\frac{\pi}{L}(i-j)\frac{\varepsilon}{2}\right)-\left(\frac{1}{(i+j)}\cos P\frac{\pi}{L}(i+j)\xi\sin Q\frac{\pi}{L}(i+j)\frac{\varepsilon}{2}\right)\right]
\end{aligned} \tag{14}$$

3. NUMERICAL ANALYSES AND RESULT DISCUSSION

Here, two cases were considered which are the moving force and moving load

Case A: The moving force

In this case he inertia effect (Coriolis, centripetal and traversed forces) were neglected in (14) yielded (15)

$$\left(m - R_0^2\right)\ddot{T}_i + 2\omega_b \dot{T} + \left(m\omega^2 \pm N + K_1\right)T_i + K_3 \int_0^L \left(\sum_0^n \sin\left(P\frac{i\pi x}{L}\right)T(t)\right)^3 \sin\left(Q\frac{j\pi x}{L}\right)dx$$

$$= \frac{Mg\sqrt{8L}}{Qmj\pi}\left(\sin\left(Qj\frac{\pi\xi}{L}\right)\sin\left(Qj\frac{\pi\varepsilon}{2L}\right)\right) \qquad (15)$$

Case B: The moving mass or load

In this case all the forces were taken into consideration in other wordthe moving forceand the loads were considered given equation (14)

Equations (14) and (15) were solved numerically using MAPLE softwarethe following numerical values $\xi = vt + \frac{\varepsilon}{2},\ \varepsilon = 0.1,\ \lambda = \frac{n\pi}{L}, \pi = 3.142,\ \omega = \frac{\sqrt{\lambda^2 EI}}{m}, L = 10,\ E = 2.07\times10^{11}, I = 2.07\times10^{-6},$ $M = 100, m = 30$ to obtain the displacement for the moving force and the moving mass w

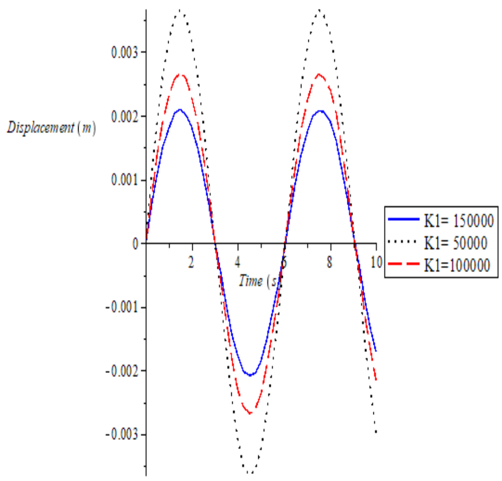


Figure 1a:Displacement against time for moving load for different values of K1 for compressive force

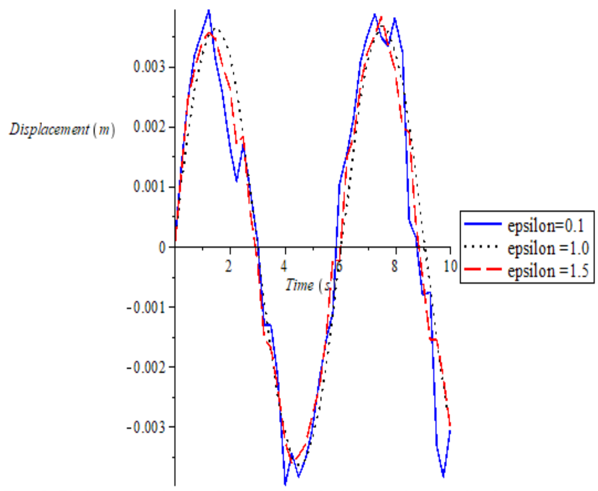


Figure2a:Displacement against time for moving loads for different values of epsilon for compressive force

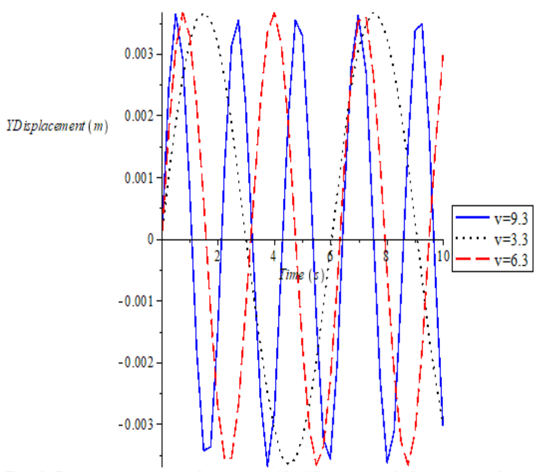


Figure3a:Displacement against time for moving load for different values of v for compressive force

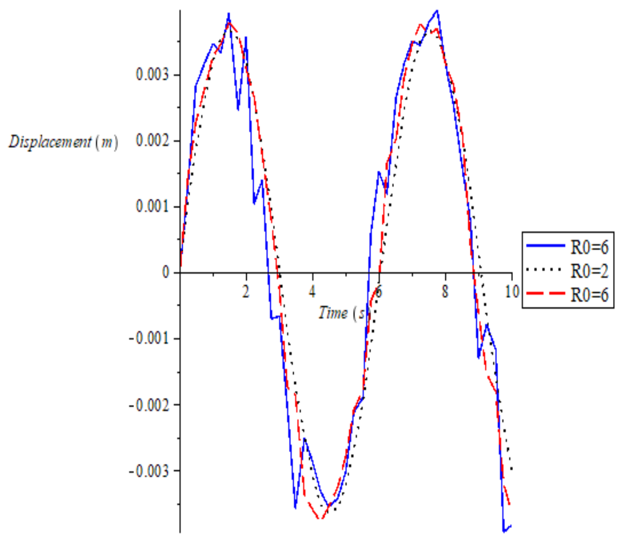
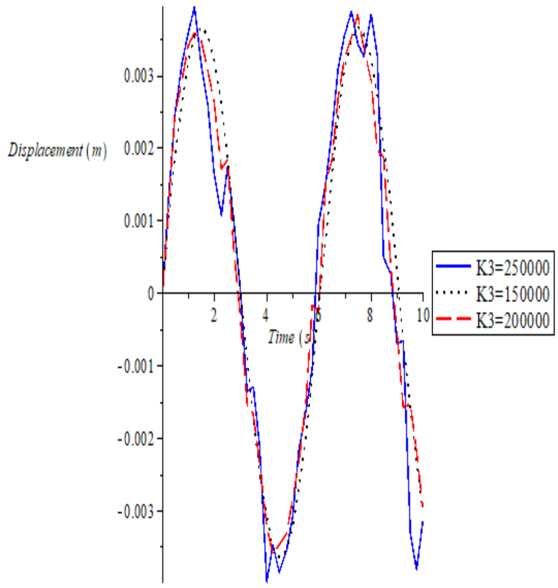


Figure4a:Displacement against time for moving load for different values of R0 for compressive force



Figur5a: Displacement against time for moving load for different values of K3 for compressive force

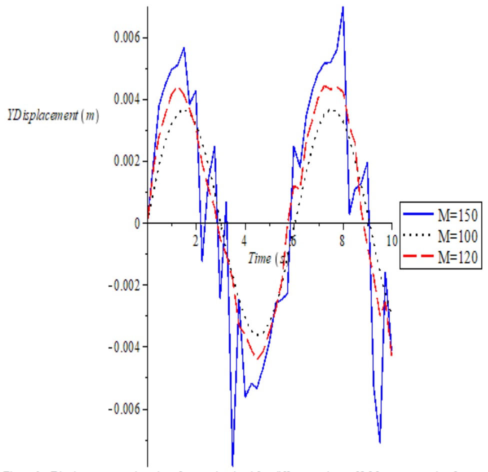


Figure6a: Displacement against time for moving load for different values of M for compressive force

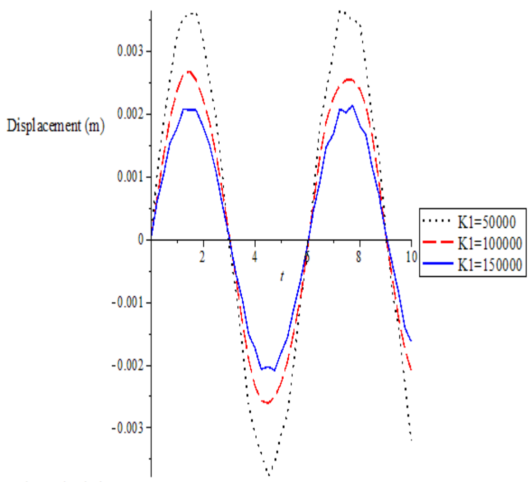


Figure 1b:Displacement against time for moving force different values of K1 for compressive force

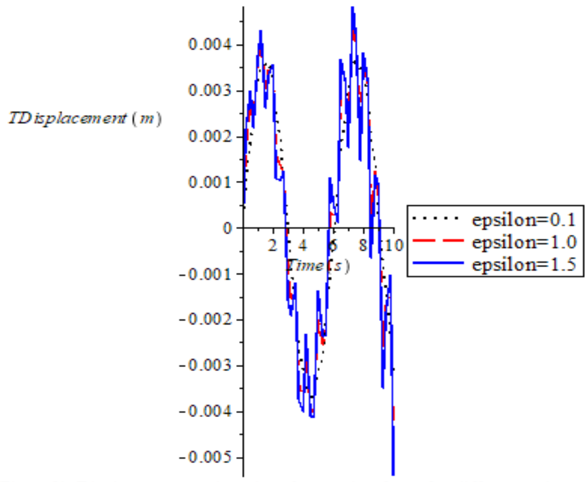


Figure2b:Displacement against time for moving force for different values of epsilon for compressive force

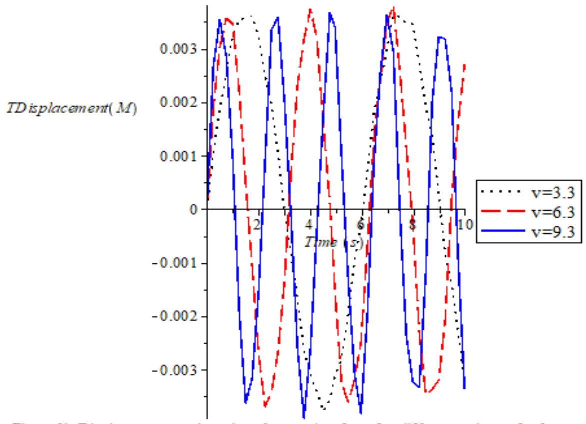


Figure3b:Displacement against time for moving forcefor different values of v for compressive force

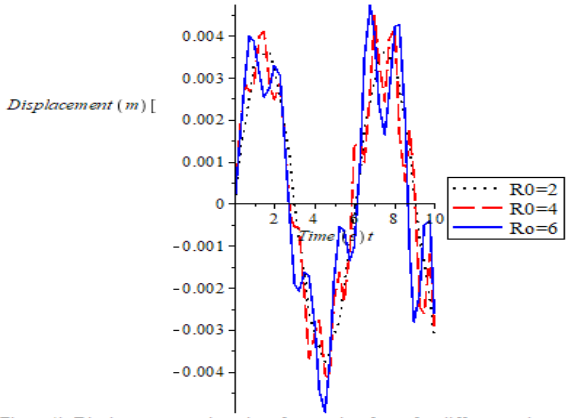


Figure4b:Displacement against time for moving force for different values of R0 for compressive force

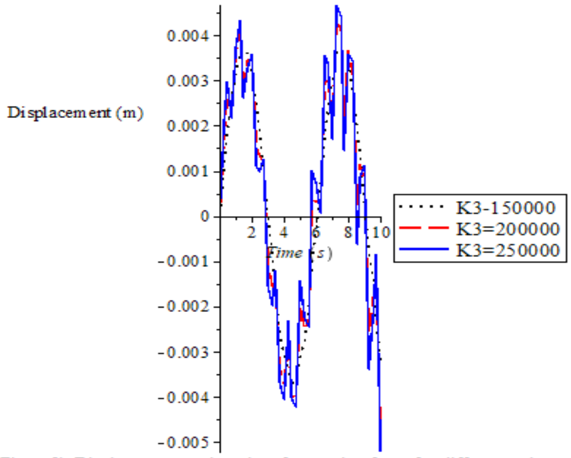


Figure5b:Displacement against time for moving force for different values of K3 for compressive force

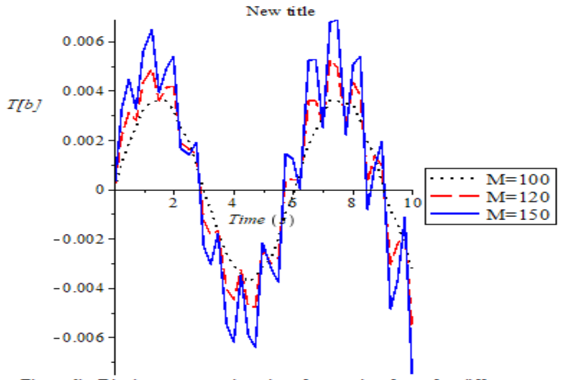


Figure6b: Displacement against time for moving force for different values of M for compressive force

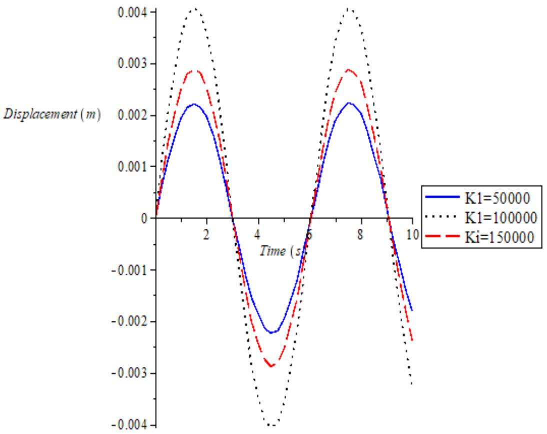


Figure1c: Displacement against time for moving loads for different values of K1 for ttensile force

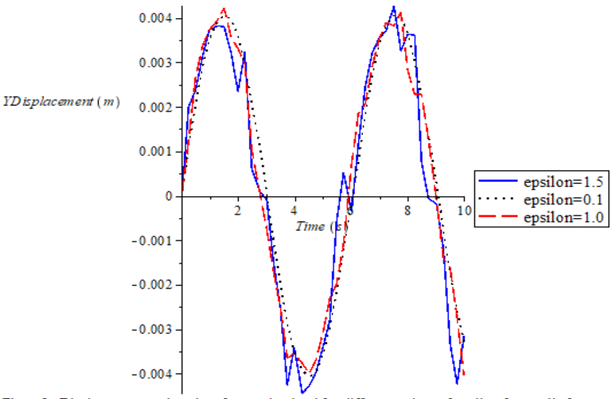


Figure2c:Displacement against time for moving load for different values of epsilon for tensile force

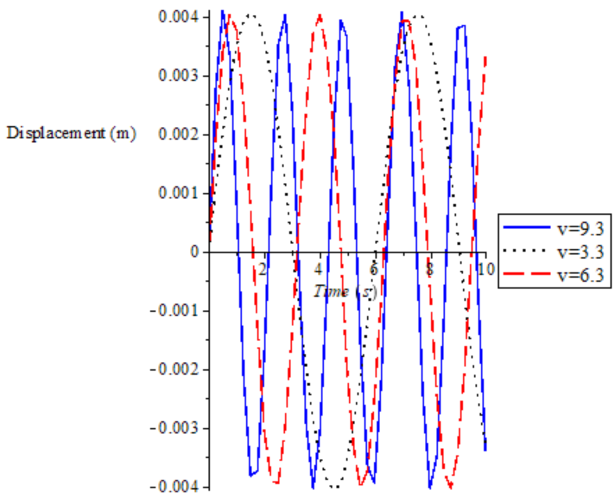


Figure3c:Displacement against time for moving load for different values of v for tensile force

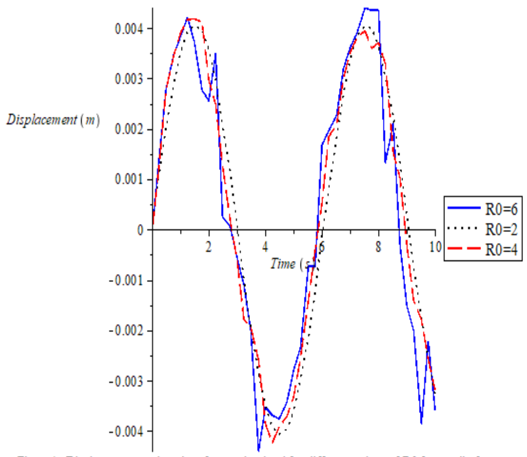
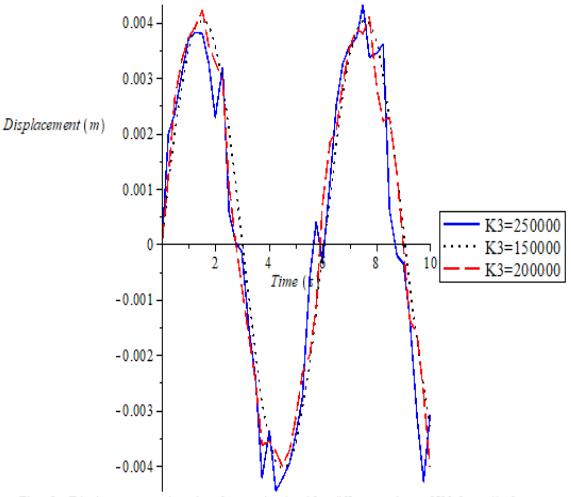


Figure4c:Displacement against time for moving load for different values of R0 for tensile force



Figur5c: Displacement against time for moving load for different values of K3 for tesile force

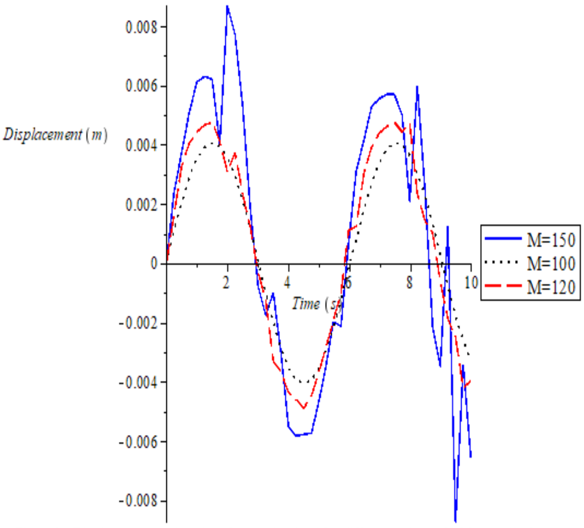


Figure6c: Displacement against time for moving load for different values of M for tensile force

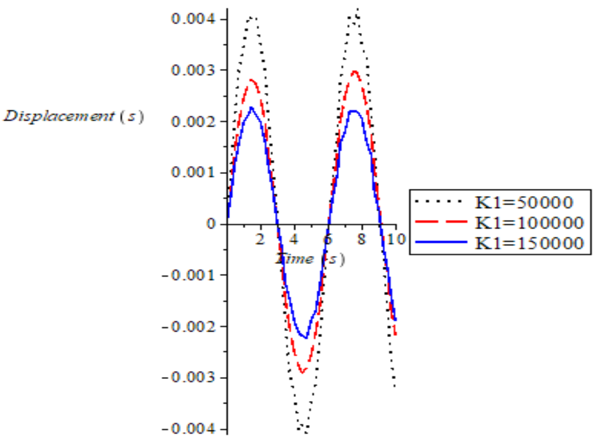


Figure1d: Displacement against time for moving force for different values of K1 for tensile force

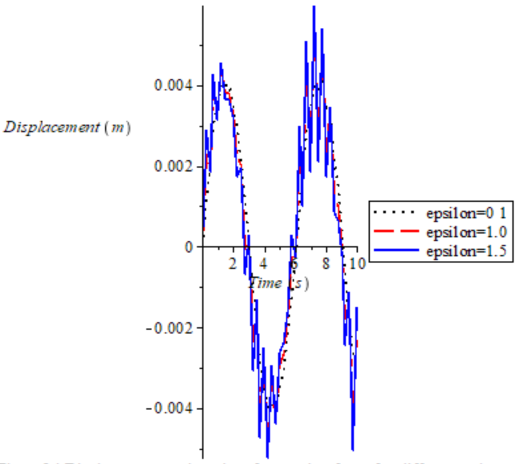


Figure2d:Displacement against time for moving force for different values of epsilon for tensile force

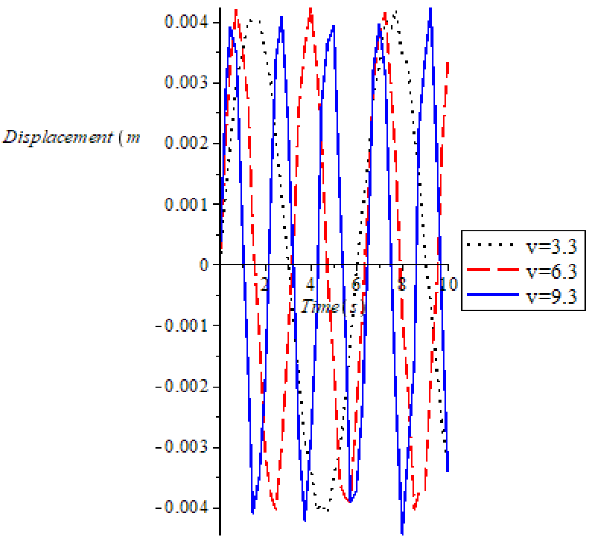


Figure3d:Displacement against time for moving force for different values of v for tensile force

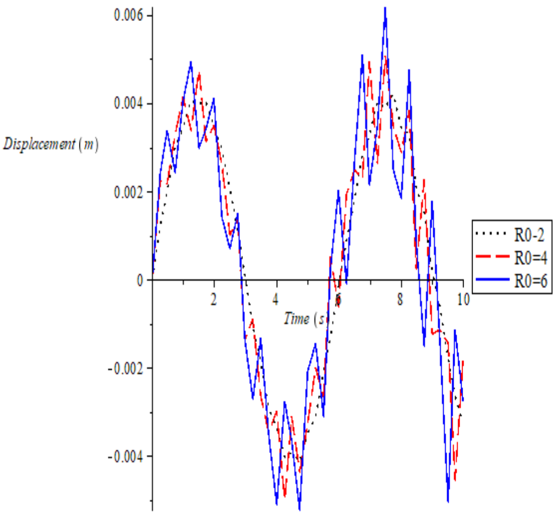
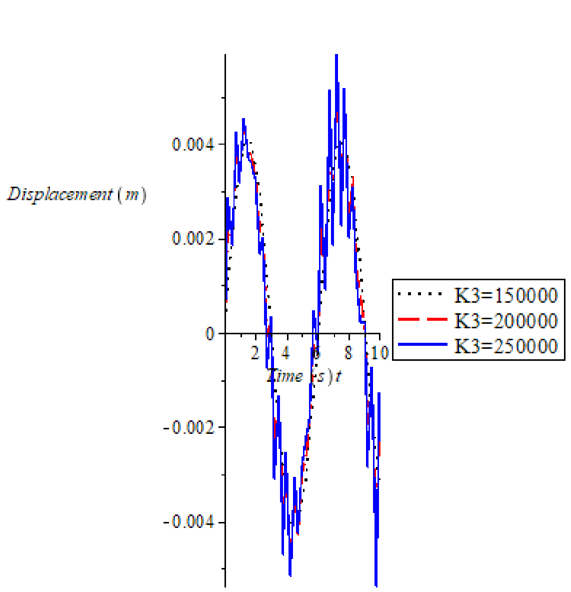


Figure4d:Displacement against time for moving forcefor different values of R0 for tensile force



Figur5d: Displacement against time for moving force for different values of K3 for tensilee force

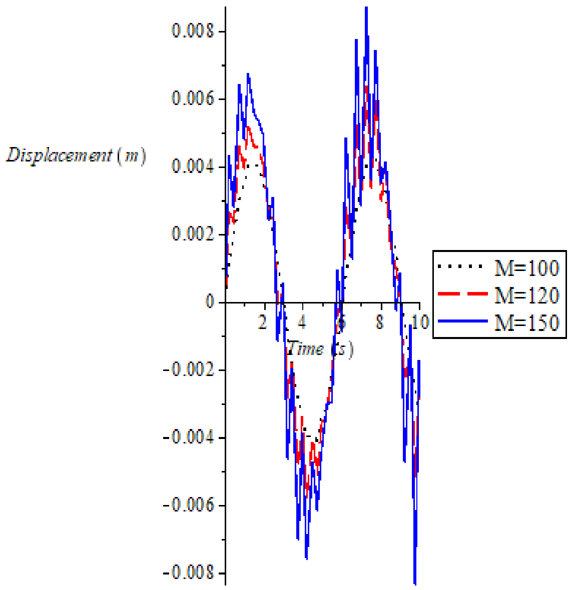


Figure6d: Displacement against time for moving force for different values of M for tensile force

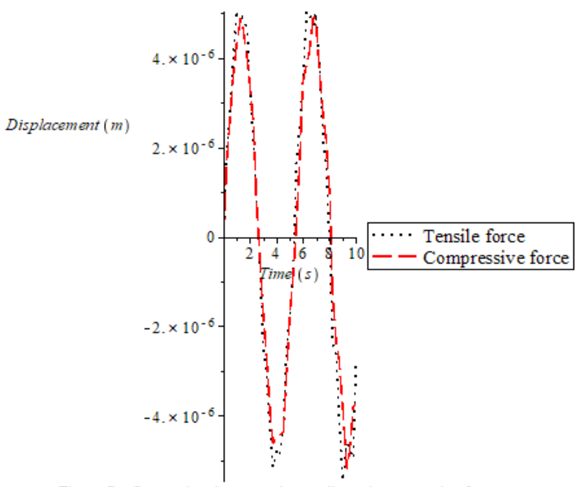


Figure 7a: Comparism between the tensile and compressive forces

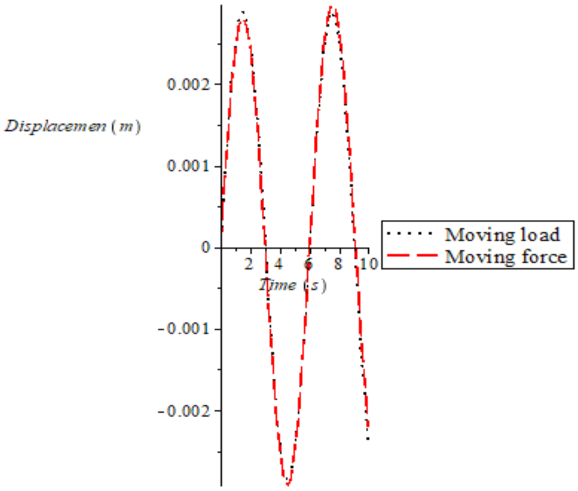


Figure 7b: Comparism between moving load and force

Figures 1a-6a and 1b-6b are graphs of displacement against time for moving loadand moving force with compressive force for different values of K_1, ε, V, K_3,N, R_0 and M. Figures 1c-6c and 1c-6d are graphs of displacement against time for movingloads and moving force with tensile force for different values of K_1, ε, V, K_3,N, R_0 and M and figures 7a is the figure 7a for comparism for tensile and compressive forces between the moving force and moving load.

CONCLUSION

Thedynamic response of Rayleigh beam with axial force to partially distributed moving loads resting on linear and nonlinear viscoelastic foundation was investigated. The behaviours of the system under consideration was determined by first reduced the governing fourth order partial differential equation for moving loads to second order differential equation by assume a solution in form of series solution and numerical method using maple software. The effect of axial force, elastic foundation both (linear and nonlinear), length of the beam velocity, radius of gyration and mass of the beam on both the moving force and the moving mass were examined with respect to both the tensile and compressive forces.It was observed that as the mass Mof the beam increases, displacement is equally increases for both the moving force and the moving mass with respect to the tensile force and the compressive force. It was also observed that as the axial force N increases, the displacement decreases for both the moving force and moving mass for only the case of tensile force while in the case of compressive force, the displacement increases as the axial force increases for both the moving force and the moving mass, the displacement increases as the length of the beam ε and velocity v increases for both the moving force and the moving mass

REFFERENCES

[1]. Beskou, N.D., Theodorakopoulos, D.D. (2011): Dynamic effects of moving loads on road pavements: A review. Soil Dynamics and Earthquake Engineering 31, 547–567.

[2]. Winkler, E. (1867): Die Lehre von der Elustizitat und Festigkeit. Dominicus, Prague.

[3]. Kerr, A.D.: Elastic and viscoelastic foundation models. Journal of Applied Mechanics 31, 491–498 (1964).

[4]. Feng, Z.H., Cook, R.D. (1983): Beam element on two-parameter elastic foundation. Journal of Engineering Mechanics Division109, 1390–1402.

[5]. Gbadeyan, J.A. and Dada, M.S. (2007); The Effects of Linearly Varying Distributed Moving Loads on Beams; Journal of Engineering and Applied Sciences 2(6): pp 1006-1011.

[6]. Gbadeyan, J.A and Agboola, O.O. (2012); Dynamic Behaviour of a double Rayleigh Beam-System due to uniform partially distributed Moving Load. Journal of Applied Sciences Research, 8(1): pp. 571-581.

[7]. Akinpelu, F.O. and Sangoniyi, S.O. (2015) Dynamics Response of simply supported axial force Euler-Bernoulli beam subjected to partially distributed moving load, International Journal of Innovation in science and Mathematics. Volume 3, Issue 2, ISSN (Online): pp 91 – 100.

[8]. Akinpelu, F.O. (2012) The response of viscously Damped Euler-Bernoulli beam to uniform partially distributed moving loads (http://www.SCiRP.org/journal/am). Applied Mathematics, 2012, 3, 199-204. Dio:10.4236/am.2012.33031.

[9]. Uzzal, R.U.A., Bhat, R.B., Ahmed, W. (2012). Dynamic response of a beam subjected to moving load and moving mass supported by Pasternak foundation. Shock and Vibration 19, 205–220.

[10]. Cao, C.Y., Zhong, Y. (2008): Dynamic response of a beam on a Pasternak foundation and under a moving load. Journal of Chongqing University 7, 311–316.

[11]. Wu, T.X., Thompson, D.J.(2004): The effects of track nonlinearity on wheel/rail impact. ProcInstMech Eng. J. Rail Rapid Transit (218, 1–16

[12]. Sapountzakis, E.J., Kampitsis, A.E. (2011): Nonlinear response of shear deformable beams on tensionless nonlinear viscoelastic foundation under moving loads. Journal of Sound and Vibration 330, 5410–5426.

[13]. Kargarnovin, M.H., Younesian, D., Thompson, D.J. (2005): Response of beams on nonlinear viscoelastic foundations to harmonic moving loads. Computers and Structures 83, 1865–1877.

[14]. Yan Yang, Hu Ding and Li-Qun Chen (2013): Dynamic response to a moving load of a Timoshenko beam resting on a nonlinear viscoelastic foundation Acta Mechanica Sinica 29(5):718–727 DOI 10.1007/s10409-013-0069-3.