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Numerical Investigation on Unsteady Casson fluid flow along a Vertical Porous Plate with Hall Current and Ion-slip effect in a Rotating System

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ABSTRACT

This article deals Hall and Ion-slip currents effect on unsteady MHD viscous incompressible electrically conducting Casson fluid along a vertical porous plate under a strong transverse magnetic field with rotational system. The governing equations are derived from Navier-Stokes equation, Energy equation and Concentration equation and boundary layer approximation has been employed. A similarity analysis introduces to obtain non-dimensional form of these governing equations. The resulting non-linear coupled partial differential equations are solved by using the explicit finite difference method. Matlab tool has been used for numerical simulation. The effect of the pertinent parameters on primary velocity, secondary velocity, temperature and concentration distributions are discussed in details as well as graphically.

Key words: MHD fluid, Heat transfer, Mass transfer, Porous medium, Rotating system, Hall current and ion-slip

INTRODUCTION

In a Newtonian fluid model, many flow characteristics are not understandable, so non-Newtonian fluid become most useful in MHD fluid flow study. Some examples of non-Newtonian fluids are salt solutions, molten polymers, ketchup, jelly, honey, soup, custard, toothpaste, starch suspensions, paints, blood, shampoo, concentrated fruit juices etc.Non-Newtonian fluid exerts a relationship between the shears tress and rate of shear strain. A shear thinning liquid that has an infinite viscosity at zero rates of shear, a yield stress below it has no flow occurs and a zero viscosity at an infinite rate of shear. Casson fluid is one of such fluids. The flow of Casson fluids in the presence of heat transfer is widely used inthe processing of chocolate, foams, syrups, toffee and many other food processing industries. Hall current and Ion-slip effects on MHD Casson fluid flow is also significant in many cases such as mechanical engineering, industrial engineering especially in the extraction of crude oil from petroleum products and polymer processing. Casson [1] was the first who introduced this model on a flow equation for pigment-oil suspensions of printing ink type. Later many other researchers have studied on the effect of transversely applied magnetic field with hall current and ion-slip with an electrically conducting fluid past a porous medium. Abid Hussanan *et al* [2] have studied analytically on theHeat Transfer in Magnetohydrodynamic Flow of a Casson Fluid with Porous Medium and Newtonian Heating. The Hall Effect in the Viscous Flow of Ionized Gas Between two Parallel Plates under TransverseMagnetic Field are investigated by Sato, H. [3]. Dash, R.K. *et al* [4] discussed on Casson Fluid Flow in a Pipe Filled with a Homogeneous Porous Medium. Hayat, T. *et al* [5] analyzed the Soret and Dufour Effects on Magnetohydrodynamic (MHD) Flow of Casson Fluid. Eldabe, N.T.M. and Salwa, M.G.E. [6] considered the Heat transfer of MHD Non-Newtonian Casson Fluid Flow between two Rotating Cylinders. Hydrodynamic Impulsively Lid-Driven Flow and Heat Transfer of a Casson Fluid has described by H. A. Attia and M. E. Sayed-Ahmed [7]. K. Ramesh and M. Devakar [8] have performed on some analytical solutions for flows of Casson fluid with slip boundary conditions. PS Gupta and AS Gupta [9] investigated on

Heat and Mass Transfer on a Stretching Sheet with Suction or Blowing. S. Pramanik [10] discussed on the Casson fluid flow and heat transfer past an exponentially porous stretching surface in presence of thermal radiation. C. S. K. Raju *et al* [11] are analyzed on the Magnetohydrodynamic Casson Fluid over an Exponentially Permeable Stretching Surface. Casson Fluid Flow near the Stagnation Point over a Stretching Sheet with Variable Thickness and Radiation are investigated by G. K. Ramesh *et al* [12]. Heat Transfer of MHD Flow of Casson Fluid due to Stretching Sheet with PST and PHF Heating Conditions have been reported by Mahantesh M *et al* [13]. R. Vijayaragavan and S. Karthikeyan [14] have studied the Heat and Mass Transfer in Radiative Casson Fluid Flow Caused by a Vertical Plate with Variable Magnetic Field Effect. MHD Flow of Casson Fluid with Slip Effects over an Exponentially Porous Stretching Sheet in Presence of Thermal Radiation, Viscous Dissipation and Heat Source/Sink have discussed by N. Saidulu and A. Venkata Lakshmi [15]. Also N. Saidulu and A. Venkata Lakshmi [16] have studied on the Slip Effects on MHD Flow of Casson Fluid over an Exponentially Stretching Sheet in Presence of Thermal Radiation. M. Eswara Rao and S. Sreenadh [17] studied the MHD Flow of a Casson Fluid over an Exponentially Inclined Permeable Stretching Surface with Thermal Radiation, Viscous Dissipation and Chemical Reaction. G. Sarojamma *et al* [18] have investigated on the MHD Casson Fluid Flow, Heat and Mass Transfer in a Vertical Channel with Stretching Walls. Soret and Dufour Effects on MHD Casson Fluid Over a Vertical Plate in Presence of Chemical Reaction and Radiation has discussed by N. Ananda Reddy and K. Janardhan [19]. J. Prakash *et al* [20] have presented on the Heat and Mass Transfer Hydromagnetic Radiative Casson Fluid Flow over an Exponentially Stretching Sheet with Heat Source/Sink. K. Pushpalatha *et al* [21] investigated on Heat and Mass Transfer in Unsteady MHD Casson Fluid Flow with Convective Boundary Conditions. Hari R. Katariaa and Harshad R. Patel [22] studied on Heat and mass transfer in magnetohydrodynamic (MHD) Casson fluid flow past over an oscillating vertical plate embedded in porous medium with ramped wall temperature. Sunita Choudhary and Mamta Goyal [23] have studied on Unsteady Mhd Casson Fluid Flow Through Porous Medium With Heat Source/Sink And Time Dependent Suction. Enhanced heat transfer in the flow of dissipative non-Newtonian Casson fluid flow over a convectively heated upper surface of a paraboloid of revolution has examined by J.V. Ramana *et al* [24]. S.S. Ghadikolaei *et al* [25] have investigated on Nonlinear thermal radiation effect on magneto Casson nanofluid flow with Joule heating effect over an inclined porous stretching sheet. R.Vijayaragavan and S.Karthikeyan are analyzed the Hall Current Effect on Chemically Reacting MHD Casson Fluid Flow with Dufour Effect and Thermal Radiation.
Hence our present investigation is to study that the "Numerical Investigation on Unsteady Casson fluid flow along a Vertical Porous Plate with Hall Current and Ion-slip effect in a Rotating System". The governing equation is concerned with the Hall current, Ion-slip effects including the Joule heating, thermal radiation, heat source and viscous dissipation. The non-linear coupled partial differential equations are very difficult to find its exact solution analytically. Hence to find an approximate solution we adopt a numerical method for this problem. We have used the explicit finite difference technique to solve the non-dimension governing equations. The results are shown graphically and discussed in detail for the velocity, temperature and concentration distribution with respect to pertinent parameters.

MATHEMATICAL FORMULATION

We consider an infinite electrically non-conducting porous plate is fixed in a laminar, viscous, incompressible, electrically conducting Casson fluid at $y=0$ along the vertically upward direction of x-axis taking with Hall current and Ion-slip into account and y-axis is normal to it. Initially the plate and fluid are at rest. Instantaneously at time $t>0$ the plate is to be move with a uniform velocity U_0. Due to the skin friction the velocity at the wall is U_0 and outside of the boundary layer is zero. Also the temperature of the plate and the concentration are raised from T_w to T_∞ and C_w to C_∞, where T_w, C_w are the species temperature and concentration at the wall also T_∞, C_∞ are the temperature and concentration outside of the boundary layer respectively. Due to the effect of Hall current and Ion-slip, the generalized Ohm's law may stated as follows:

$$\mathbf{J}+\frac{\beta_e}{B_0}(\mathbf{J}\wedge\mathbf{B})=\sigma(\mathbf{q}\wedge\mathbf{B})+\frac{\omega_e\tau_e\beta_i}{B_0^2}(\mathbf{J}\wedge\mathbf{B})\wedge\mathbf{B}$$

where $\mathbf{B}$, $\mathbf{J}$, $\mathbf{q}$, σ, β_i, ω_e, τ_e are the magnetic field vector, the current density vector, the fluid velocity vector, the conductivity of the fluid, the Hall parameter,cyclotron frequency and electron collision time respectively. Here $\omega_e\tau_e=\beta_e$ may treat as Hall parameter. In Cartesian coordinate $\mathbf{B}=\hat{i}B_x+\hat{j}B_y+\hat{k}B_z$, $\mathbf{J}=\hat{i}J_x+\hat{j}J_y+\hat{k}J_z$ and $\mathbf{q}=\hat{i}u+\hat{j}v+\hat{k}w$. A constant magnetic field $\mathbf{B}$ is applied to the plate acting along the y-axis so that $\mathbf{B}=(0,B_0,0)$. It is assume that the plate is extended to infinite length, then all the physical variables in the problem are function of y and t alone.Therefore the equation of continuity gives $v=-v_0$ everywhere in the flow. The Physical model and Coordinate systemis shown in the following Fig. 1.

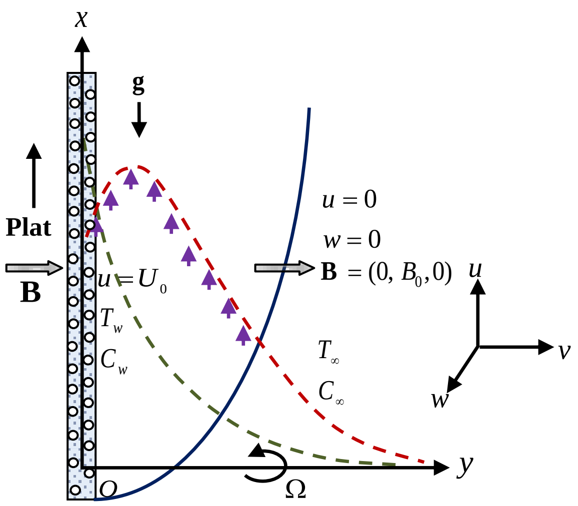

Fig. 1 Physical model and Coordinate System

The rheological equation of state for the Cauchy stress tensor of an isotropic and incompressible flow of a Casson fluid is written as

$$\tau_{ij} = \begin{cases} 2\left(\mu_b + \dfrac{p_y}{\sqrt{2\pi}}\right)e_{ij} & \text{when } \pi > \pi_c \\ 2\left(\mu_b + \dfrac{p_y}{\sqrt{2\pi_C}}\right)e_{ij} & \text{when } \pi < \pi_c \end{cases}$$

where τ is the shear stress, μ_b is known as plastic viscosity of the non-Newtonian fluid, p_y is known as yield stress of the fluid, mathematically expressed as $p_y = \mu_b\sqrt{2\pi}/\beta$, where β is the parameter of the casson fluid. $\pi = e_{IJ}e_{IJ}$, (i.e. π is the product of the component of deformation rate with itself, here e_{IJ} is the (i, j)th component of deformation rate), π_c is the critical value based on the non-Newtonian model. Dynamic viscosity of casson fluid is defined as $\mu = \mu_b + p_y/\sqrt{2\pi}$ so that kinematic viscosity may define as $\upsilon = \upsilon_b(1+1/\beta)$ where $\upsilon_b = \mu_b/\rho$. According to the above-mentioned assumptions along with the Boussinesq and boundary layer approximations, the governingequations of Casson fluid and generalized Ohm's law with Hall current and ion-slip influence are given by

$$\frac{\partial u}{\partial t} - v_0\frac{\partial u}{\partial y} = g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty) + \upsilon_b\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}\frac{(1+\beta_e\beta_i)u + \beta_e w}{(1+\beta_e\beta_i)^2 + \beta_e^2} - 2\Omega_0 w \tag{1}$$

$$\frac{\partial w}{\partial t} - v_0\frac{\partial w}{\partial y} = \upsilon_b\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 w}{\partial y^2} + \frac{\sigma B_0^2}{\rho}\frac{\beta_e u - (1+\beta_e\beta_i)w}{(1+\beta_e\beta_i)^2 + \beta_e^2} + 2\Omega_0 u \tag{2}$$

$$\frac{\partial T}{\partial t} - v_0\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} + \frac{\sigma B_0^2}{\rho C_p}\frac{u^2 + w^2}{(1+\beta_e\beta_i)^2 + \beta_e^2} + \frac{\upsilon_b}{c_p}\left(1 + \frac{1}{\beta}\right)\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] - \frac{Q}{\rho C_p}(T - T_\infty) \tag{3}$$

$$\frac{\partial C}{\partial t} - v_0\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} + D_T\frac{\partial^2 T}{\partial y^2} - K_1(C - C_\infty) \tag{4}$$

with the boundary conditions:

$$\begin{aligned} &u = U_0,\ w = 0,\ T = T_w,\ C = C_w \ \text{ at } \ y = 0 \\ &u = 0,\ w = 0,\ T = T_\infty,\ C = C_\infty \ \text{ at } \ y \to \infty \end{aligned} \tag{5}$$

where ρ is the density of the fluid, σ is the electric conductivity of the fluid, k is the thermal conductivity, C_p is the specific heat at the constant pressure, β_T is the coefficient of the volume expansion, β_C is the volumetric of expansion with concentration, Q is the variable heat source parameter, g is the acceleration due to gravity, β is the parameter of the casson fluid. Due to the consideration of the thermal radiation, the Rosseland application simplifies the radiative heat flux q_r as follows

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \quad (6)$$

where, σ^* is the Stefan Boltzmann constant and k^* is the absorption coefficient. By assuming that the temperature difference within the flow is such that T^4 may be expanded in a Taylor series about T_∞ with neglecting higher orders it gives, $T^4 = 4T_\infty^3 T - 3T_\infty^4$. Therefore,

$$q_r = -\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial T}{\partial y} \quad (7)$$

By using equation (3) in (7) reduces to

$$\frac{\partial T}{\partial t} - v_0\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{1}{\rho C_p}\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2} + \frac{\sigma B_0^2}{\rho C_p}\frac{u^2 + w^2}{(1+\beta_e\beta_i)^2 + \beta_e^2} + \frac{\upsilon_b}{c_p}\left(1+\frac{1}{\beta}\right)\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] - \frac{Q}{\rho C_p}(T - T_\infty) \quad (8)$$

Similarity Analysis

To make the dimensionless form of the above equations we introduce the following non-dimensional variables:

$$y^* = \frac{yU_0}{\upsilon_b},\ u^* = \frac{u}{U_0},\ w^* = \frac{w}{U_0},\ t^* = \frac{tU_0^{\ 2}}{\upsilon_b},\ \theta = \frac{T - T_\infty}{T_w - T_\infty},\ \phi = \frac{C - C_\infty}{C_w - C_\infty} \quad (9)$$

Removing the asterisk sign, we obtain the non-dimensional form of the Momentum equation, Energy equation and Concentration equations as follows:

$$\frac{\partial u}{\partial t} - V_0\frac{\partial u}{\partial y} = G_r\theta + G_m\phi + A_0\frac{\partial^2 u}{\partial y^2} - MA_3 u - (MA_4 + R)w \quad (10)$$

$$\frac{\partial w}{\partial t} - V_0\frac{\partial w}{\partial y} = A_0\frac{\partial^2 w}{\partial y^2} + MA_3\, w - (MA_4 - R)u \quad (11)$$

$$\frac{\partial \theta}{\partial t} - V_0\frac{\partial \theta}{\partial y} = A_5\frac{\partial^2 \theta}{\partial y^2} + A_2 J_h\left(u^2 + w^2\right) + A_0 E_c\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] - Q_h\theta \quad (12)$$

$$\frac{\partial \phi}{\partial t} - V_0\frac{\partial \phi}{\partial y} = \frac{1}{S_c}\frac{\partial^2 \phi}{\partial y^2} + S_0\frac{\partial^2 \theta}{\partial y^2} - K_c\,\phi \quad (13)$$

The corresponding boundary conditions are given as follows:

$$\begin{aligned} &u = 1,\ w = 0,\ \theta = 1,\ \phi = 1 \quad \text{at} \quad y = 0 \\ &u = 0,\ w = 0,\ \theta = 0,\ \phi = 0 \quad \text{at} \quad y \to \infty \end{aligned} \quad (14)$$

Where, $R = \frac{2\upsilon_b\Omega_0}{U_0^{\ 2}}$ is the Rotational parameter; $G_r = \frac{g\beta_T\upsilon_b(T_w - T_\infty)}{U_0^{\ 3}}$ is the thermal Grashof Number; $G_m = \frac{g\beta_C\upsilon_b(C_w - C_\infty)}{U_0^{\ 3}}$ is the Modified /Mass Grashof Number; $M = \frac{\sigma\upsilon_b B_0^2}{\rho U_0^{\ 2}}$ is the Magnetic parameter; $V_0 = \frac{v_0}{U_0}$ is the Suction parameter; $P_r = \frac{\rho C_p\upsilon_b}{k}$ is the Prandtl Number; $Q_r = \frac{16\sigma^* T_\infty^3}{3k^* k}$ is the Radiative Parameter; $E_c = \frac{U_0^2}{c_p(T_w - T_\infty)}$ is the Eckert Number; $S_c = \frac{\upsilon_b}{D_m}$ is the Schmidt Number; $J_h = ME_c$ is the Joule Heating Parameter; $K_c = K_1\frac{\upsilon_b}{U_0^2}$ is the Chemical Reaction Parameter ; $Q_h = \frac{Q\upsilon_b}{\rho C_p U_0^2}$ is the Heat Source Parameter; $S_0 = \frac{D_T}{\upsilon_b}\frac{(T_w - T_\infty)}{(C_w - C_\infty)}$ is the Soret Number and $A_0 = 1 + \frac{1}{\beta}$; $A_1 = (1 + \beta_e\beta_i)$; $A_2 = \frac{1}{(1+\beta_e\beta_i)^2 + \beta_e^2}$; $A_3 = \frac{(1+\beta_e\beta_i)}{(1+\beta_e\beta_i)^2 + \beta_e^2}$; $A_4 = \frac{\beta_e}{(1+\beta_e\beta_i)^2 + \beta_e^2}$; $A_5 = \frac{1+Q_r}{P_r}$ are the constant coefficients of the above equations.

METHOD OF SOLUTION

To solve the governing non-linear coupled dimensionless partial differential equations (10) to (13) with the associated initial and boundary conditions (14). In fact for this set of equations, it is not possible to find its exact solution and hence we solve these equations by using the ***explicitfinite difference method***. The finite difference schemes for the equations (10)-(13) are as follows:

$$\frac{u_j^{k+1}-u_j^k}{\Delta t}-V_0\frac{u_j^k-u_{j-1}^k}{\Delta y}=G_r\Theta_j^k+G_m\Phi_j^k+A_0\left[\frac{u_{j+1}^k-2u_j^k+u_{j-1}^k}{(\Delta y)^2}\right]-MA_3u_j^k-(MA_4+R)w_j^k$$

$$\frac{w_j^{k+1}-w_j^k}{\Delta t}-V_0\frac{w_j^k-w_{j-1}^k}{\Delta y}=A_0\left[\frac{w_{j+1}^k-2w_j^k+w_{j-1}^k}{(\Delta y)^2}\right]+MA_3w_j^k-\left[MA_4-R\right]u_j^k$$

$$\frac{\Theta_j^{k+1}-\Theta_j^k}{\Delta t}-V_0\left[\frac{\Theta_j^k-\Theta_{j-1}^k}{\Delta y}\right]=A_5\left[\frac{\Theta_{j+1}^k-2\Theta_j^k+\Theta_{j-1}^k}{(\Delta y)^2}\right]+A_2J_h\left[(u_j^k)^2+(w_j^k)^2\right]+A_0E_c\left[\left(\frac{u_j^k-u_{j-1}^k}{\Delta y}\right)^2+\left(\frac{w_j^k-w_{j-1}^k}{\Delta y}\right)^2\right]-Q_h\Theta_j^k$$

$$\frac{\Phi_j^{k+1}-\Phi_j^k}{\Delta t}-V_0\left[\frac{\Phi_{j1}^k-\Phi_{j-1}^k}{\Delta y}\right]=\frac{1}{S_c}\left[\frac{\Phi_{j+1}^k-2\Phi_j^k+\Phi_{j-1}^k}{(\Delta y)^2}\right]+S_0\left[\frac{\Theta_{j+1}^k-2\Theta_j^k+\Theta_{j-1}^k}{(\Delta y)^2}\right]-K_c\Phi_j^k$$

Here, the subscript '*j*' refers to *y* and '*k*' to time. The corresponding boundary conditions (14) are given as follows:

$$\left.\begin{array}{l} u_L^k=1\ ,\ w_L^k=0\ ,\ \Theta_L^k=1\ ,\ \Phi_L^k=1 \quad \text{at} \quad L=0 \\ u_L^k=0\ ,\ w_L^k=0\ ,\ \Theta_L^k=0\ ,\ \Phi_L^k=0 \quad \text{at} \quad L\to\infty \end{array}\right\} \text{ for all } k$$

Skin-friction, Rate of heat transfer and Rate of mass transfer

Theshear stress at wall is theskin-friction; the non-dimensional form of the local primary shear stress in x-direction is given by the relation $\tau_{Lx}=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$ and the local secondary shear stress in z-direction is $\tau_{Lz}=\mu\left(\frac{\partial w}{\partial y}\right)_{y=0}$. The temperature at the plate, the Nusselt number are investigated for the different values of the various parameter. In terms of Nusselt number therate of heat transfer is given by $Nu_L=-\mu\left(\frac{\partial\theta}{\partial y}\right)_{y=0}$. Similarly for the concentration at the plate, the Sherwood number is investigated for the different values of the various parameters. In terms of Sherwood number the rate of mass transfer is given by $Sh_L=-\mu\left(\frac{\partial\phi}{\partial y}\right)_{y=0}$.

GRAPHICAL RESULTS AND DISCUTION

The results are shown the influence of the pertinent non-dimensional parameters namely Magnetic parameter (M), Casson parameter (β), the Hall parameter (β_e), the Ion-slip parameter (β_i), the Prandtl number (P_r), Suction parameter (S), Radiation parameter (Q_r), Chemical reaction parameter (K_c), Heat source parameter (Q_h), Schmidt number (S_c), and Soret number (Thermal diffusion parameter S_0) on the velocity, temperature and the concentration distribution. We consider the values of $\beta=2.0,\ \beta_e=0.2,\ \beta_i=2.0,\ P_r=0.71,\ G_r=5.0,\ G_m=10.0,$ $Q_r=0.5,\ S_c=0.22,\ V_0=0.2,\ K_c=1.0,\ S_0=1.0,\ Q_h=4.0,\ M=2.0,\ E_c=0.01,\ J_h=M*E_c$ and $R=0.2$ are fixed as common for all cases except the varied values of respective parameter.

Time sensitivity test

To obtain the steady state solution, the computations have been carried out for different time $\tau=1.0, 1.5, 2.0, 4.0, 6.0, 8.0$ and 10.0 on the velocity, temperature and on concentration profile are shown in Fig. (2)-(5). It observed from those figure that the profiles are very little change after $\tau=6.0$, therefore the solutions for all variables, $\tau=10.0$ are taken essentially as the steady state solution.

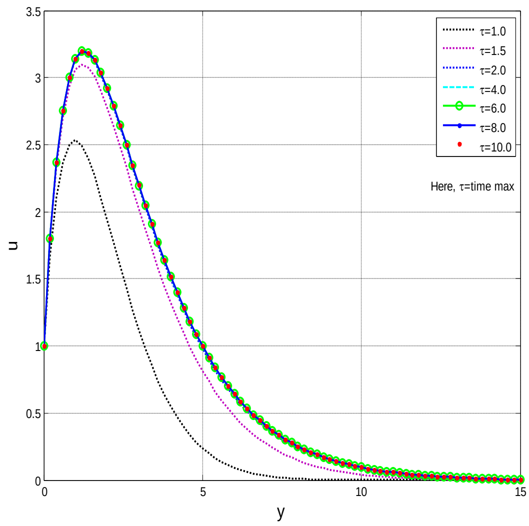

Fig. 2 Time sensitivity for primary velocity u

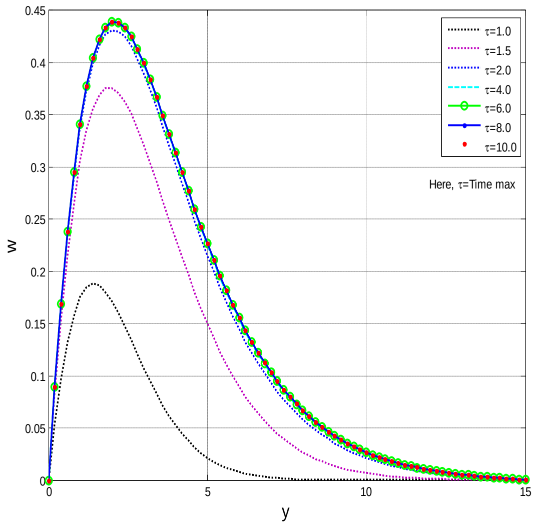

Fig. 3 Time sensitivity for secondary velocity w

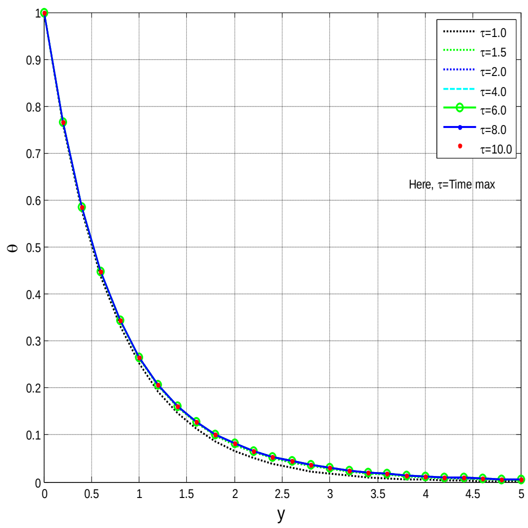

Fig. 4 Time sensitivity for Temperature θ

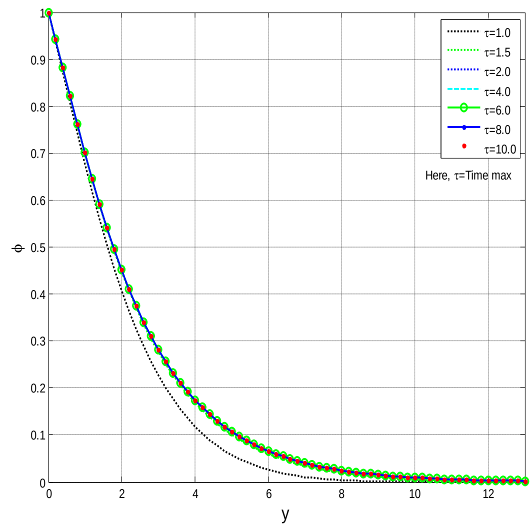

Fig. 5 Time sensitivity for Concentration ϕ

Primary and secondary velocity distribution

Figs. (6)-(15) elucidate the influence of pertinent parameters on the primary and secondary velocity distributions.We are now pay attention to the effects of Casson, Magnetic, Hall, Ion-slip, Rotational and Suction parameters on velocity profiles. Fig. 6 is described the effect of the Casson parameteron velocity profiles. The primary velocity u is increased in a certain interval $0 < y < 3.5$ (approx.), thereafter it has minor decreasing effect with the increase of Casson parameter. Similarly the secondary velocity w has increasing effect in an interval $0 < y < 4.5$ (approx.), and then it has negligible decreasing effect with the increase of Casson parameter. It is observed from Fig. 7 that, the primary and secondary both velocities have large decreasing effect with the increase of Magnetic parameter. Fig. (8)–(10) show the effect of the Hall parameter, Ion-slip parameterand Rotational parameter on velocity profiles respectively. Here, in all the cases u & w both velocities are increased with the increase of Hall, Ion-slip and Rotational parameters respectively. But from Figs. (11)-(13), we have seen that the primary and secondary both velocities are decreased with the increase of suction parameter, Prandtl number and Chemical reaction parameter respectively. Both velocity components u & w are large increasing effect with the increasing value of the thermal and mass Grashof numbers, which are shown in Figs. (14) and (15). Note that from all Figs. (2)-(15) that, the primary velocity are more effective than the secondary velocity.

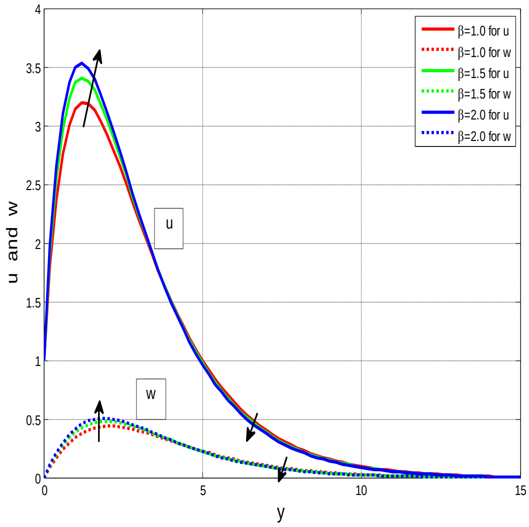

Fig. 6 Primary and Secondary velocity profiles for different values of Casson parameter β

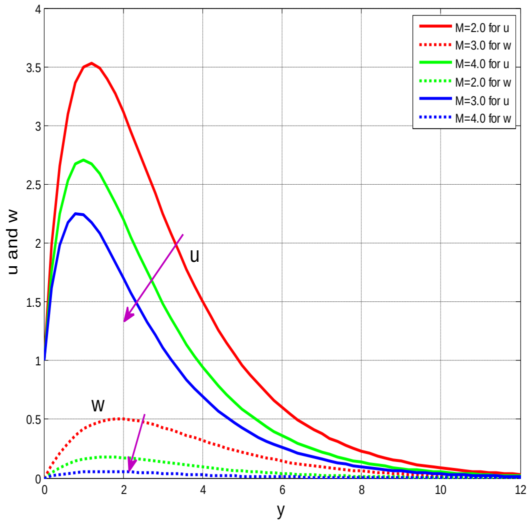

Fig. 7 Primary and Secondary velocity profiles for different values of Magnetic parameter M

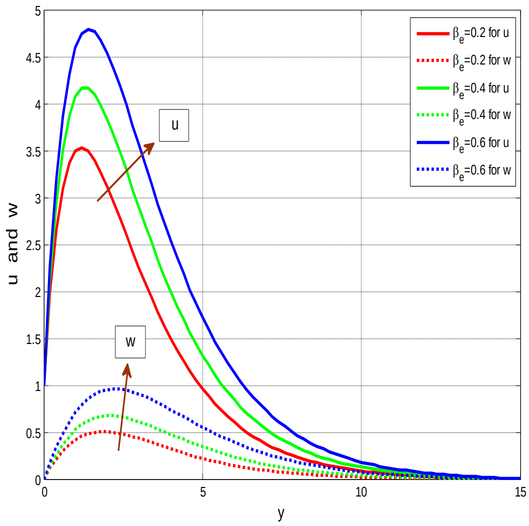

Fig. 8 Primary and Secondary velocity profiles for different values of Hall parameter β_e

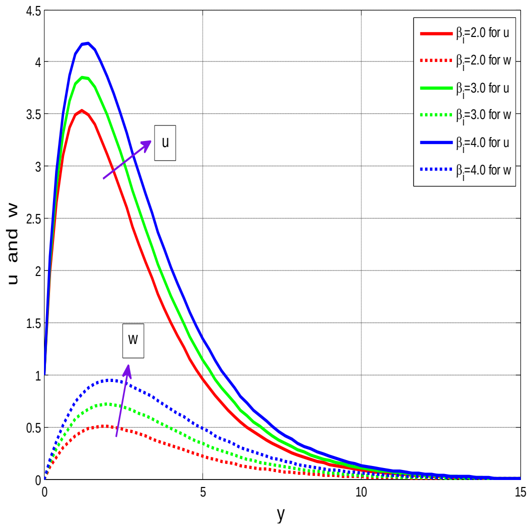

Fig. 9 Primary and Secondary velocity profiles for different values of Ion-slip parameter β_i

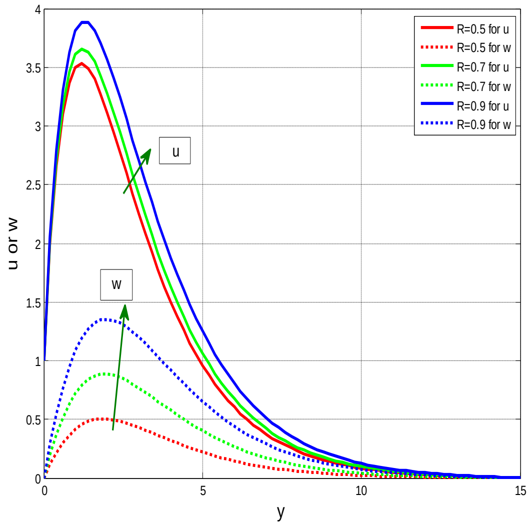

Fig. 10 Primary and Secondary velocity profiles for different values of Rotational parameter R

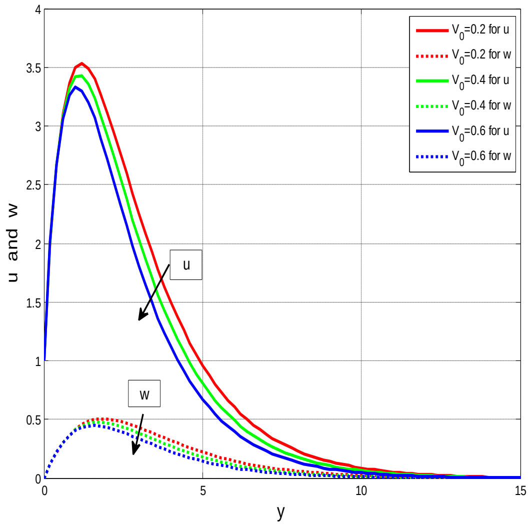

Fig. 11 Primary and Secondary velocity profiles for different values of Suction parameter V_0

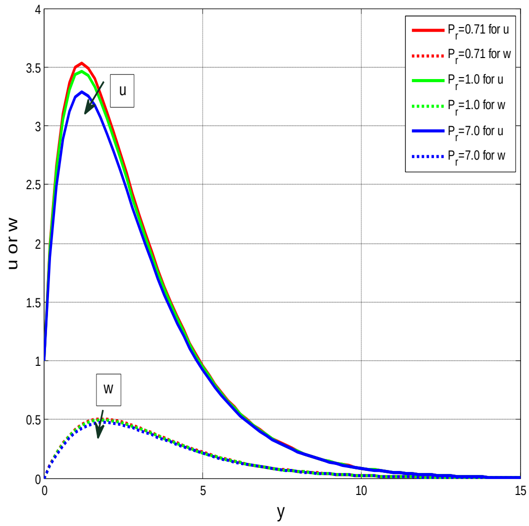

Fig. 12 Primary and Secondary velocity profiles for different values of Prandtl number P_r

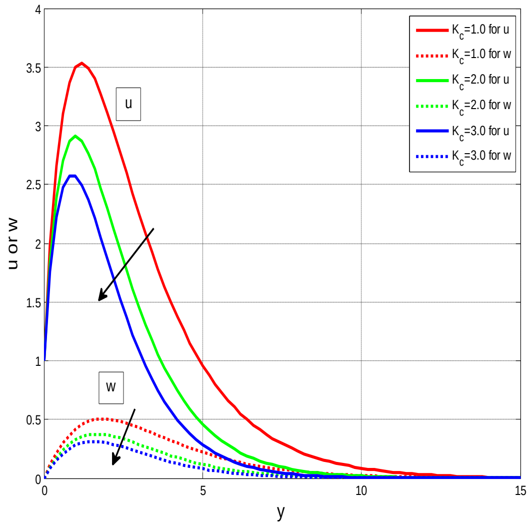

Fig. 13 Primary and Secondary velocity profiles for different values of Chemical reaction

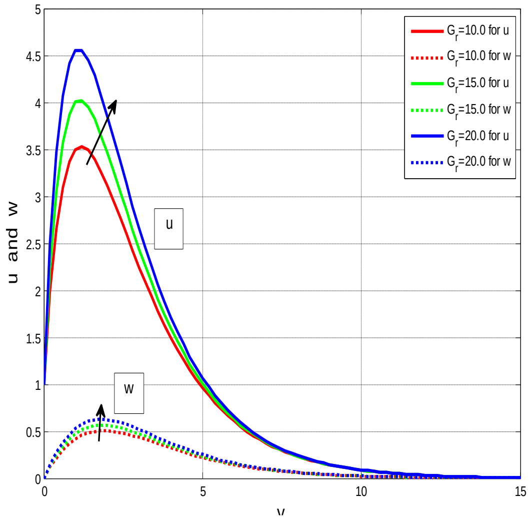

Fig. 14 Primary and Secondary velocity profiles for different values of thermal Grashof number

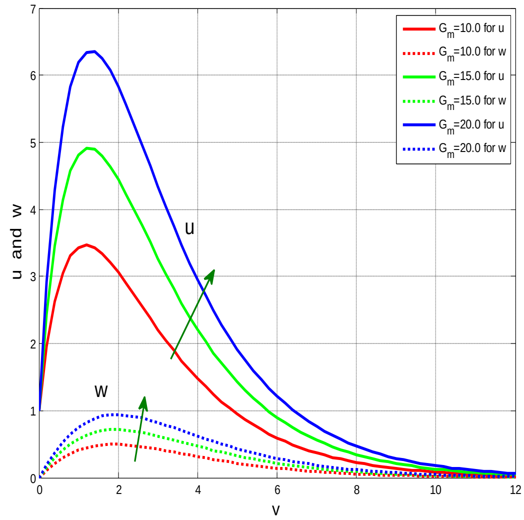

Fig. 15 Primary and Secondary velocity profiles for different values of mass Grashof number G_m

Temperature and Concentrationdistribution

Figs. (16)-(21) elucidate the influence of pertinent parameters on the Temperature and Concentration distributions.We are now pay attention to the effects of Casson, Magnetic Hall parameter, Soret number (Thermal diffusion parameter), Chemical reaction parameter and Schmidt number on Temperature and Concentration profile. We observed from Fig.16 that, the temperature is minor increase with the increase of Casson parameter, on the other hand the concentration is negligible effect on the Casson parameter. But Fig. 17 indicates that the temperature is very minor decreasing effect and the concentration is negligible effect with the increase of Hall parameter. We have seen from Fig.18 that the temperature is minor increasing effect and the concentration is increasing effect with the increase of thermal diffusion parameter. Figs.(19) and (20) are illustrate that the temperature has minor decreasing effect but the concentration has large decreasing effect with the increase of the Chemical reaction parameter and Schmidt number respectively. From the last figure (21), we observed that the temperature has minor decreasing effect but on the other hand the concentration is negligible increasing effect with the increase of magnetic parameter.

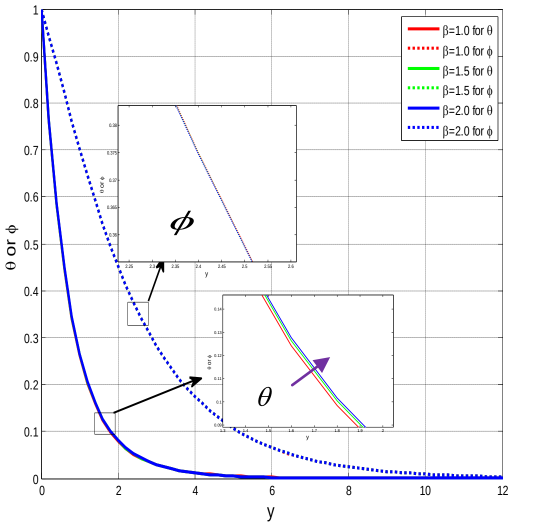

Fig. 16 Temperature and Concentration profile for different values of Casson parameter β

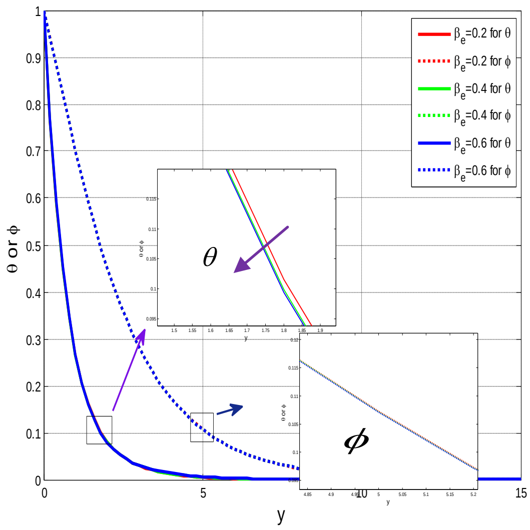

Fig. 17 Temperature and Concentration profile for different values of Hall parameter β_e

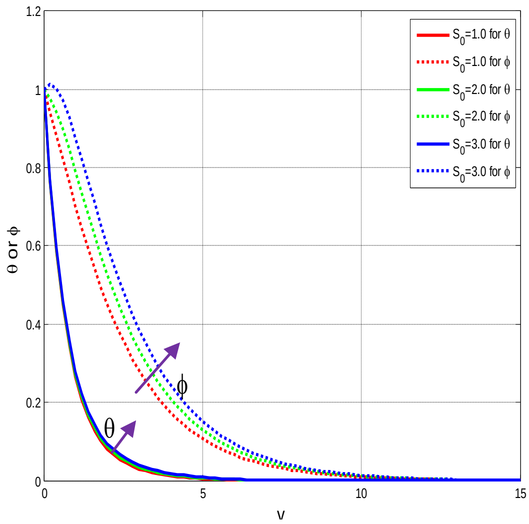

Fig. 18 Temperature and Concentration profile for different values of Soret number S_0

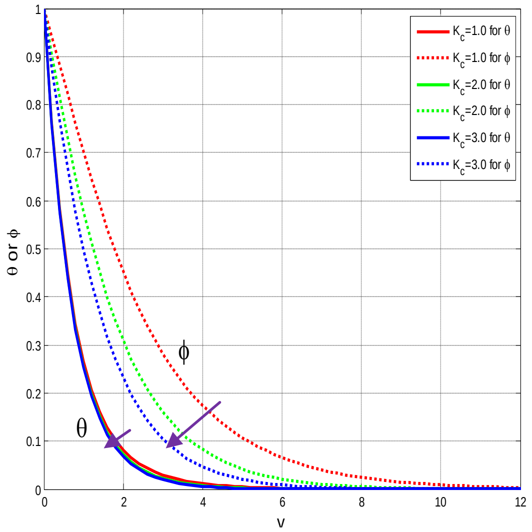

Fig. 19 Temperature and Concentration profile for different values of Chemical reaction parameter K_c

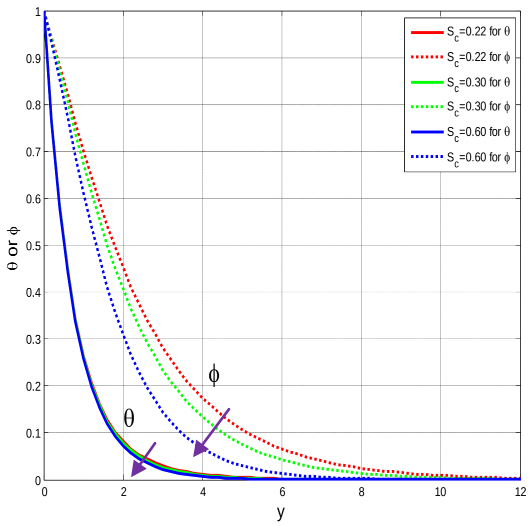

Fig. 20 Temperature and Concentration profile for different values of Schmidt number S_c

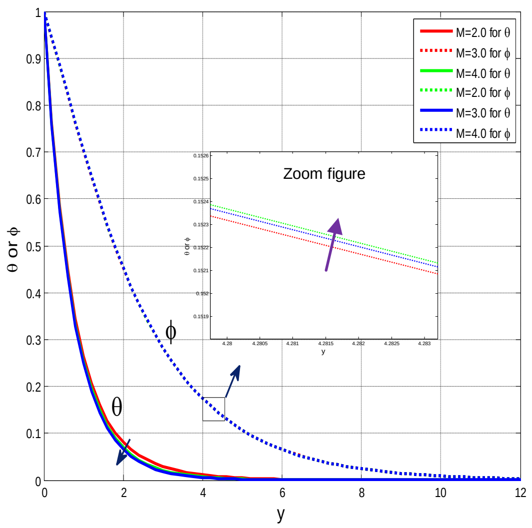

Fig. 21 Temperature and Concentration profile for different values of Magnetic parameter M

CONCLUTIONS

In the present study, we are investigated numerically that the influence of Hall current and Ion-slip on MHD unsteady Casson fluid past along a vertical porous plate with rotation. The non-linear coupled governing equations are solved numerically and the main findings can be summarized as follows:

[1] The primary velocity u and also the secondary velocity w increases with the increase of β_e, β_i, R, G_r, G_m while it decreases with the increase of M V_0, P_r, K_c.

[2] The temperature θ increases with the increase β, S_0 while it decreases with the increase of β_e, M, S_c, K_c.[3] The concentration φ increases with the increase M, S_0 while it decreases with the increase of S_c, K_c.

The accuracy of this work is qualitatively good in case of all the flow parameters.

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