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Development of Mathematical Equations for Computing Head Loss across Spun and String-Wound Filters for Drip Irrigation Systems

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ABSTRACT

Filtration is needed in drip irrigation systems to prevent emitter clogging, but the incorporation of filters causes a significant pressure drop that eventually reduces the efficiency of the system. Equations available to describe head loss due to filters used in micro irrigation systems were developed mainly for screen, disc and sand filters, not for the most recent spun and string-wound filters. A new mathematical model to describe head loss across spun and string-wound filters has been developed using dimensional analysis. Buckingham π-theorem was used to derive 6 dimensionless parameters (π-terms) from 8 variables that were found to have effect on the head loss for both filters. $\Delta H = \frac{Q^2 C}{D^4} \times K \times \left(\frac{V}{D^3}\right)^a \times \left(\frac{\varphi}{D}\right)^b \times \left(\frac{D\mu}{CQ}\right)^c \times \left(\frac{g D^5}{C^2}\right)^d \times \left(\frac{\rho}{C}\right)^e$. *Model coefficients and exponents of the π-terms were generated for each filter type used in the research. The models were compared against observed head loss values gotten from laboratory tests. Models developed could explain 97.0% and 93.0% variation in the data for String and Spun filters respectively, at 95% significant level. Statistical diagnostic on the new models revealed no obvious pattern in its residual plot, in addition to other statistical parameters that revealed high precision and accuracy.*

Key words: Mathematical model, dimensional analysis, filter, pressure drop, string-wound, irrigation systems

INTRODUCTION

Drip irrigation systems use small diameter plastic lateral lines fitted with emitters or drippers at predetermined spacing to apply water either to the surface or beneath the soil surface, near the base of the plants [1, 2, 3]. The drip emitters are one of the main devices of a drip irrigation system, because the water application efficiency is directly dependent on the application uniformity of the drip emitters. Emitters on a drip irrigation system laterals are small and are easily clogged by sediments, algae or scales (chemical precipitates) present in the irrigation water [4 - 7]. Spun and String-Wound filters are typical filter types used in drip irrigation systems to prevent clogging of drip lines and emitters. However, incorporation of filters into the drip irrigation system causes a significant pressure drop that eventually reduces system efficiency [8, 9, 10]. Change in pressure is dependent on the specific filter design characteristics, and the quality of irrigation water [11, 12]. Change in pressure distribution becomes more complex when effluents or other sources of irrigation water with low quality are used due to the increased chance of clogging. Therefore, it is necessary to account for the head loss due to the effect of the filter when designing drip irrigation systems [13, 7].

Different works have studied the filtration process and head loss across filters. Some of these works typically studied the filtration process and head loss across the filters using parameters related to filtration cake characteristics, which are difficult to estimate because of variations that occur during the filtration cycle [11, 14]. Dimensional analysis and dimensionless parameters are helpful tools for the investigation of this type of complex hydraulic problem. Thus, Puig-Bargue´s*et al.* [12] and Duran-Ros*et al.* [13] developed two different models to compute head loss across sand, screen and disc filters in effluent filtration in micro irrigation systems using dimensional analysis. Elbana*et al.* [10] and Wu *et al.* [15] also developed mathematical models for computing head loss across sand media and screen filters respectively, using the Buckingham π- theorem method of dimensional analysis.

The equations available to describe the operation of filters used in micro irrigation systems were developed mainly for screen, disc and sand filters, not for the most recent spun and string-wound filters. The study was designed to investigate the use dimensional analysis to generate independent dimensionless parameters for describing head loss and experimentally investigate the effect of each independent dimensionless parameter on head loss across different spun and string-wound filters and, second, to generate constants for the independent dimensionless terms that have effect on head loss and compare the results thereof with those from laboratory experiments.

MATERIALS AND METHODS

Experiments were carried out using spun and string-wound propylene filter cartridges (Lenntech Water Treatment Solutions) micron rating numbers: 0.5, 5, and 10µm with each filter having a cartridge length of 25.4 cm and diameter of 11.43cm. Suspended matter consisted of sediment drawn from a fresh water ponds and the University of Ibadan water treatment plant. Three ranges of total suspended solids concentration were prepared. The ranges were 100 mg/l, 150mg/l and 200mg/l of suspended solids concentrations. These ranges were selected because they are typical concentrations of the water used for irrigation [11].

Water in the tank containing the homogenous suspension was stirred vigorously at regular intervals of 5 minutes andpumped using a 0.5hp centrifugal pump through the filtration set-up. The inlet pressure was set at 300kPa pressure regulator. A low pressure gauge (Wika Instrument, LP) with an accuracy of ±1.0% measured the pressure at the inlet and outlet of each filtration system. Water pressure before and after the filtration unit was recorded at 5 minutes intervals using two pressure gauges at the filter inlet and outlet, respectively. The difference in reading between inlet and outlet pressures was taken as the head loss across the filter. Flow rates were calculated as filtered volume of water per unit time.Values of 0.001 Pa.s and 998 kgm^3 were used for water viscosity and density, respectively. Turbidity readings of the filtered water were measured in situ using a Hach 2000 spectrophotometer set at a wavelength of 450nm. The filter was changed when turbidity reading exceeded 300 NTU. Effluent samples were taken for total suspended solids (TSS) tests at the start, middle and end periods of the experiment. TSS was determined in the laboratory by weighing the solids retained in a glass fibre filter in line with the BSS standard 312.

Obtaining dimensionless groups

Previous studies have shown that several variables have some influence on head loss across screen filters, and the main factors impacting on filter head loss can be grouped into three categories. These are the Filter body structure parameters, filter medium physical parameters and filtering fluid parameters [12, 13, 18]. The following 8 variables met the criteria for selection in this study: concentration of suspended solids (C), water density (ρ), water viscosity (µ), diameter of suspended solids (D), filtered volume (V), discharge (Q), filtration level (ϕ), and acceleration due to gravity (g). Considering these variables together with the dependent variable (ΔH), the following relationship can be established:

$$f(\Delta H, C, \rho, \mu, D, V, Q, \phi, g) = 0 \quad (1)$$

A dimensional analysis was set using Buckingham's π-theorem to generate the independent dimensionless parameters. The recurring set of variables consisted of the suspended solids concentration (C), discharge (Q) and diameter of suspended solid particles (D). The dimensionless parameters generated were:

$$\pi_1 = \frac{\Delta H D^4}{Q^2 C} \quad \pi_2 = \left(\frac{V}{D^3}\right)^a$$

$$\pi_3 = \left(\frac{\varphi}{D}\right)^b \quad \pi_4 = \left(\frac{D\mu}{CQ}\right)^c$$

$$\pi_5 = \left(\frac{gD^5}{C^2}\right)^d \quad \pi_6 = \left(\frac{\rho}{C}\right)^e$$

The dimensionless parameters from π_1 to π_6 were related according to Equation 2 in an attempt to predict and compute head loss across screen filters.

$$f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6) = 0 \quad (2)$$

Equation 2 can be written in terms of ΔH as follows:

$$\Delta H = \frac{Q^2 C}{D^4} \times K \times \left(\frac{V}{D^3}\right)^a \times \left(\frac{\varphi}{D}\right)^b \times \left(\frac{D\mu}{CQ}\right)^c \times \left(\frac{gD^5}{C^2}\right)^d \times \left(\frac{\rho}{C}\right)^e \quad (3)$$

Where K is an empirical coefficient and a, b, c, d and e are empirical exponents.

Equation 3 is the new mathematical model for head loss across the filters. The values of the coefficient (K) and exponents (a, b, c, d and e) are determined for each of the two types of filters used in the experiment through a nonlinear regression analysis.

Statistical Analysis

Data from the physical parameters of the water and filters were used to calculate the different dimensionless groups and were statistically correlated with Equation 3 using the non-linear regression procedure of the Sigmaplot 11.0 statistical package.

RESULTS AND DISCUSSIONS

Table 1 presents results of values of coefficients and exponents for the nonlinear dimensionless equation that was developed. These values were generated using the sets of experimental data. The table showed the values of the coefficient and exponent for two filter types: Spun and String-Wound. The newly derived mathematical model with the corresponding values for each filter is shown in equations 4 and 5.

String-Wound Filter:

$$\Delta H = \frac{Q^2 C}{D^4}\ 1.0 \times 10^{-4} \left(\frac{V}{D^3}\right)^{0.6730} \left(\frac{\varphi}{D}\right)^{-0.1877} \left(\frac{D\mu}{CQ}\right)^{0.1994} \left(\frac{gD^5}{Q^2}\right)^{1.1191} \left(\frac{\rho}{C}\right)^{0.7343} \quad (4)$$

Spun Filter:

$$\Delta H = \frac{Q^2 C}{D^4}\ 1.82 \times 10^{3} \left(\frac{V}{D^3}\right)^{1.6279} \left(\frac{\varphi}{D}\right)^{-0.0547} \left(\frac{D\mu}{CQ}\right)^{1.5695} \left(\frac{gD^5}{Q^2}\right)^{1.4570} \left(\frac{\rho}{C}\right)^{-0.5897} \quad (5)$$

Predicted head losses for each filter were calculated using the newly generated coefficients and exponents. This was then compared with the measured head losses obtained from the practical experiments. Table 2 showed results of the comparison between the predicted and observed values of head losses using the generated coefficients and exponents. It is apparent in table 2 that the developed relationship is found to be adequate at 95% confidence level. The model F-values of 238.106 and 294.68 at ‘‘Prob> F’’ less than 0.0001) for string-wound and spun respectively, implies that the relationship is significant. There is only a 0.01% chance that an F-value this large could occur due to errors.

The modelshave a high multiple coefficient of determination (R^2) for both filter types, with the spun filter having the highest value of 0.9723 string-wound having 0.9659. The R^2 measures the percentage of variation in head-loss that can be explained by the independent variables; thus, it is an indication of how well the nonlinear dimensionless model fits the data.

The effect is a corresponding increase in R^2 and it approaches 100%. This can be very misleading, as the model while appearing to fit the data very well, would produce poor predictions. Therefore, the adjusted multiple coefficient of determination (Adj. R^2) is used to account for such a situation. This is because the adj. R^2 unlike the R^2 accounts not only for the sum of squares total and the sum of squares error, but also for the appropriate degrees of freedom for each variable. Thus, it does not change with increase in the number of variables. Since the adj. R^2 is high for both filters (between 0.9653 and 0.9573), it is an indication of how effective this model is in predicting head-loss.

Similar works by Puig-Burgues *et al* [12], Duran-Ross *et al* [13], and Wu *et al* [16] used these statistic parameters to test the models they developed. Their multiple coefficients of determination reached 0.979, 0.984 and 0.951 (Adj. R^2) respectively. Further laboratory tests to validate their models supported the interpretations given by these statistic parameters. Apparently, since this new model has a high adj. R^2 of 0.9534, 0.969 and 0.9619 for GAC, spun and string filters respectively, it suffices for a precise and accurate head loss prediction for the filters.

In addition to the coefficients of determination, the coefficient of variation (CV) expressed as a percentage, shows how far apart the data points are from the model line. The spun filter has the least variation of 15.29% while the string has a CV of 15.79%. Furthermore, the “Adeq precision” statistic measures the signal to noise ratio with the least desirable value being 4 [17]. Both filters have a high signal to noise ratio which indicates that there is a low underlying uncertainty in the model.

The head loss predictions obtained from equations 4 and 5 were compared with the data obtained in the practical experiments for both filter types. As shown in Figures 1 and 2, there is a strong positive linear correlation between the model (predicted) and the measured data.

Table 3: Values of coefficient and exponents for the model

Filter Type	Coefficient K	Exponents				
		a	b	c	d	e
Spun	1.0×10^{-4}	0.6730	-0.1877	0.1994	1.1191	0.7343
String-wound	1.82×10^{3}	1.6279	-0.0547	1.5695	1.4570	-0.5897

Confidence Interval = 95%.

Table 4: Results of regression analyses for the model

Filter Type	Std. Dev.	R^2	Mean	Adj. R^2	Pred. R^2	Adeq. Precision	CV%	F Ratio	P
Spun	0.12	0.9723	9.15	0.9690	0.9653	45.79	15.29	238.11	<0.0001
String	1.33	0.9659	8.43	0.9619	0.9573	61.09	15.79	294.68	<0.0001

Confidence Interval = 95%.

Figure 3 (A & B) give the plots of the regression residuals against the predicted head-loss for each filter. The plots reveal no pattern in the residuals as head loss increased. This implies that the errors have a constant variance which validates the normality assumption. This normality assumption further supports the results in Table 4 which indicates that the model is effective.

According to [18], the deviations In Figures 4 and 5 do not appear to be significant, consequently, it can be concluded that the residuals are normally distributed. All the above consideration indicates the adequacy of the developed relationship.

A good model accounts for systematic movement in the process, leaving out a series of uncorrelated, purely random errors which are assumed to be normally distributed with a zero mean and a constant variance. Since these assumptions have all been met for this model, it implies that the model so developed will be effective in predicting head losses.

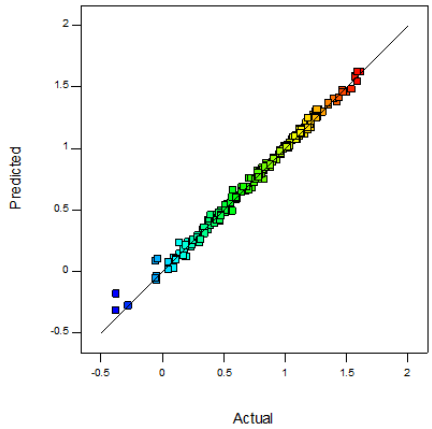


Fig. 1 Predicted vs. actual head-loss for Spun filter

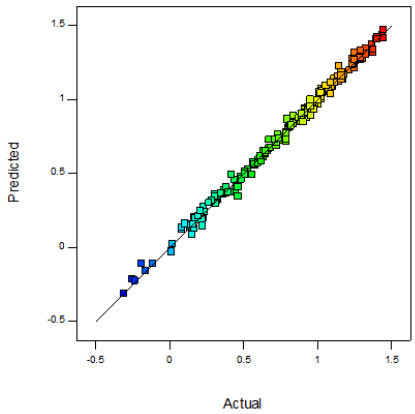
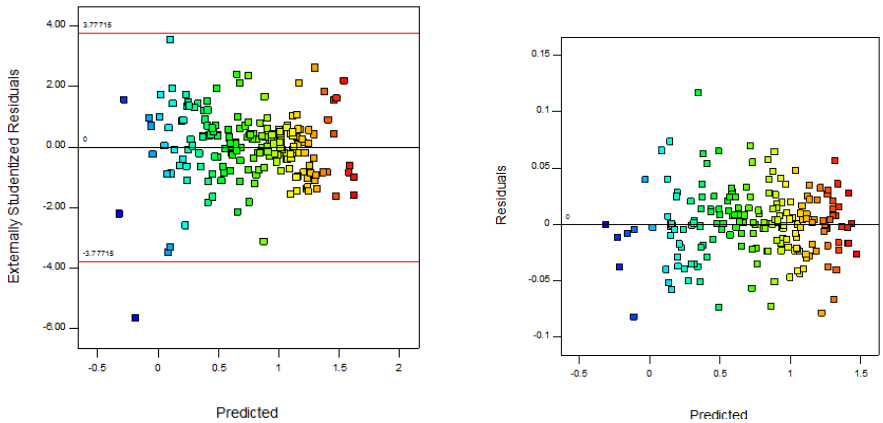


Fig. 2 Predicted vs. actual head loss for String-wound filter



(A) Spun **(B) Strin**

Fig. 3 Plot of the regression residuals against the predicted head loss

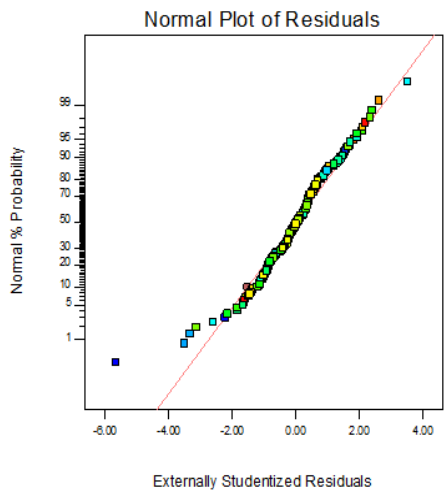


Fig. 4 Normal probability plot for spun filter

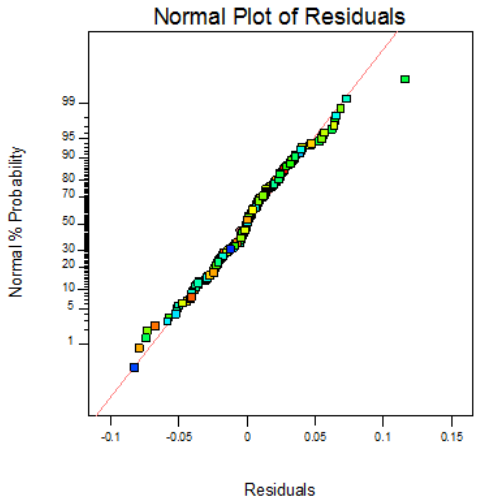


Fig. 5 Normal probability plot for String Filters

Effect of Variable Interactions on Head-Loss

Regression analysis showed that the interaction of suspended solid particles with respect to volume, filtration levels relative to volume, suspended solids in relation to filtration level were significant. Figure 7 showed the response for the interactive factors: filtration level and volume. The shape indicates that there was a significant interaction between the parameters of filtration and volume on head loss for both filters. A trend can be observed in which the rise in head loss

becomes steeper with decreasing filtration level as volume increases. For the spun filter at a filtration level of 0.5 microns, the head loss rise is highest (52 kPa), whereas at 10 microns, it is smallest (1.0 kPa). This trend is in line with the findings of Wu *et al.* [16], Doran-Ross *et al.* [13], and Adin and Elimelech [19].

The interaction between suspended solids concentration and volume in Figure 8 also indicated that at lower suspended solids concentration, head losses were at a minimum. Clearly, the trend is similar to that of Figure 7. The net result is the corresponding rise in head loss over time [20, 12, 15].

Figure 9 depicts the interaction between suspended solids concentration and filtration level which is significant ($p < 0.0001$). A combination of high suspended solids concentration and small filter pore diameter will result to a high head loss. Expectedly, minimum values of Head loss can be obtained when the water is free of suspended solids and high filter pore size is used.

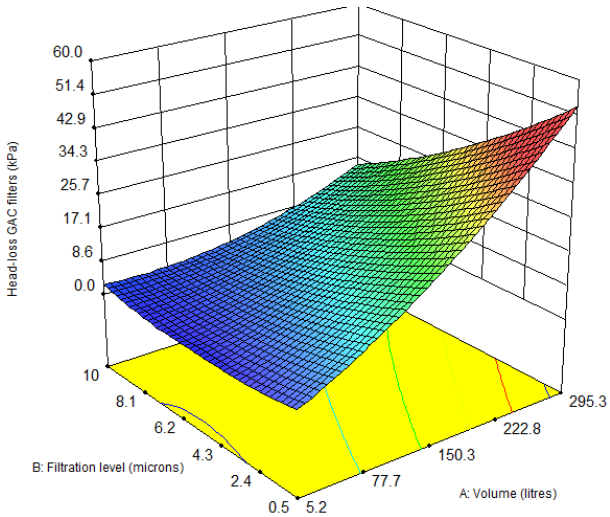


Fig. 7 Response of head loss to filter pore size and Volume interaction

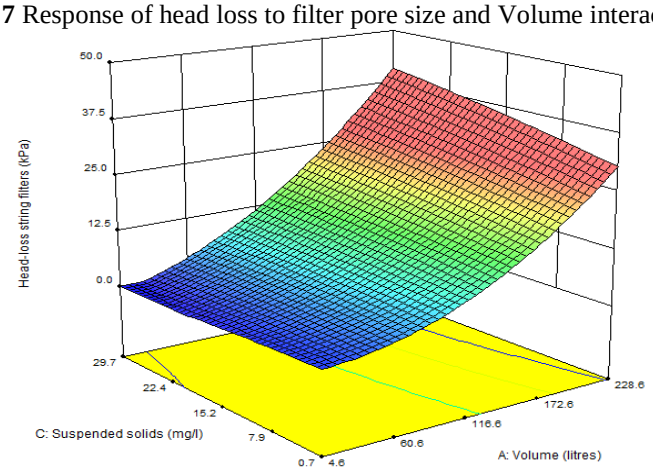


Fig. 8 Response of Head loss to Suspended solids and Volume interaction

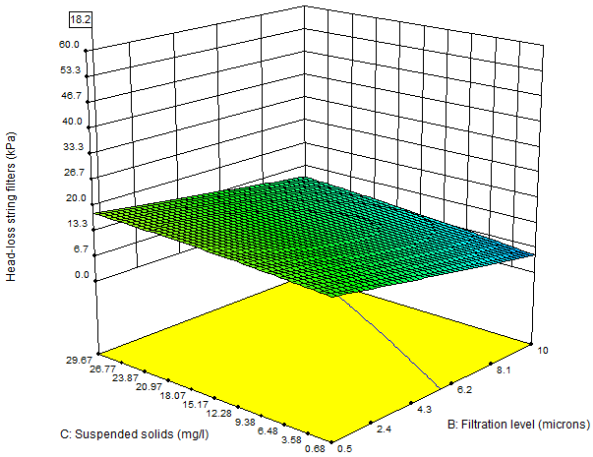


Fig. 9 Response of Head loss to Suspended solids and filtration level interaction

CONCLUSIONS

Six dimensionless parameters (π-terms) which formed a nonlinear dimensionless model for predicting head-loss across screen filters were developed in this study. These dimensionless parameters are ratios of 8 variables that were found to have effect on the head loss phenomenon for spun and string-wound polypropylene filters. The variables from which the π-terms were derived includes the filter pore size, concentration of suspended solids in the water to be filtered, diameter of the suspended solids, volume of filtered water, discharge across the filters, water density, dynamic viscosity of water and acceleration due to gravity. The F-value for the model indicated that the non-linear dimensionless data fits the model. Also, with the high adjusted coefficient of determination R^2 for GAC, spun and string filters. The model can be used for a precise and accurate head loss prediction for the filters, with low underlying uncertainty in the model due to high signal to noise ratio.However, results have shown that lower suspended solids give minimal head loss when associated with high values of filtered volume.

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