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Mixed Integer Linear Fractional Programming Model for Producer Multi-Product Supply Chain Optimization Problem under Uncertainty

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ABSTRACT

A vast amount of mathematical models were developed for supporting agricultural production structure optimization decisions. In this paper, we have presented a mixed integer linear fractional programming model and presented a case study for the agricultural products produce policy of Bangladesh. This study, a decision making model has been proposed for agricultural products production planning under uncertainty. Agricultural products are in general cost expensive with high risk in profitability due to its fluctuating prices and natural calamities. Natural calamities are a key factor for small-scale farmers. We have also developed a model by collecting data from small-scale farmers of some selected area in Bangladesh and analyzed their profit under a situation of uncertainty. The formulated model is solved applying A Mathematical Programming Language (AMPL) with appropriate solver (Minos). If the uncertainty probability increase or decrease, the profit changes accordingly.

Key words: Mixed integer linear fractional program, Optimization, AMPL, Uncertainty, Bangladesh.

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1. INTRODUCTION

Agriculture sector plays an important role in overall economic development of Bangladesh. The agricultural sector (crops, livestock, forests and fishing) contributes 14.74 percent to the country's GDP (BBS), provides employment about 41percentof the labor force according to Quarterly Labor Force Survey 2015-16. Moreover, agriculture is the source of wide range of consumer demanded agricultural commodity markets, especially in rural areas. Agriculture's share of GDP declined from 62 percent in 1975 to 13.41 percent in 2017, but agriculture's share of total employment has not declined as much. The declining share of agriculture in GDP should not be construed to reflect a diminishing role of agriculture in the overall growth of the economy or in poverty reduction. Notably, the service sector has expanded at a rapid pace at this stage of economic transformation. Much of the growth in the services sector relates to the marketing and processing of agricultural products resulting from rapid commercialization and diversification in agriculture. The agriculture sector is dynamic, changing with demand of the people, availability of technology and change of management practices. A vast amount of opportunity to develop of agriculture sector in Bangladesh, but natural disasters is one of the main sources of crisis for small-scale farmers. Every year natural calamities such as floods, droughts and cyclones reason extensive damage to crops, houses, households and community assets, which causes all kind of production cost increase per unit products and decrease overall profit and also can lead to decrease in livelihood opportunities for the small-scale farmers. Thus, it requires regular adjustment with different planning and development programs. The country has much potential, yet it faces many challenges including vulnerability to climate change. For planning and sustainable development purposes, a diagnostic study of Bangladesh Agriculture is required in order to foster growth of this important sector harmonizing with the management of natural resources and addressing the challenges.

In agriculture production policy, a producing pattern or allocation of land to different type of products varies with farmer's perspective of his land holding. These problems allocation of land for different products, maximization of products, maximization of profit, minimization of production cost are addressed in agriculture management system. This study, described as single objective MILFP model. In agriculture sector, profit or loss also depend on fluctuating demand, supply and pricing of a particular product with minimization of cost of cultivation needed for that product.

Supply chain management (SCM) is the systematic analysis and educated decision-making within the different business functions of an organization resulting in smooth and cost-effective flows of resources – material, information, and money. In other words, it is the coordination and synchronization of the flow of resources in the network of suppliers, manufacturing facilities, distribution centers and customers. These network elements form the different echelons of the supply chain. The supply chain is the network that sources raw material from suppliers, transforms it into finished products at the manufacturing facilities, and distributes the finished products to the final customers through the distribution centers. These activities constitute the individual business functions of the company's supply chain network.

In the global competitive market, the importance of Supply Chain Management (SCM) increases day by day. To maximize the profit and minimize the cost are the main goals, so it is important to make the model optimal for both consumer and a manufacturer. An efficient supply chain system operates under a strategy to minimize costs by integrating the different functions inside the system and by meeting customer demands in time.

2. REVIEW OF LITERATURE

The supply chain of an organization consists of different functions at each planning stage. According to Ganeshan and Harrison [1], these functions can be broadly classified in the following four categories – location, production, inventory, and transportation. Each of these functions plays a major role in the overall performance of the supply chain. So it becomes essential to execute each one of them in an optimal manner to ensure an efficient supply chain performance.

Facility Location Problems (FLP), which are typically used to design distribution networks, involve determining the sites to install resources, as well as the assignment of potential consumers to those resources. Drezner *et al*. [2] briefly described FLP the location of manufacturing plants, the assignment of ware houses to these plants, and finally the assignment of retailers to each warehouse. Jose et al. Moreover, Nagurney [3] derived a relationship between supply chain network equilibrium and transportation network equilibrium Jose et al. [4] presented mixed integer type linear programming to solve a capacitated vehicle routing problem minimizing number of vehicle and travelling time. They implemented the model to a real life problem of a distribution company and solved it numerically.

In domain of agricultural production system, where uncertainty and vagueness play a major role in decision making, several researchers such as Sher and Amir [5], Sumpsi et al. [6], Sarker et al. [7], Vasant [8] used various programming techniques for a farm planning problem. Kruse and Meyer [9] attracted researchers to study agricultural crop planning with stochastic values as stochastic linear programming problem to address such problems. Itoh et al. [10] Considered a problem of crop planning under uncertainty assuming profit coefficients are discrete random variables and proposed a model to obtain maximum and minimum value of gains for decision maker. Toyonaga et al. [11] Studied a product planning problem with fuzzy random profit coefficients. Sharma et al. [12] Studied FGP for agricultural land allocation problem and proposed an annual agricultural plan for different crops.

Further the assessment of reliability for satisfying (or risk of violating) the system constraints under uncertainty which described, Charnes and Cooper [13], Infanger and Morton [14]. There is a range of agricultural applications based on the technique of interval programming or probabilistic programming, including scheduling replacements in sow farms determining the optimum production and marketing decisions on crop farm Kaiser and Apland [16]. Hurter A.P., Martinich J.S., [17], described facility location problem and the theory of production, Lien and Hardaker [18] examining impacts of subsidy scheme and utility function on portfolio choice in agriculture. However, few studies were reported to incorporate above two methods within a general optimization framework to adjust agricultural production structure for maximizing farmers' interests and simultaneously ensuring national food security.

Among these, Charnes and Cooper [13] described a transformation technique, which transforms the Linear Fractional Program (LFP) into equivalent linear program. This method is quite simple but needs to solve two-transformed model to obtain the optimal solution. Fractional programming problems have become a subject of wide interest since they arise in many fields like agricultural planning, financial analysis of a firm, location theory, capital budgeting problem, supply chain, portfolio selection problem, cutting stock problem, stochastic processes problem. From time to time survey papers on applications and algorithms on fractional programming were developed by various authors. In addition, fractional programming has benefited from advances in generalized convexity and vice versa. Further, Charnes and Cooper transformation reduces the linear fractional program into linear program and then an optimal solution to the problem could be obtained easily.

In this study, producer multi-product supply chain optimization problem under uncertainty is formulated as a MILFP model which maximizes the ratio of return on investment and at the same time analyzed the profit for some uncertainty probabilities. Using Charnes and Cooper [13] transformation, the formulated MILFP model was solved by AMPL. Finally, a numerical example along with the sensitivity of labor cost is considered to estimate the performance of the model.

Rest of the paper we have organized as follows. We have presented data collection in section 3. In section 4, **a** mathematical formulation of MILFP model is presented. The section has three subsections, discussed basic notations, parameters and decision variables. Solution approaches and result discussion in section 5. Finally, in section 6, the conclusions and contributions of this study are discussed.

3. DATA COLLECTION

We developed our MILFP model by collecting data for Agricultural products optimization in randomly selected samples of 150 small-scale farmers from three districts (Mymensingh, Kishorgonj and Jamalpur) in Bangladesh. We collected detailed information on Agricultural activities including availability of land, water, labor, fertilizer, capital and farm machinery. We have collected total costs of production such as land rental costs, seeds costs, labor costs, fertilizer costs, water costs, holding costs, transportation costs, packing costs and personnel costs. After that calculated profit dividing total return by total investment. We have also collected secondary data from Bangladesh Bureau of Statistics (BBS), Directorate of Agricultural Marketing (DAM), Statistics Department of Bangladesh Bank (SDBB), The Bangladesh Journal of Agricultural Economics (BJAE), NGOs reports and Newspapers.

FORMULATION OF MILFP MODEL

Before mathematical formulation of MILFP model, we have discussed basic notations, parameters and decision variables that are relevant with our work in this study.

Basic Notations and Parameters

Sets:

L: Set of production locations indexed by j;

P: Set of products indexed by i;

Parameters:

u_{ij} The price of i^{th} product at j^{th} location ($/kg).

v_{ij} Yield of i^{th} product at j^{th} location (kg/ha).

l_{ij} Labor Requirement of i^{th} product at j^{th} location (ha).

b_{ij} Labor cost of i^{th} product at j^{th} location ($/unit).

w_{ij} The amount of water need of i^{th} product at j^{th} location (ha).

y_{ij} Water cost of i^{th} product at j^{th} location ($/unit).

f_{ij} Fertilizer Requirement of i^{th} product at j^{th} location (kg/ha).

c_{ij} The price of unit raw materials for i^{th} product at j^{th} location ($/unit).

a_{ij} The amounts of raw materials need to produce i^{th} product at j^{th} location ($/unit). .

t_{ij} Unit transportation cost of raw materials for i^{th} product at j^{th} location ($/unit).

p_{ij} The production cost of i^{th} product to j^{th} location at ($/unit).

h_{ij} Unit holding cost of i^{th} product from j^{th} location for some given unit of time ($/unit-time).

g_{ij} Fertilizer cost of i^{th} product at j^{th} location ($/unit).

p_i Uncertainty probability of i^{th} product

Decision Variables

$$z_l = \begin{cases} 1, \textit{if location } j \textit{ is used,} \\ 0, \textit{else} \end{cases}$$

Assumptions

The basic assumptions of the model are as follows:

- There are no temporal changes in soil physical and chemical properties
- Each manufacturing facility is able to produce all of the products
- The selling price for a product may vary from retailer to retailer depending on the discussions, order size, locations etc.
- The objective function and all constraints are linear

MILFP Model

In this subsection, we formulated the equivalent mixed integer linear fractional programming model that estimates the total profit as well as optimal allocation and distribution. The objective function is the ratio between return and investment.

The objective function is:

$$Maximize = z1/z2 \tag{1}$$

Where,

$$\sum_{i=1}^{m}\sum_{j=1}^{n}(u_{ij}v_{ij})x_{ij} = z1 \tag{1a}$$

$\sum_{j=1}^{n}\sum_{i=1}^{m}(z_j a_{ij} + p_i l_{ij} b_{ij} + f_{ij} g_{ij} + w_{ij} y_{ij} + p_i a_{ij} t_{ij} + a_{ij} c_{ij} + p_i v_{ij} h_{ij}) = z2,$	(1b)

Constraints:
These constraints restrict the use of available resources such as land, labor, fertilizer and water. For utilization of available resources, the following relationships are used:

$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} \le A$	(2)
$\sum_{i=1}^{m}\sum_{j=1}^{n} b_{ij} x_{ij} \le L$	(3)
$\sum_{i=1}^{m}\sum_{j=1}^{n} w_{ij} x_{ij} \le Q$	(4)
$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij} \le F$	(5)
$x_{ij}, a_{ij}, b_{ij}, c_{ij}, w_{ij}, u_{ij}, v_{ij}, h_{ij}, H_{ij}, t_{ij}, f_{ij} \ge 0$, z_j is binary	(6)

Where i the type of product is i_1, i_{21}, i_3 represent wheat, rice and vegetables; j is the produce location of that product. A is the total farm land available for products of location j; L is the total labor available for products of location j; Q is the total water available for products of location j and F is the total fertilizer available for product of location j.
The constraints help define the interrelationships among the decision variables and the agriculture production conditions. In detail, constraints (1a) express the total return and (1b) express the total investment production of the total products; constraint (2) indicate the total land allocated to different products should be less than or equal to the total land area available; constraints (3), (4), (5) indicate that the maximum requirement of such factors of production as labor, water and fertilizer must be less than or equal to the regional resource available. The last equation (6) is the nonnegative constraints. This program has accomplished on a Core-I3 machine with a 3.60 GHz processor and 4.0 GB RAM.

4. SOLUTION APPROACH AND RESULT DISCUSSION

We have to solve the formulated MILFP model by using Charnes and Cooper transformation.
For any r≥0, let the present decision redefined as the following form:

$$z_j = rx_j, \; for\; j = 1, \dots, n$$
$$z_{ij} = rx_{ij}, \; for\; j = 1, \dots, n \; and\; i = 1, \dots, m$$

Since r≥ 0, x_l is {0, 1}; therefore, z_l becomes {0, r}. Again x_{ij} is non- negative, so z_{ij} is also becoming non-negative. Hence, the formulated MILFP model can be reformulated as follows:
Maximize:

$\sum_{i=1}^{m}\sum_{j=1}^{n}(u_{ij} v_{ij}) x_{ij} \Big/ (\sum_{j=1}^{n}\sum_{i=1}^{m}(z_j a_{ij} + p_i l_{ij} b_{ij} + f_{ij} g_{ij} + w_{ij} y_{ij} + r\, p_i a_{ij} t_{ij} + a_{ij} c_{ij} + p_i h_{ij} v_{ij})), \forall j$	(7)

Subject to

$\sum_{i=1}^{m}\sum_{j=1}^{n} z_{ij} \le rA$	(8)
$\sum_{i=1}^{m}\sum_{j=1}^{n} b_{ij} * z_{ij} \le rL$	(9)

$\sum_{i=1}^{m}\sum_{j=1}^{n} w_{ij} * z_{ij} \leq rQ$	(10)
$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} * z_{ij} \leq rF$	(11)
$z_{ij}, b_{ij}, c_{ij}, w_{ij}, a_{ij}, u_{ij}, v_{ij}, p_{ij}, l_{ij}, h_{ij}, t_{ij},\ r \geq 0, z_l \text{ is binary}$	(12)

In order to analyze the effectiveness of the proposed models, we consider a numerical example, which consisting 7 production locations and 5 products. Due to the volume details data optimal solution are not presented in this study. If readers are interested then they can contract with the corresponding authors. Fig.1. shows that, MILFP model provides optimal locations of the farmer; product-2 is the most profitable, which produce in locations 1, 4 and 7. Also product-3 is profitable in locations 3 and 5. Product-4 is not optimal at any location. When production cost of any product of any location were high, then that location do not produce that product.

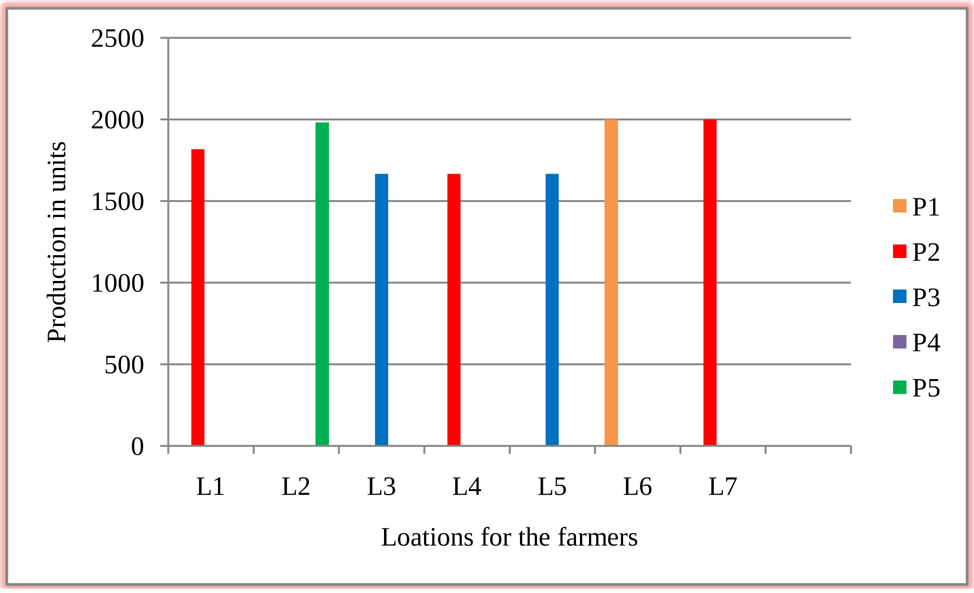

Fig. 1 Produce of different products at different locations

Also we have presented profits comparison for different uncertainty parameters. Natural disasters are one of the main sources of crisis for small-scale farmers. Every year natural calamities such as floods, droughts and cyclones reason extensive damage to crops, houses, households and community assets, which causes all kind of production cost increase per unit products and decrease overall profit and also can lead to decrease in livelihood opportunities for the small-scale farmers. Due to the volume details optimal solution are not presented in this paper. If readers are interested then they can contact with the corresponding authors. Various uncertainties profit comparisons has presented in Fig. 2.

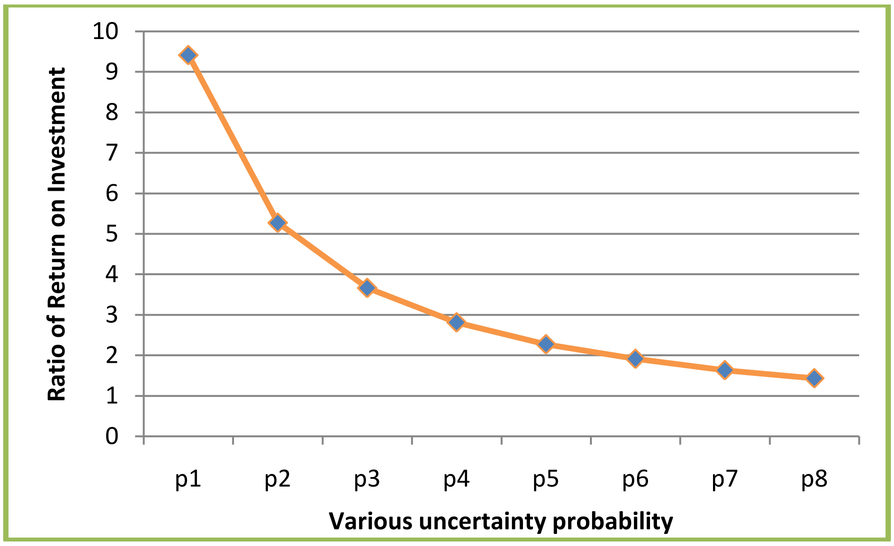

Fig. 2 Profit comparison under uncertainty

From Fig. 2, it is clear that, if we increase uncertainty probability then profit has decreased gradually. The amount has been calculated in dollar. Therefore natural calamities are a key factor for small-scale farmers. It has direct impact on its profit or loss. The effect of natural calamity and it's seemed to us that it will be very terrible for the producer if profit decreases in such a way. Finally, we can see that the producers/farmers have taken chariness for the big natural calamity but they do not protect their products against big natural calamity. In this study, it is clear that farmers bear most of the risk portion, but they are deprived from the actual price of the products. Intermediaries of the agriculture market are gainer of the maximum portion of the profit. In the next section, we have drawn a conclusion about our work.

5. CONCULUSION

In this paper, the MILFP model is developed for the integrated supply chain network which is solved by AMPL using the suitable Charnes and Cooper transformation. We developed an MILFP model and analyzed the production policy of Agricultural products in Bangladesh. We collected one year data to develop this model. The formulated MILFP model is to maximize the ratio of total income on total cost and also to optimize most profitable production locations, which satisfied most of the customer demands. Finally, we can analyze the profit for the uncertainty parameters. Some of the significance findings can be summarized as follows:

Firstly, the illustrated numerical example apparently shows that the MILFP model shows (Fig.1.) product-2 is the most profitable, which produce in locations 1, 4 and 7. Also product-3 is another profitable product which produces in locations 3 and 5. Product-4 is not optimal at any location. Secondly, for the increase of uncertainty probability then the profit has decreased gradually (Fig.2.). We are confident that this is the first work with Agricultural products production policy in Bangladesh. We hope that this work will be helpful to the researchers, producers of agricultural products and common readers to know about the system of profitable agricultural products production.

The future research of our interest is to optimize the whole system of the supply chain of agricultural products in Bangladesh, like a producer- local wholesaler-urban wholesaler-retailer and consumer. Also, formulate the coordination model among the whole system. Further, comparison this MILFP model with MILP model for various parameters.

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