



## Punching Shear Failure of Reinforced Concrete Flat Slab System- A Review

Georgewill V.A., Ngekpe B.E., Akobo I.Z.S. and Jaja G.W.T.

Rivers State University, Port Harcourt, Nigeria

### ABSTRACT

*This study reviewed punching shear failure of flat slab system. Firstly, the merits of flat slab system compared to normal framed structure were appraised. Due to complex mechanisms that occur during punching shear failure, it has been a subject of intense experimental, analytical and experimental investigations. This study focuses more on finite element models developed by previous researchers. Two concrete failure criteria namely: Total strain cracked model (T-S model) and damaged plasticity models (DPM) were compared. One of the merits of T-S model is that cracked in concrete is modelled using a constitutive relationships. T-S captures the residual strength of cracked concrete in the post-cracked regime. Finite element models available showed that DPM implemented in ABAQUS has been widely used in the prediction of punching shear failure. The application of finite element method has led to significant understanding of punching shear phenomenon.*

**Key words:** Punching shear, Concrete, Damaged plasticity model, T-S model, failure

### 1. INTRODUCTION

Slab-column connections are solid concrete slabs of uniform depth that transfers load directly to the supporting columns without the aid of beams or capitals or drop panels [1]. In recent times, flat plates are probably the most commonly used slab system today in the construction of multi-storey reinforced concrete hotels, motels, apartment houses, hospitals, and dormitories due to its architectural and structural merits over the normal frame structure [2].

Flat slab system has architectural and constructive advantages over the normal frame structure such as: simple formwork, no beams, simplifying under-floor services outside the drops, has minimum structural depth. Therefore, the application of the of flat slab system becomes more economical than the normal frame.

However, the main disadvantage of flat plate system is the risk of brittle punching failure at the slab-column connection caused by the transfer of shear and unbalanced moments [3]. This vulnerability of flat slab to punching shear has been a subject of intense experimental, theoretical and analytical investigations since 1960s [4]. Punching shear is a local failure phenomenon that occurs due to concentration of local stresses within the proximity of supporting column. Although, the punching shear capacity of reinforced concrete flat plates can be increased by various means, their applicability is often limited, e.g., traditional shear reinforcing by means of stirrups is applicable only to slabs with the depth greater than 150 mm according to ACI Committee 318-05 [4].

Moreover, this type of structure when subjected to seismic lateral loads, shear stresses in the slab intensifies due to an unbalanced moment, crack occurs within the slab in the proximity of the column. The cracks spreads through the depth of the slab at different angular degrees to the bottom of the slab which leads to punching shear failure of the slab along the crack propagation.

Finite element method has been used extensively to investigate punching shear failure. For instance, [6] applied "Non-orthogonal cracks in a smeared finite element model", to predict punching shear failure. Menetrey [7] and Hallgren, [8] have applied two-dimensional rotationally symmetric elements model to predict punching shear failure.

Success in numerical modeling for punching shear cannot be achieved without adequate understanding of material behavior and failure criteria for concrete and steel. The type of failure criterion adopted has a significant effect on the punching shear behavior in addition to the type of finite element analysis used. Researchers such as Elgabry and Ghali [9] and Moehle [10] have studied the linear elastic finite element method of moment transfer between slab and column

with no justifiable assumption of elastic behavior of concrete. They concluded that linear finite element analysis can be used to study prescribed boundary conditions in research but cannot provide comprehensive account of concrete behavior, because concrete as a quasi-brittle material, does not obey elastic law. Eder et al [11] conducted numerical and experimental investigations on punching shear of a hybrid flats slab with shear heads. Ngekpe, B.E [12], also carried out experimental and numerical simulations on the Punching Shear Failure of Reinforced concrete flat slabs supported on steel edge column, using Total strain crack” implemented in MIDAS FEA to study the contributions of shearhead and steel column to the punching shear capacity of flat slab at the edge connection.

Teixeira et al. (2015) investigated the punching shear failure of the flat slab made with Self Compacting Concrete (SCC). Their findings revealed that the higher the aspect ratio led to higher fracture energies. Consequently, the slab presented a higher load carrying capacity without the punching shear failure occurring. It was observed that no significance changes occurred on the carrying capacity for both serviceability and ultimate limit states in relation to increment in the pre-cracking region but did not consider the effect of the lateral loads on the slab-column connection and its post-cracking behaviour

This paper presents a comprehensive review on various finite element models for the prediction of punching shear failure of slab-column connection.

## 2. CONCRETE CONSTITUTIVE MODELS

### 2.1 Damaged Plasticity Model (ABAQUS)

This method employs two versions of coupled inelastic models involving damage and plasticity which are both based on the concept of effective stress acting on the undamaged skeleton of the material and involve permanent strains when maximum limit is reached. The damaged plasticity model in the commercial FE program ABAQUS includes the effect of moderate confining pressure and irreversible damage, focusing on failure mechanisms characteristic for quasi-brittle materials especially, concrete. In addition, different responses were predicted in tension and compression with respect to yield strengths (in tension understood as elasticity limit), hardening/softening and stiffness degradation. Stiffness evolution during cyclic loading and rate sensitivity are included in the model. Based on the equations 1 and 2:

$$\Sigma = (1 - \Omega)D^E E^E \dots\dots\dots 1$$

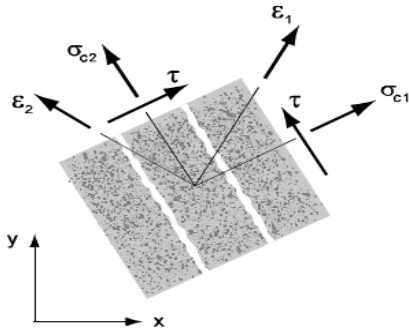
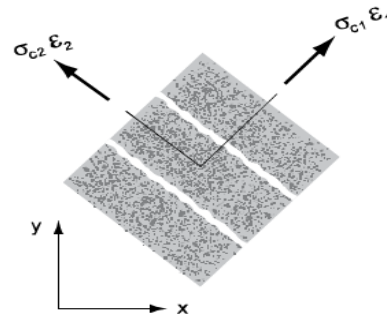
$$E = E^E + E^P \dots\dots\dots 2$$

but the isotropic damage parameter ( $\omega$ ) is here the function of the effective stress and two hardening parameters for tension and compression.

At the ultimate load the punching shear cone is visible due to the sudden opening of the cracks. Concrete damaged plasticity model assumes that the cracking initiates when the maximum principal plastic strain is positive. The orientation of the cracks is considered to be perpendicular to the maximum principal plastic strains and thus, the direction of the cracking is visualized through the maximum principal plastic strains. The yielding of the flexural reinforcement has occurred at the failure in both test and FEA. The tensile longitudinal reinforcement yielded under the column at the failure load. The maximum tensile principal stresses are shown for the two surfaces of the slab at the failure. The tensile principal stresses can be used in FEA in order to show the cracking patterns.

### 2.2 Total Strain Crack Model

The MIDAS FEA [14] adopts the total strain crack model which is categorized under the smeared crack model. The smeared crack is further divided into fixed crack and rotating crack models. Fixed crack model assumes that the axes of cracks remain fixed once crack axes are defined. It has been observed that fixed crack accurately reflects the physical characteristics of the crack phenomena. The fixed crack model is capable of simulating the physical behavior of concrete cracks more accurately than rotating crack model because it evaluates both the normal stresses and shear stresses on the crack surface as depicted in figures 1a and 1b. It is also particularly suitable for modelling shear behavior of concrete. Rotating crack model assumes the direction of cracks rotates continuously depending on the changes in the axes of principle strains when cracks are initiated. In both cases of the fixed and rotating models, the first crack at the integral points always initiates in the direction of the principal strains. Concrete materials display isotropic properties prior to cracking and anisotropic properties after cracking. MIDAS considers the properties of concrete as orthotropic materials after cracking as well as the normal stresses and shear stresses.

**Fig. 1a:** Fixed crack model [14]**Fig. 1b** Rotating crack model [14]

A constitutive model based on the total strain expresses the stress as a function of the strain. This phenomenon is known as hypo elasticity, when loading and unloading behaviour is along the same stress-strain path. The basic concept of the Total strain model is that stresses are computed with respect to the crack directions. In Midas FEA, loading and unloading takes place on separate stress-strain paths, especially unloading is modeled with a secant slope. For secant implementation, for both compression and tension, the secant modulus always passes through the origin. This elastic damage approach is dissimilar to elastic-plasticity where the stiffness is fixed, independent of loading and unloading. The tensile behaviour of concrete is considered linear elastic until the peak strength is attained. The constitutive model of total strain is established on the modified compression field theory of Vecchio and Collins [15]. MIDAS implemented it with a three 3D models based on the theory proposed by Selby and Vecchio [16].

### 3. FINITE ELEMENT MODELS

Nuno et al [17] studied punching shear failure of orthogonally reinforced flat slab systems using both experimental and three-dimensional nonlinear finite element methods. The finite element analysis was carried out using ATENA, commercially available FEA software. The results of both experimental and numerical studies correlated adequately with EC2 and MC2010 codes equations for punching shear capacity of slab-column connection. Based on the validity of the finite element method, it was used to simulate thirteen slabs models, and developed an equation to predict the punching capacity was proposed. Results showed the higher reinforcement ratio resulted in significant stiffness after cracking in the compression zone which leads to brittle failure. The FEA predicted punching load was proportional to  $f_c^{0.41}$  which is an average value of EC2 and MC2010 codes. The increase in cracking load was commensurate with the concrete strength and the post cracking patterns were almost similar in stiffness. The increase of the thickness of the slab-column dimensions also increased in predicting punching strength. It is also proposed an equation to predict the punching capacity, based on EC2 equation but with a change in the size effect parameter, where fracture mechanics parameters are introduced, namely, structure brittleness. This parameter allows taking directly into account the type and size of aggregates.

Oliver [18] compared both brittle Cracking Model and the Concrete Damaged Plasticity Model and the results show that the Concrete Cracking Model of ABAQUS/Explicit does not seem to be a suitable constitutive model to model missile impacts on reinforced concrete slabs using solid 3D meshes. The constitutive model in principle allows the setting of a failure strain (brittle failure strain) leading to the removal of element from the Finite Element model when their strain has reached that value.

Shu et al [19] conducted the punching shear failure of reinforced concrete (RC) slabs without shear reinforcement using nonlinear finite element (FE) analysis implemented in DIANA 9.5, commercially available finite element software. The total strain crack model was used to model concrete behaviour. The Total Strain rotating crack model was used due to its merit over the fixed crack. Parameters governing punching shear failure such as: fracture energy, tensile response of concrete and lateral confinement for the compressive strength of concrete. The model was further refined using experimental results such as the load carrying capacity, load-deflection response and crack pattern. Parametric study shows that the referenced finite element model predicted with high accuracy the measured Load – deflection response of the slab-column connection. The result of the finite element analysis correlated well with the experimental results and Eurocode 2 equation for punching shear capacity.

Ngekpe B. E (2016) applied 'Total strain crack model' implemented in Midas FEA to predict punching shear failure of a reinforced concrete slab supported on steel edge column without shear reinforcement. The nonlinear finite element model was validated using the experimental results of a slab-column connection without shear reinforcement which failed in punching. There was a good correlation between the experimental and numerical results. Results confirmed that the adopted numerical model is capable and reliable in predicting punching shear capacity of the edge connections. The results also revealed that ACI 318-05 equation slightly underestimated the punching shear capacity at edge connections.

Ngekpe et al [20] investigated the performance of a modified shearhead assembly for edge steel column embedded in reinforced concrete slab. The authors adopted the Total Strain Crack model as failure criterion for concrete. The model was validated using previous research work of Eder et al. [11], following the success of the validation scheme, the model

was adopted to predict the behavior of the tested slab-column connection with a modified ACI318-14 shear-head assembly. Results showed that the shear-head contributed appreciably to the punching shear resistance of the connection. Eder et al [11] conducted numerical and experimental investigations on punching shear of a hybrid flat slab with shearheads. The study focused on the contribution of shearhead to punching shear capacity of the interior slab-column connection not transferring unbalanced moment. The shearhead was designed based on the ACI 318-05 code recommendation. The shearhead was welded to the tubular steel column and inserted between the layers of the reinforcement. It was observed that the shearhead deformed plastically before punching failure occurred.

M.A. Eder et al (2011) investigated the behaviour of ductile shearheads for linking reinforced concrete flat slabs to interior tubular steel columns. The structural response of the proposed shearheads was compared to the conventional ACI-type shearheads that is fully embedded in the slab. The proposed shearhead was designed as a dissipative element which yields in shear before punching failure occurs in the slab.

Genikomsou and Polak [21] examined the punching shear failure modes of interior slab-column connections without shear reinforcement under static and pseudo-dynamic loadings using both experimental and Nonlinear Finite element methods. Concrete failure was modelled using coupled damaged plasticity model implemented in ABAQUS. The results showed that damaged plasticity model can adequately predict the punching shear response of the slabs.

Wosatko et al [22] applied damage-plasticity models for the prediction of shear failure of reinforced concrete slab-column connections. The numerical model was validated using previous published experimental work on punching shear connection. The authors adopted two regularized numerical models of concrete, established within elastic-plastic damage theories which include gradient damage and rate-dependent plasticity models. Results revealed that properly calibrated damage-plasticity models can be applied to the prediction of shear behaviour and failure in RC slabs. Results also showed that Finite element is very sensitive to the type of tensile model used to represent concrete tensile behaviour; this is very significant because punching shear is initially by tensile cracking. It was observed that with regularization damage zones representing cracks are not localized, which may introduced difficulty in the formation of punching cone after the initial stage of flexural fracture.

Bompa and Elghazouli [23] studied the performance of hybrid reinforced concrete flat slabs with shear-heads at connections to steel columns, through several numerical and parametric studies. Damage-plasticity constitutive model was used to model concrete failure. Comparison of the numerical simulations against the test results shows close correlations in terms of ultimate strength, deformations and stress levels in the constituent elements of hybrid members and was followed by couple of parametric equations on key structural features for hybrid flat slabs with steel shear-heads. The results of the investigations identified three modes of failure based on the interaction between the shear-head and surrounding concrete which led to the development of improved analytical models for predicting the response as well as the ultimate strength of the structural elements. In addition, recommendations were given for the determination of shear-head dependent parameters, required for practical design purposes, with particular focus on the embedment length and shear-head sizes. The suggested expressions for assessing the shear-head elemental features presents a more reliable design approach in comparison with existing methods and are suitable for effective practical application and implementation in marked procedures. In particular, the results indicated that more effective application can be obtained from intermediate rather than relatively short or long shear-heads provided that the depth is at least half the slab thickness. Regarding the wide range of salient parameters considered within the parametric studies, which were subsequently used for validation, the analytical expressions proposed in this study presented a more reliable design approach in comparison with existing approaches. The suggested equations for determining the required shear-head properties and predicting the ultimate strength of the embedding hybrid members are suitable for direct practical application and implementation in organized sequence.

Teixeira et al [13] carried out numerical simulations of the punching behavior of centrally loaded steel fibre reinforced self-compacting concrete (SFRSCC) flat slabs based on Reissner–Mindlin hypothesis. The slab was discretized into differently assumed plane stress state in each layer to simulate the continuous damage induced by cracking. The results of the numerical simulation showed that smeared crack constitutive models is more appropriate than discrete crack models in the simulation of crack propagation in FRC structures, mostly, in specimens with high degree of redundant supports. The different specimen content of hooked-end steel fibres (0, 60, 75 and 90 kg/m<sup>3</sup>) and concrete strengths of 50 and 70 MPa tested and numerically modelled, shows that due to deficiencies in the automated procedure of placing the fibres into the concrete mixer, the slabs reinforced with 60 and 75 kg/m<sup>3</sup> and a compressive strength of 50 MPa showed relatively low content of fibres (around 30 kg/m). Also, higher aspect ratio led to higher fracture energies and consequently, the slab presented a higher load carrying capacity without the punching shear failure occurring. In terms of the shape of the second branch of the crack constitutive law, it was observed that when the second post-cracking stress point is 1.25 f<sub>ctm</sub> (tensile strain hardening), a significant increase occurred in the carrying capacity both at the serviceability and ultimate limit states.

Aikaterini S. Genikomsou et al [24] adopted the damaged plasticity model analyzed in ABAQUS in calibrating previous material parameters to investigate the effect on the boundary conditions and modified slab-column connection of the compressive membrane action in flat concrete slabs to compare results from isolated specimens and continuous floor systems. The tested slabs simulated and analyzed in ABAQUS shows accuracy in punching shear capacity of a continuous slab being higher when compared to the capacity of a conventional isolated simply supported slab. The

experimental and numerical results are in good agreement in terms of load-deflection response and crack patterns. Also, numerical results obtained showed that the continuous slabs have higher punching shear strength with lesser deflections in relation to the isolated simply supported slab and the continuous slabs shows smaller concentration of crack widths and patterns around the column area only. The comparative results between the design codes shows that EC2 gives higher punching shear resistance for all slabs than the predicted provisions ACI with the recommendation that for more effective, accurate, rational and cost effectively strengthening techniques in assessing existing structures, the continuous membrane action effect be included in the design codes.

#### 4. MECHANICAL AND ANALYTICAL MODELS

Hallgren [8] investigated the punching shear resistance and the structural behavior of high strength concrete slabs. The study recommended an advanced model of Kinnunen and Nylander [25] model of punching shear for proportionally loaded slabs without shear reinforcement. The two failure criteria were based on the ultimate tangential concrete strain. While the criterion by Kinnunen and Nylander [25] consists of semi-empirical expressions, the criterion by Hallgren [8] is derived with fracture mechanics and accounts for the concrete brittleness and size effect. Hallgren's model is based on the equilibrium of the forces acting upon a slab segment from a circular slab similarly with the model by Kinnunen and Nylander [25]. Except for the resultants mentioned in the previous section, Hallgren [8] introduces a dowel force  $D$  in the reinforcement. The slab segment is carried by a truncated wedge which has some similarities with the compressed conical shell in the model by Kinnunen and Nylander [25] but with varying tangential strains in the compressive zone [26]. Theoretical investigation on the punching shear strength of interior slab-column connections made of steel fiber reinforced concrete (FRC). In the steel FRC slab-column connection, the shear force applied to the critical section was resisted by both the compression zone and the tension zone at the critical section. The shear capacity of the compression zone was defined by considering the interaction between the shear and the normal stresses developed at the critical section. The shear capacity of the tension zone was defined by considering the post-cracking tensile strength of FRC. By using the shear capacity, a new strength model for the punching shear strength of steel FRC slab-column connections was developed. The proposed strength model was verified using existing test results and showed very good accuracy. For convenience in design, a simplified design equation was also developed

Kinnunen & Nylander [25] carried out several punching shear tests on circular concrete slabs without shear reinforcement, loaded along the perimeter with uniformly distributed load and proportionally supported on a column. The experimental result showed that formation of concrete sections similar to stiffened frame is due to the conical shear and the radial cracks. Based on the experimental result, Kinnunen and Nylander [25] proposed a punching shear model for predicting the failure load at punching. The model consists one of the slab sections bounded by the shear crack and two radial cracks with the angle  $\Delta\phi$  in-between with the assumption that the section revolves around the centroid of rotation placed at the base of the shear crack and that the slab section are supported by a compressed conical shell between the column perimeter and the base of the resultant shear crack.

Ménétrey P. [7] concluded that punching failure is due to tensile failure of concrete along the inclined punching crack and is not due to compressive failure of concrete and that increasing the reinforcement ratio implies increasing the punching load. Size effect was also investigated, resulting in a nominal shear stress decrease with increasing thickness of the slab.

Muttoni [4] investigated the Punching Shear Strength of Reinforced Concrete Slabs without Transverse Reinforcement on the basis of the opening of a critical shear crack which enhanced the proposition of new failure criteria for punching shear basically on rotation of the slab. The adopted method exactly predicted experimental results of punching shear failures, in slabs with low reinforcement ratios. The application requires the knowledge of the load-rotation relationship of the slab, for which a simple mechanical model is proposed. The resulting approach is shown to give better results than current design codes, with a very low coefficient of variation (COV). Parametric studies revealed that the adopted scheme precisely predicts numerous influential factors of punching shear recently observed in testing as size effect decreasing nominal shear strength with increasing size of the member. The study proposed a method to calculate the punching strength as a function of the effective depth of the slab, the size of the column, the flexural reinforcement ratio, the yield strength of the reinforcing steel, the concrete strength, the maximum aggregate size, and the span-depth ratio of the slab. This method gives very good results when compared with Accounting for the proposed failure criterion and load-rotation relationship of the slab, the punching shear strength of a flat slab is shown to depend on the span of the slab, rather than on its thickness as often proposed.

Ngekpe B.E [12] conducted an experiment using the nonlinear finite element analysis 'Total strain crack model' implemented in Midas FEA was applied to model to examine the punching shear failure of a reinforced concrete slab supported on steel edge column with and without shear reinforcement. The validation of experimental results and the parametric results of the slab-column connection correlation confirms that the adopted numerical model is capable and reliable in predicting punching shear capacity of the edge connections of the plate. The results corresponded to ACI 318-05 equation which significantly underestimates punching shear capacity at edge connections.

Osman et al [27] studied on punching shear analysis of reinforced concrete flat slabs introduces the results of nonlinear finite element analyses. Model verification has been done Simulation of experimental data results with other research results prior to the parametric study for model verification. Several finite element models were developed to carry out the

parametric study on interior, edge, and corner slab-column connections to investigate the effect of concrete compressive strength, load eccentricity to slab thickness ratio and column dimensions to slab thickness ratio ( $b/t_s$ ) on the behavior of the slab-column connections. The results of the numerical simulations were consistent with the available experimental results of the ultimate load and related deflection. Column dimension to slab thickness ( $b/t_s$ ) of the experiment indicates a near greater influence on punching resistance, while the ACI codes provision, underrates the strength of interior, edge and corner slabs by residual increment when compared with ANSYS FEM results. Substantial statistical and numerical differences of punching shear force analysis of the slab specimens used when related to codes: ACI 318-05, CEBFIP MC 90, and BS 8110-9, allowed for accurate prediction. Apart from a special FEA, there is no simplified method in any code that can predict the punching shear stresses for a highly eccentrically loaded slab-column and that an accurate FEA model is key to mitigating the punching shear failure of column-slab connections transferring moments

Ozbolt & Bazant [28] investigated the numerical smeared fracture analysis non-local micro-crack interaction approach found out that the results of finite element analysis are shown to be mesh insensitive, and good convergence is obtained. Cracking damage is found to localize into a volume whose size and shape depend on the macroscopic concrete properties as well as the current stress-strain state. Although the damage is considered to be tensile on the micro-level, due solely to mode I micro-cracks, the new non-local model can describe well not only mode I fracture tests but also complex shear-dominated and mixed-mode types of failure such a diagonal shear, and can do so for the same values of material parameters (which was not the case for previous nonlocal models). Most importantly, the new nonlocal model can correctly capture the size effect of quasi-brittle fracture, in approximate agreement with Bazant's size effect law

#### 4. CONCLUSION

Punching shear failure has been a subject of intense experimental, analytical and numerical investigations. To understand punching shear failure, extensive experimental investigations have been carried out by many authors which led to the development of several mechanical and analytical models. However, most mechanical models led to poor prediction of punching shear failure. In this study, two types of failure criteria were examined. It was observed that the damaged plasticity models (DPM) has been used by many authors compared to Total strain crack model (TS-model). It was observed that residual tensile strength of concrete was not properly captured using DPM. The total strain crack model used the smeared crack model and modelled concrete cracking via constitutive relationships. Concrete nonlinear parameters such as: tensile strength, compressive strength and shear modulus are modelled in logical sequence. Based on the literatures reviewed, there the need for further applications of TS- model for predicting punching shear failure of Flat slab to column connection.

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