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**Research Article** 

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# Estimating the Residual Life of Cutting Tool in Turning Process under Variable Cutting Conditions

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# ABSTRACT

The economic importance of machining process has encouraged to study regarding cutting tool life among many researchers so as to increase the product quality, machining productivity, process stability and machining safety while decrease the overall production costs. In metal cutting process, failure of cutting tool is responsible for significant downtime, which not only effects the production time and cost but also lowers the productivity. The downtime occurs due to cutting tool failure alone is responsible for about 20% of overall downtime. Thus, it is recommended to have an accurate and reliable residual tool life estimation methodology. The aim of this work is to calculate the residual life of a worn cutting tool which is used for turning AISI D2 steel bar under variable cutting condition. The proportional hazard model with Weibull distribution as baseline hazard is used to model the failure time data of cutting tool. Remaining useful life of cutting tool has been estimated using Mean Residual Life (MRL) function. The results obtained through the model have been verified experimentally and it shows quite a good agreement.

**Key words:** Mean Residual Life, Cutting tool life, Survival analysis, Weibull distribution Proportional hazards model, Cutting tool Reliability

# **1. INTRODUCTION**

Aim of modern manufacturing industry is producing economical and reliable products along with good quality. Quality of machining product is generally referred as good surface finish and accuracy in dimensions, which is highly dependent on the condition of cutting tool. Besides product quality, tool failure puts high impact on machining system and productivity. Moreover tooling cost accounts for a significant part of cost of machining. Hence, non-optimum use of cutting tool can effect both product quality and manufacturing economy. In practice, many times either cutting tools are not used up to their full life or cutting tool are kept on using beyond their effective life. Both underestimation and overestimation of tool life effects overall productivity and cost. Thus, having an idea regarding tool replacement time based on residual life estimation is highly necessary for a machining system.

Till now many researchers have developed various models for estimating cutting tool life and TCM (Tool Condition Monitoring) strategy. The very first model was developed by Taylor [1] which is still widely used. However, this model is found to be confined for specific tool-workpiece combinations and small range of cutting speed [2]. Different model were proposed for indirect tool life estimation based on correlation between cutting force and tool wear [3-6]. Likewise, several models have been developed based on correlation of different monitoring signals such as temperature, surface roughness, vibration power consumption etc. with tool wear [7]. Though these methods are questionable in regard to randomness of toll wear [8]. Thus it necessitates development of model based on probability analysis. Various model based on probability distribution such as normal, lognormal, Weibull etc. were proposed for describing the probabilistic behaviour of toll wear. Different

tool replacement strategy were developed based on reliability analysis of cutting tool [9].Proportional hazards model (PHM), an effective tool for survival analysis, which is mostly used in medical applications, has been introduced by Mazzuchi [10] in machining field, for tool life assessment. The main feature of this model is that it takes into account time depending aging phenomenon of tool along with effect of the process parameters. Aramesh et al. [11] estimated the remaining tool life under variable cutting conditions corresponding to different state of tool wear using PHM.

The aim of this work is to find out residual life of worn tool which is used for turning AISI D2 steel under variable cutting parameters. The PHM is used to model the cutting tool hazard rate for different cutting conditions in which cutting speed, feed rate and depth of cut are considered as model covariates. Subsequently, reliability function and failure density function are obtained which are used in Mean Residual Life (MRL) equation for estimating residual life of cutting tool.

#### 2. MODELLING OF MEAN RESIDUAL LIFE FUNCTION BASED ON PROPORTIONAL HAZARD MODEL

### 2.1. The COX Proportional Hazard Model

D. R. Cox [12] introduced the proportional hazards model (PHM) in 1972. This PHM is in one of the most popular models in the field of survival analysis due to its flexibility and simplicity in evaluating the effects of various model covariates which influence the failure time of a component or system. Hence, the proportional hazards model is extensively used in the field medical, engineering etc. for analysing failure time data and condition based monitoring (CBM).

The PHM is based on the concept that the failure rate of an engineering component dependents on the operating time as well as the operating conditions. Hence, the PHM is suitable for survival analysis of cutting tool because tool life is extensively dependent on machining conditions (e.g. speed, feed rate, depth of cut etc.) and at the same time cutting tool has an aging effect with the time of machining [13].

The hazard function for PHM is expressed by the equation as follow:

$$h(t) = h_0(t) . \exp \sum_{i}^{n} (x_i C_i) n = \text{number of variables}$$
(1)

The hazard function consists of the product of two parts. The first part  $h_0(t)$  is termed as the baseline hazard, which is a time dependent function. It represents the aging characteristic of cutting tool. The second part  $e^{\sum_{i=1}^{m} x_i C_i}$ , which is an exponential function of model covariates  $(C_i)$  along with corresponding model

parameters  $(x_i)$ 

In the place of baseline hazard function, we can assume any of the standard distribution such as Weibull, lognormal, normal, exponential, etc. However, it has been found that Weibull distribution is the best model for fitting failure time data of cutting tool [14]. Weibull distribution is widely used in time dependent failure analysis because of its flexibility in fitting variety of data. Hence, for this work, the Weibull distribution is considered for modelling baseline hazard which is used to present the time to failure data of cutting tools. The failure rate or hazard rate of a cutting tool operating under constant cutting condition (i.e. when speed, feed and depth of cut remain constant throughout the full life of a cutting tool) is given as follows:

$$h_0(t) = \frac{\beta}{\mu} \left(\frac{t}{\mu}\right)^{(\beta-1)} \tag{2}$$

Where  $\beta$  represents the shape parameter and  $\mu$  represents the scale parameter and the tool life t, is taken as continuous random variable. By substituting the Weibull baseline function in Eq. 1, it will reduce to:

$$h_0(t) = \frac{\beta}{\mu} \left(\frac{t}{\mu}\right)^{(\beta-1)} \exp\left(\sum_{i=1}^m x_i C_i\right)$$
(3)

The expression for calculating the cutting tool reliability at certain point of time when subjected to variable cutting condition is given by Eq. 4. This equation can be obtained from the relationship between hazard function and survival function as follow:

$$R(t) = \exp\left[-\int_{0}^{t} h(t)dt\right] = \exp\left(-\left(\frac{t}{\mu}\right)^{\beta} \exp\left(\sum_{i=1}^{m} x_{i}C_{i}\right)\right)$$
(4)

Similarly, the expression of failure density function f(t), cutting tool subjected to variable cutting condition is given by the Eq. 5which can be obtained from the relationship between survival function and failure density function and as follow:

$$f(t) = \frac{\beta}{\mu} \left(\frac{t}{\mu}\right)^{(\beta-1)} \exp\left(-\left(\frac{t}{\mu}\right)^{\beta} \sum_{i=1}^{m} x_i C_i\right)$$
(5)

Where  $C_1$ ,  $C_2$  and  $C_3$  are the model covariates representing the speed and feed respectively. To obtain a balanced convergence of model parameters  $(x_i)$ , covariate values are used in normalized form, which are presented as follow:

$$C_1 = \frac{V - V_{mean}}{V_{max} - V_{min}}, C_2 = \frac{f - f_{mean}}{f_{max} - f_{min}} \text{ and } C_3 = \frac{d - d_{mean}}{d_{max} - d_{min}}$$

The PHM parameters ( $\beta$ ,  $\mu$ ,  $x_1$ ,  $x_2$  and  $x_3$ ) can be found out through the maximum likelihood estimation (MLE) [15] method, using any standard statistical software like SPSS, Stata, SAS etc. Once the PHM parameters are estimated, the next step is to construct proportional hazard model by substituting the parameter values, for obtaining survival function and failure density function. Using these functions, the last step is to estimate useful life of cutting tool with the help of Mean Residual Life (MRL) function. Given that a tool has already been used for a certain time  $t_0$ , then residual life of the tool from that time  $t_0$ , can be estimated from MRL equation as follow:

$$MRL(t_{0}) = \frac{\int_{0}^{\infty} t f(t) dt}{R(t_{0})} - t_{0}$$
(6)

#### 2.1. Maximum Likelihood Estimation

Three commonly used methods for estimating the parameters are the least-squares methods, the graphical method and the maximum likelihood estimation (MLE) method [16]. In this study the maximum MLE method is used to estimate the parameters of proportional hazard model. MLE is known to be very useful, practical and powerful technique for estimating parameters. Likelihood function can be written as:

$$L(\lambda) = \prod_{i \in F} f(t_i, \lambda) \prod_{i \in C} R(t_i, \lambda)$$
(7)

Where,  $\lambda$  is used to represent the parameter  $(x_1, x_2 \text{ and } x_3)$  which is need to be estimated. Probability density function is represented by  $f(t_i, \lambda)$  and  $R(t_i, \lambda)$  represents the reliability function. *C* and *F* are censored dataset and failure dataset, respectively. The Parameter  $\theta$  can be found out by solving the equation as follows:

$$\frac{\partial}{\partial\lambda} \ln[L(\lambda)] = 0 \tag{8}$$

The reliability function and probability density function are substituted into Eq. 7, so Weibull likelihood function is reduced to as follow:

$$L(\lambda) = \prod_{i \in F} \left[ \frac{\beta}{\mu} \left( \frac{t}{\mu} \right)^{(\beta-1)} \exp\left( x_1 C_1 + x_2 C_2 + x_3 C_3 \right) \right]_{i \in C+F} \left[ \exp\left( -\left( \frac{t}{\mu} \right)^{(\beta-1)} \exp\left( x_1 C_1 + x_2 C_2 + x_3 C_3 \right) \right) \right] (9)$$

Taking logarithm to the both sides of the Eq. 9, it is reduced to as follow:

$$\ln L(\lambda) = \ln \left[\frac{\beta}{\mu} \left(\frac{t}{\mu}\right)^{(\beta-1)}\right] + \sum_{i=1}^{n_f} \left(x_1 C_1 + x_2 C_2 + x_3 C_3\right) - \sum_{i=1}^{n} \exp \left[-\left(\frac{t}{\beta}\right)^{\beta} \exp\left(x_1 C_1 + x_2 C_2 + x_3 C_3\right)\right] (10)$$

Applying the maximum likelihood estimation technique, quasi-differential coefficient  $ln[L(\lambda)]$  can be used to find out the parameters. Subsequently, three nonlinear equations can be obtained as follows:

$$\frac{\partial}{\partial x_1} \ln[L(\lambda)] = C_1(x_1C_1 + x_2C_2 + x_3C_3) - C_1 \exp(x_1C_1 + x_2C_2 + x_3C_3) \int_0^t \frac{\beta}{\mu} (\frac{t}{\mu})^{(\beta-1)} dt = 0$$
(11)

$$\frac{\partial}{\partial x_2} \ln[L(\lambda)] = C_2 (x_1 C_1 + x_2 C_2 + x_3 C_3) - C_2 \exp(x_1 C_1 + x_2 C_2 + x_3 C_3) \int_0^t \frac{\beta}{\mu} (\frac{t}{\mu})^{(\beta - 1)} dt = 0 \quad (12)$$

$$\frac{\partial}{\partial x_3} \ln[L(\lambda)] = C_3(x_1C_1 + x_2C_2 + x_3C_3) - C_3 \exp(x_1C_1 + x_2C_2 + x_3C_3) \int_0^t \frac{\beta}{\mu} (\frac{t}{\mu})^{(\beta-1)} dt = 0 \quad (13)$$

After solving these three nonlinear equations the parameters of PHM  $(x_1, x_2 \text{ and } x_3)$  will be obtained.

# **3. EXPERIMENTATION**

Cylindrical bar of AISI D2 steel is used as workpiece material. Dry turning operating have been performed on CNC turning centre as shown in Fig. 1. TiN coated tungsten carbide inserts having specification CNMG 120408, have been used for conducting experiments.

The value cutting parameters are selected from the range provided by the tool manufacturer. The machining parameters (also model covariates) with their levels and their values are listed below in Table 1.



Fig. 1 Turning centre



Fig. 2 Upright materials microscope

Table -1 Process variables with levels

Machining parameters	Cutting speed (m/min)	Feed rate (mm/rev)	Depth of cut (mm)			
Level 1	60	0.15	0.30			
Level 2	100	0.25	0.50			

Experiments have been performed based on the full factorial design of experiment considering three factors with two levels as presented in Table 2.

Table -2 Design of experiment									
Exp. Run	xp. Run   No. of Repetitions   Cutting speed (m/min)   Feed rate (mm/rev)   Depth of cut (n								
1	2	60	0.30	0.25					
2	2	60	0.15	0.50					
3	2	100	0.15	0.25					
4	2	100	0.30	0.50					
5	2	100	0.15	0.50					
6	2	100	0.30	0.25					
7	2	60	0.30	0.50					
8	2	60	0.15	0.25					

Each experimental run has been repeated twice using sixteen number of new cutting tools in order to include random nature of cutting tool life data. While turning, sequential observation (measurement of tool flank wear) has been carried out until the pre-defined threshold flank wear limit ( $VB_{max} = 0.2 \text{ mm}$ ) is reached. For example, the data of sequential wear measurements for experimental run 1 is shown in Table 3. Tool wear has been measured using upright materials microscope as shown in Fig. 2. The same procedures have been continued for all experimental run. Fig.3 shows the evolution of flank wear for different experiment runs.

Sequential	Machining	Tool Wear (mm)		
Measurement no.	Time (s)	<b>Repetition 1</b>	<b>Repetition 2</b>	
1	0	0	0	
2	180	0.059	0.062	
3	360	0.077	0.082	
4	540	0.086	0.099	
5	720	0.101	0.111	
6	900	0.119	0.132	
7	1080	0.135	0.155	
8	1260	0.167	0.191	
9	1440	0.212	0.242	

 Table -3 Sequential measurements of tool wear for experimental run 1





(a) Run 1 (b) Run 2 (c) Run 3 (d) Run 4 (e) Run 5 (f) Run 6 (g) Run 7 (h) Run 8

The cutting tool fails when the tool becomes dull and no longer operates within acceptable quality. The common way of quantifying the tool time to failure (TTF) is to put a limit on the maximum acceptable flank wear( $VB_{max}$ ). Critical tool wear limit is chosen based upon the economic aspect of the machining process and product. As, once a tool gets worn out significantly, it degrades the product quality in terms of the surface roughness and dimensional inaccuracy. At the same time, the non-optimum use of cutting tool leads to increase in tooling cost which in turn increases the overall machining.

In this study, the critical flank wear level equal to 0.2 mm, is selected as the failure criteria this. Sixteen number of new tool inserts were used in this study for eight experiment runs (each of which was repeated two times). Failure time (t) corresponding to the threshold limit ( $VB_{max} = 0.20$  mm) of each tool insert is calculated by interpolating between last two observations ( $i_{th}$  and  $(i + 1)_{th}$ ) by using the following equation:

$$\frac{t - t_i}{t_{i+1} - t_i} = \frac{VB_{\max} - VB_i}{VB_{i+1} - VB_i}$$
(14)

Here it is assumed that the evolution of tool flank wear to be linear as shown in Fig. The similar procedure was followed for other experimental runs to acquire the full dataset layout. The values of tool life for each experimental run using new tool are presented in Table 4.

Exp. Run	Tool Life	Exp. Run	Tool Life
	<b>(s)</b>		<b>(s)</b>
1	1392	9	292
2	1291	10	283
3	2026	11	237
4	1854	12	228
5	352	13	1134
6	342	14	1225
7	145	15	2336
8	134	16	2242

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Table -4	Tool Life	data from	experiment

#### 4. RESULTS AND DISCUSSION

#### 4.1. Estimation of PHM Parameters

Experimental data set has been used to obtained model parameters. Well-known statistical software R was used to estimate parameters of baseline hazard i.e. the parameters of Weibull distribution ( $\beta$ , $\mu$ ). The remaining three parameters( $x_1, x_2$  and  $x_3$ ) of PHM were estimated through another popular statistical software, SPSS. All the four estimated parameters are presented in Table 5.

Table -5 Estimated model parameters						
β	$\beta$ $\mu$ $x_1$ (for speed) $x_2$ (for feed) $x_3$ (for dep					
1.162	999.208	4.261	1.643	0.238		

#### 4.2. Development of the mean residual life function

Once the parameters of the PHM are estimated, the next step is to develop the mathematical equations by substituting these parameters in the equation of reliability and probability density function. After substituting the parameters, the equation of reliability and probability density function are reduced to as below:

$$h(t) = \left[\frac{1.162}{998.208} \left(\frac{t}{998.208}\right)^{(1.162-1)} \exp\left(4.261C_1 + 1.643C_2 + 0.238C_3\right)\right]$$
(15)

$$R(t) = \exp\left[-\left(\frac{t}{998.208}\right)^{1.162} \exp\left(4.261C_1 + 1.643C_2 + 0.238C_3\right)\right]$$
(16)

$$f(t) = \frac{1.162}{998.208} \left(\frac{t}{998.208}\right)^{(1.162-1)} \exp\left[-\left(\frac{t}{998.208}\right)^{1.162} \exp\left(4.261C_1 + 1.643C_2 + 0.238C_3\right)\right] (17)$$

The values of the covariates are normalized so as to know the effect of speed, feed and depth of cut on the hazard function As the speed, feed and depth of cut value are taken in the range of (60 - 100), (0.15 - 0.30) and (0.25 - 0.5), respectively, the normalized value of the covariates will be obtained as follows:

$$C_1 = \frac{V - 80}{100 - 60}, C_2 = \frac{f - 0.225}{0.35 - 0.15} \text{ and } C_3 = \frac{d - 0.375}{0.50 - 0.25} \text{ where } C_1, C_2, C_3 \in [-1, 1]$$

The remaining life of cutting tool till failure while operating under any random cutting parameters combination, can be calculated by substituting the calculated probability density function and its corresponding reliability function in Eq. 9. This model is capable of calculating the remaining tool life at any arbitrarily chosen cutting parameters combination.

$$MRL(t_{0}) = \frac{\int_{t_{0}}^{\infty} t \frac{1.162}{998.208} (\frac{t}{998.208})^{(1.162-1)} \exp\left[-(\frac{t}{998.208})^{1.162} \exp\left(4.261C_{1} + 1.643C_{2} + 0.238C_{3}\right)\right] dt}{\exp\left[-\left(\frac{t_{0}}{998.208}\right)^{1.162} \exp\left(4.261C_{1} + 1.643C_{2} + 0.238C_{3}\right)\right]} - t_{0}$$
(18)

#### 4.3. Cutting Tool Reliability and Hazard Rate under Variable Cutting Conditions

From Eq. 16, the reliability function has been plotted for each of experimental run as shown in Fig. 4. The effect of machining conditions on the reliability function can be seen by comparing between two different runs.





For example, from the comparison between the experimental run 1 and 6 (where cutting speed is variable while feed and depth of cut are constant) and also between the experimental run 2 and 7 (where feed is variable while cutting speed and depth of cut are constant), it can been seen that the effect of speed on cutting tool reliably is much higher than that of feed.

From Eq. 17, the hazard rates have been plotted for each of experimental run as shown in Fig. 5. The effect of different machining parameters on the hazard rate can be seen by comparing between two different runs. As in Fig. 3, the maximum risk of failure is when machining under high cutting speed and feed rate.



#### 4.4. Estimation of Residual Life

After estimating the distribution parameters and constructing the reliability model, next step is to calculate expected residual life. For example, let a cutting tool has been used while machining a part for  $t_0 = 540$  (s) under machining conditions: cutting speed = 70 (m/min), feed rate = 0.20 (mm/rev), depth of cut = 0.3 (mm). Then, mean residual life of can be calculated by the Eq. 19, which tells the information about how long the same tool can be used for further matching under the same matching condition before replacing with a new one. These calculations were done with the help of Matlab software.

$$MRL(540) = \frac{\int_{540}^{\infty} t \frac{1.162}{998.208} (\frac{t}{998.208})^{(1.162-1)} \exp\left[-(\frac{t}{998.208})^{1.162} \exp\left(4.261C_1 + 1.643C_2 + 0.238C_3\right)\right] dt}{\exp\left[-\left(\frac{540}{998.208}\right)^{1.162} \exp\left(4.261C_1 + 1.643C_2 + 0.238C_3\right)\right]} - 540$$
(19)

#### 4.5. Model Validation

To check the accuracy of the presented model, two validation tests have been performed based on randomly selected cutting parameters within the predefined range, which are presented in Table 6. For each validation test, the predicted remaining life estimated from the model was compared with the result of the validation test as presented in Table 7. 

Table -o Experimental runs for model validation						
Validation Test	Cutting speed (m/min)	Feed rate	Depth of			
		(mm/rev)	cut (mm)			
1	70	0.20	0.3			
2	90	0.25	0.4			

Table -7	Com	parison	between	estimated	and ex	perimental results
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Val. Test	Consumed life (s)	Residual life from model (s)	Residual life from experiment (s)	Percentage error (%)
1	540	872.961	824	5.82
2	240	346.427	322	7.45

It can be noticed that, the difference between the estimated and experimentally obtained remaining tool life is less than 8%. Both of the predicted residual cutting tool life close to the experimentally obtained remaining tool life. The error can be minimized if more numbers of tool life data (may be either experimentally collected or from previous history in the case of machining industry) are used for fitting to a distribution model. Because for such case, estimated parameters would have been more accurate.

#### **5. CONCLUSION**

In this work, a methodology based on proportional hazard model (PHM) is introduced for residual tool life estimation, which is intended for optimize cost of machining and maximise the availability time of machine tool. The PHM is found to be appropriate to obtain the operational reliability and the failure density function of cutting tool, as it considers both the time dependent degradation phenomenon and the effect varying process parameters. Using these model, useful remaining life of a cutting tool can be estimated for any desire cutting condition. Hence, decision regarding optimum cutting tool replacement time can be taken.

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