



## DYNAMIC ANALYSIS OF FOOTBRIDGE TO EUROCODE (CASE STUDY OF LEVENTIS FOOTBRIDGE ABA ROAD PORT HARCOURT)

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### ABSTRACT

*This research project presents a dynamic structural analysis of footbridges as per Eurocode (Case Study of Leventis footbridge Aba Road Port Harcourt). The footbridge was simulated using commercial finite element software, Midas FEA. The crowd-footbridge is modelled as a structural oscillator to which some external load is applied. Therefore, the crowd is taken as imposed load rather than as a dynamical system. The Eigenvalue analysis was carried out to obtain the natural frequency  $\omega_n$ , is obtained as the eigenvalue of the characteristics equation and the corresponding mode shape is obtained as the eigenvector of the characteristic equation. The values are compared to Eurocode specification for comfort criteria on footbridge. Time history analysis also was carried out. This method was used to construct time histories of such variables as: displacement and acceleration by calculating the response at the end of a succession of very small time steps. The peak deck acceleration is obtained and compared to Eurocode specification for comfort criteria on footbridge. From the time history graph, it was observed that the maximum acceleration of the footbridge is  $0.56\text{m/s}^2$  which is greater than the Eurocode standard specification of  $0.5\text{m/s}^2$  for maximum comfort. This implies that the footbridge acceleration obtained is not satisfactory and so some measures need to be taken in order to remediate this effect and control the vibrations. Some measures to reduce vibration is recommended.*

**Key words:** Acceleration, Dynamics, Footbridge, Natural frequency, Vibration

### 1. INTRODUCTION

Footbridges (or pedestrian bridges) are structures whose primary purpose is to carry pedestrians over a physical obstacle. They are not designed for vehicular traffic but for pedestrians, cyclists and even animals. The popular case of the London Millennium Bridge which was opened in 2000 experienced synchronous lateral excitation exerted by crowded pedestrians [2]. This has triggered investigations of crowd-structure interaction on footbridges. The Footbridge was closed soon after opening due to lateral swaying experienced as uncomfortable by the pedestrian [9]. The excessive vibration induced by the crowd-structure interaction phenomenon also called Synchronous Lateral Excitation (SLE) by pedestrian who walk on crowded footbridges have attracted increasing attention in the last two decades. While designing footbridges, human induced loads became an important parameter to consider since the dynamic effect of pedestrian load can cause uncomfortable and excessive vibrations due to its low frequency.

The phenomenon of the vulnerability of footbridges to vibration also occurs due to resonant vibration effect of the structure with fundamental frequencies close to the frequency of the pedestrian walking load. In this case, the dynamic load will have a great effect on the functionality of the bridge as the human beings are very sensitive to vibration levels [1].

To remedy such vibration issues due to dynamic loads induced by pedestrians on modern footbridges, it is therefore pertinent that the structure are designed according to recommendations of the standards which requires an analysis of the super structures in the two limit states: Ultimate Limit State (ULS) and Serviceability Limit State (SLS). Few researchers have investigated the dynamic analysis on footbridges around the world.

[8] reported on application of EN 1990/AI vibration serviceability limit state requirements for steel footbridges. According to the report, a brief characteristic of vibration comfort criteria requirements for footbridges according to EN 1990/AI and their comparison with recommendations of different authors and guidelines was presented.

[4] proposed a mathematical model and a computational approach to study the complex multi-physical nonlinear coupled system that results from the interaction between a moving platform and the moving pedestrians. The mathematical model accounts for several nonlinear features of the problem. It also considers the effects of discontinuities in the crowd flow such as obstructions, traffic jams, stop-and-go phenomena. They recommended that microscopic or mesoscopic models of the pedestrian flow could be included in the framework of crowd-structure interaction.

[5] developed a realistic load model to incorporate the effects induced by people walking in the dynamical response of footbridges. The method is based on the motion of equations including coefficients of the rate of pedestrian's lateral force, the pedestrian's density, the rate of synchronized pedestrians to large vibration amplitude. The proposed prediction model was validated by field measured data of two bridges that suffer from lateral vibration.

[7] studied the phenomenon of excessive pedestrian-induced lateral vibrations and full-scale-measurements of some selected bridges under crowd loading. Self-excited pedestrian force was determined via an extensive experimental campaign and a stochastic load model has been proposed for modelling of the frequency and amplitude dependent pedestrian-induced lateral forces of footbridge.

According to [6] the fundamental parameters governing the intensity of vibration perception by humans are: Vibration amplitude, Frequency characteristics of vibrations, direction of the vibration, vibrations impact time (exposure time), repeatability of the vibration, human activity.

[10] investigated the lateral vibration of footbridges by synchronous walking. A dynamic model was then proposed to evaluate the pedestrians' synchronized dynamic forces. According to the available literatures results reveals that cable supported bridges have both structural rationality and elegant features and are very attractive.

Prior to static analysis, dynamic structural analysis is equally required and to determine the effect of dynamic loads on the structure's dynamic response and the influence on users. To satisfy serviceability requirement, while carrying out dynamic analysis of footbridges, it should be verified that the proposed design provides a sufficiently high level of comfort. In order to ensure user comfort and safety and prevent outright structural failure many footbridges today are equipped with tuned mass damper (also known as harmonic absorber).

### 1.1. Research Significance

To carry out dynamic analysis on the leventis footbridge subject to human-induced dynamic loads and compared to the comfort criteria recommended by Eurocode. The aim would be employed using the following objectives:

- To carry out free vibration analysis (Eigenvalue analysis) on the footbridge to obtain the natural angular frequency  $\omega_n$ .
- To carry out time history analysis on the footbridge to obtain variables such as: The deck acceleration, displacement, velocity using forward integration in the time domain.
- To compare the variables such as the natural angular frequency and deck acceleration of the footbridge with that recommended in Eurocode.

## 2. THEORETICAL BASIS

### 2.1. Eigenvalue Analysis

Eigenvalue analysis was employed to undertake free vibration analysis of the bridge. Conventionally, vibration in a structure is analysed as a 3-D damped dynamical system. In this analysis damping and excitation are not considered.

$$M\ddot{u}(t) + Ku(t) = 0 \quad (1)$$

where K = Stiffness matrix of structure

M = Mass matrix of structure

U (t) = Displacement vector of structure

$\ddot{U}(t)$  = Acceleration vector.

If the displacement vector u is presumed to be a linear combination of mode-shape vectors, characterised by the mode shape matrix  $\Phi$  and the combination factors for the selected modes are defined by a vector of time-functions, Y(t). Therefore, the displacement vector becomes:

$$u = \Phi Y(t) \quad (2)$$

Substituting equation (2) into (1)

$$M\Phi\ddot{Y} + K\Phi Y = 0$$

Midas FE

$$y_m(t) : \cos(\omega_n t + \beta_m)$$

A defined the time function Y(t) as;

$$Y(t) = \{y_1(t) \dots y_m(t) \dots y_n(t)\}^T$$

Where n is the total number of degrees freedom in the system.

The combination factors  $y_m(t)$  are assumed as harmonic functions in time defined as

For implementation in Midas FEA, the second derivative  $\ddot{y}_m(t)$  with time of the harmonic function can be written as inverse multiplication with a constant factor  $\lambda_m = \omega_n^2$  of the original function,  $y_m(t)$ .

$$\ddot{y}_m(t) = -\lambda_m y_m(t) \quad (3)$$

Consequently, this assumption can be used to transform equation (3) into

$$(-M\ddot{\phi} + K\phi)Y = 0 \quad (4)$$

Where the matrices  $\lambda$  and  $\phi$  are assembled as

$$\lambda = \begin{bmatrix} \lambda_1 & & \\ & \lambda_m & \\ & & \lambda_n \end{bmatrix}$$

$$\lambda_m = \omega_n^2 \quad (5)$$

$$\phi = [\phi_1 \dots \phi_m \dots \phi_n] \quad (6)$$

Equation (4) is valid for every harmonic function, this implies that every  $y_m(t)$ , can be transformed into

$$K\phi_m - \lambda_m M\phi_m = 0 \quad (7)$$

Equation (7) represents Eigenvalue problem, which must satisfy the condition of equation (8) and from this, natural angular frequency is obtained as the Eigenvalue of the characteristic equation in equation (8).

$$|K - \lambda_m M| = 0 \quad (8)$$

The velocities of the structure are expressed in terms of natural frequencies,  $f_m$  (cycle/time).

$$\omega_n = 2\pi f_m \quad (9)$$

## 2.2. Time History Analysis

Time history analysis was performed using the direct integration method. In this approach, the total analysis time range is sub-divided into a number finite steps and numerical integration of the dynamic equilibrium equation is performed at each time step.

This method is particularly suitable for describing a dynamic system experiencing nonlinearity of stiffness of damping several methods are used to perform numerical integration but MIDAS uses the average acceleration method of the Newmark- $\beta$  method in which the acceleration  $\ddot{U}(t)$  in the time range  $t_i < t < t_{i+1}$  is assumed constant at the average of  $\ddot{U}_i$  and  $\ddot{U}_{i+1}$  in equation (10)

$$\ddot{U}_{(t)} = \frac{\ddot{U}_i + \ddot{U}_{i+1}}{2} = \text{Constant} \quad (10)$$

Consequently, the velocity and displacement at  $t = t_{i+1}$  are expressed as

$$\dot{U}_{i+1} = \dot{U}_i + \frac{\ddot{U}_i + \ddot{U}_{i+1}}{2} \Delta t \quad (11)$$

$$U_{i+1} = U_i + \dot{U}_i \Delta t + \frac{\ddot{U}_i + \ddot{U}_{i+1}}{4} \Delta t^2 \quad (12)$$

Let's use the integration variables of the Newmark-  $\beta$  method to characterise equations (11) and (12).

$$\dot{U}_{i+1} = \dot{U}_i + (1 - \gamma)\Delta t \ddot{U}_i + \gamma \Delta t \ddot{U}_{i+1} \quad (13)$$

$$U_{i+1} = U_i + \Delta t \dot{U}_i + \left(\frac{1}{2} - \beta\right) \Delta t^2 \ddot{U}_i + \beta \Delta t^2 \ddot{U}_{i+1} \quad (14)$$

Where  $\beta = 0.25$ ,  $\gamma = 0.5$

Rearranging equation (14) gives the acceleration at the end of a successive time-step;

$$\ddot{U}_{i+1} = \frac{1}{\beta \Delta t^2} \left\{ U_{i+1} - U_i - \Delta t \dot{U}_i - \left(\frac{1}{2} - \beta\right) \Delta t^2 \ddot{U}_i \right\} \quad (15)$$

Equation (15) is substituted into equation (13) and gives the following expression for the velocity at the end of the time-step

$$\dot{U}_{i+1} = \frac{\gamma}{\beta \Delta t} U_{i+1} - \frac{\gamma}{\beta \Delta t} U_i + \left(1 - \frac{\gamma}{\beta}\right) \dot{U}_i + \left(1 - \frac{\gamma}{2\beta}\right) \Delta t \ddot{U}_i \quad (16)$$

Equations (15) and (16) are substituted back into the dynamic equation of motion and rearranged for the displacement response  $U_{i+1}$  at the end of the increment as follows;

$$\left(\frac{1}{\beta \Delta t^2} M + \frac{\gamma}{\beta \Delta t} C + K\right) U_{i+1} = F + M \left[\frac{1}{\beta \Delta t^2} U_i + \frac{1}{\beta \Delta t} \dot{U}_i + \left(\frac{1}{2\beta} - 1\right) \ddot{U}_i\right] + C \left[\frac{\gamma}{\beta \Delta t} U_i + \left(\frac{\gamma}{\beta} - 1\right) \dot{U}_i + \left(\frac{\gamma}{2\beta} - 1\right) \Delta t \ddot{U}_i\right] \quad (17)$$

## 3. NUMERICAL SIMULATION

### 3.1. Description of the Footbridge (Structural Model)

Detailed geometrical information on structural model of the footbridge is presented in section 1.1.1 of chapter 1 but reiterated here for proper discussion. The foot bridge investigated in this research project is the leventis footbridge Aba Road, Port Harcourt. The bridge is supported by five piers (that is, columns) at 7m interval each and is currently used for pedestrian crossing. The footbridge is made of reinforced concrete all through and spanning 33m lengthwise. The pier is 5.625m high with 0.6m diameter. The deck is supported by a main girder along the longitudinal direction of the beam. The beam is also made of reinforced concrete and is 1m wide, 0.6m deep and 2m long. The bridge width is 2m with a 0.15m thick concrete slab.



Photo 1: Leventis footbridge Aba Road Port Harcourt

### 3.2. Computational Model

The footbridge was simulated using a commercial finite element software, Midas FEA. The entire footbridge was discretized into 56775 elements and 69407 Nodes. A full scale model of the bridge was carried out. The piers were modelled using 3D 8-noded parametric solid element. The deck was modelled using 3D solid element defined by the elastic failure criterion. The embedded reinforcements in the deck was modelled using embedded reinforcement concept and was defined using the von mises failure criterion. The base of the piers were restrained in the translational and rotational degrees of freedom.

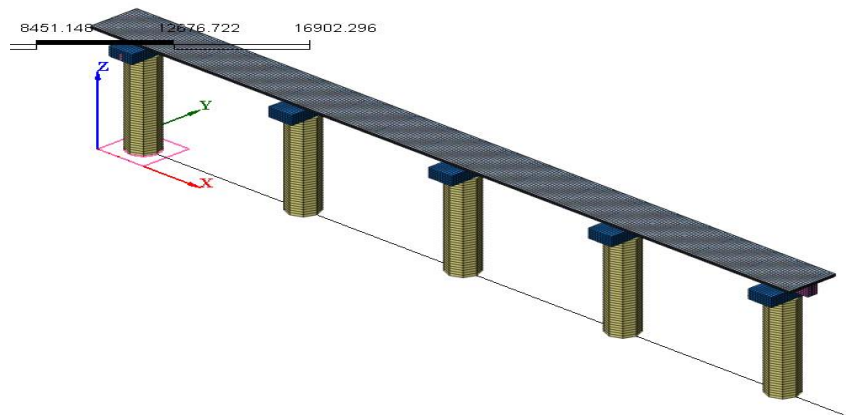


Fig. 1 Model of Leventis footbridge Aba Road, Port Harcourt

### 3.3. Modelling of the Dynamics Actions Induced by Walking Pedestrians

Various models were examined but the model proposed by Figueiredo *et al* [5] was adopted in this paper. The load mode 1 is described as such: In this model, the dynamical forces that represent the walking loads were estimated using equation 13. It is applied on the basis that only one resonant load harmonic was applied on the footbridge's highest modal amplitude point at midspan. It is assumed that the excitation frequency is equal to the footbridge fundamental frequency.

$$F(t) = P\alpha_i \cos(2\pi f_s t) \quad (18)$$

Where:

- P: individual weight taken as 700-800N
- $\alpha_i$ : dynamic coefficient for the  $i$ th harmonic force component
- $i$ : step frequency harmonic multiple
- $f_s$ : step frequency
- $t$ : time in seconds.

## 4. ANALYSIS OF RESULTS

### 4.1. Natural Frequencies and Mode Vibrations

The footbridge's natural frequencies were with the aid of the finite element method simulations. Figure 2 presents the natural frequencies and mode shapes for the footbridge when the footbridge freely vibrates in a particular mode, it moves up and down with a certain configuration or mode shape.

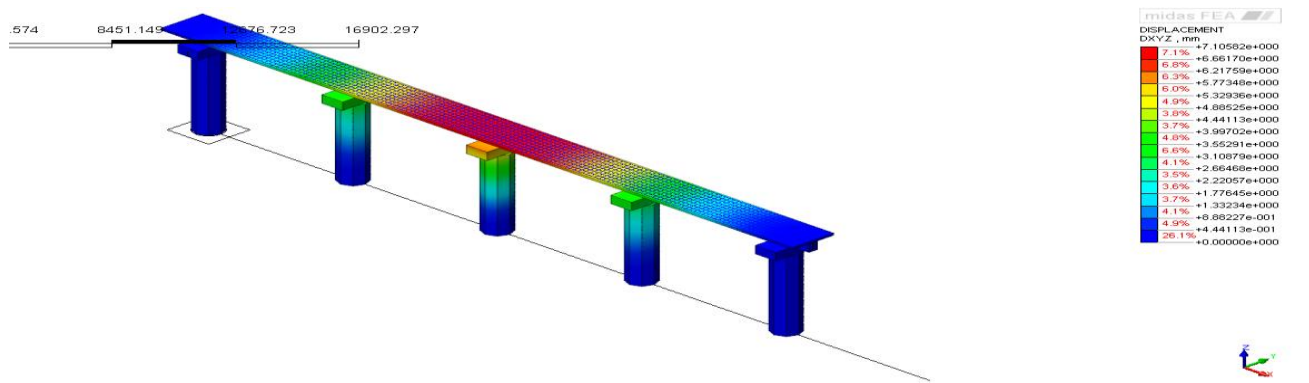
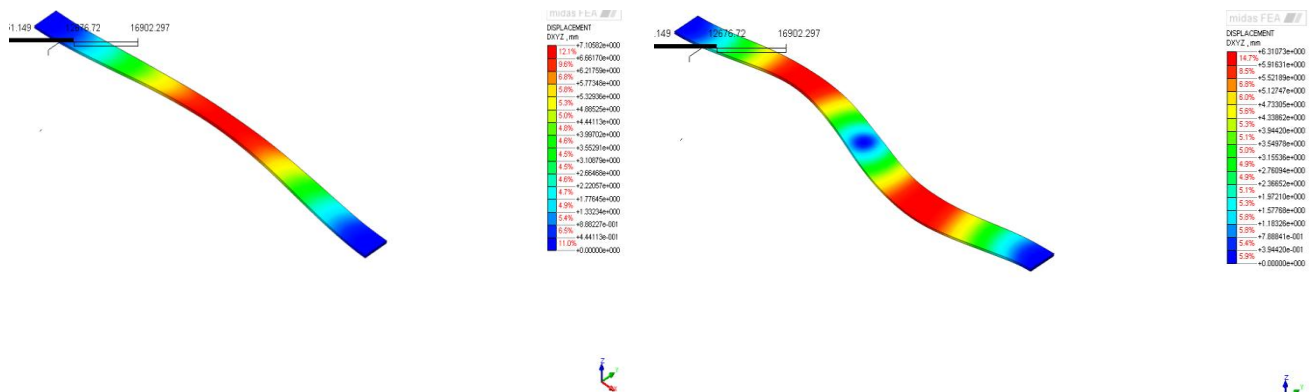
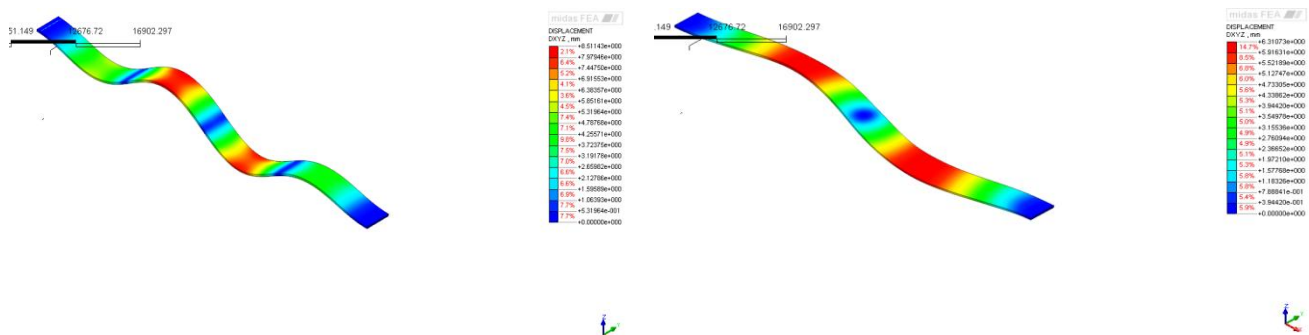


Fig. 2 Displacement contour of the footbridge



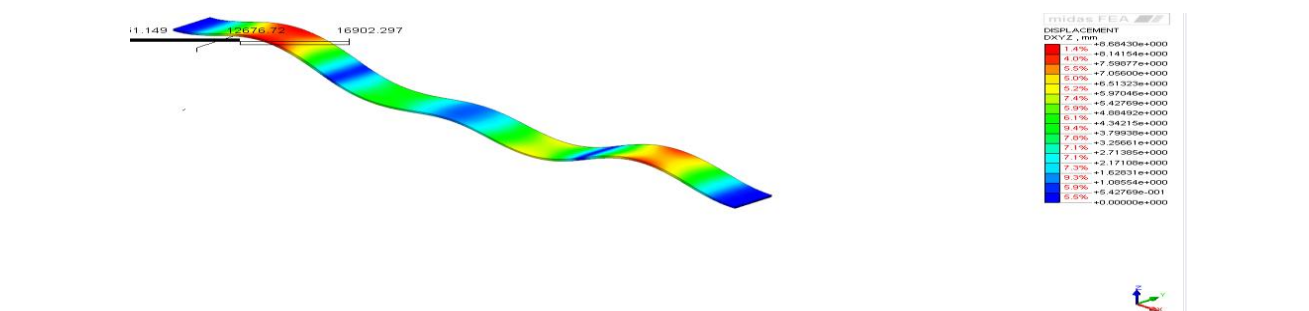
Mode shape associated with the 1<sup>st</sup> natural frequency:  
 $f_{01} = 9.14 \text{ Hz}$

Mode shape associated with the 1<sup>st</sup> natural frequency:  
 $f_{02} = 16.0 \text{ Hz}$



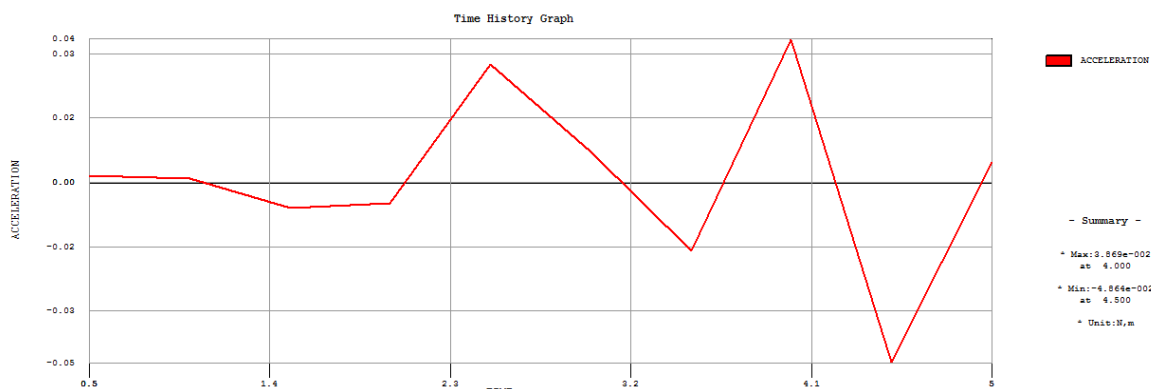
Mode shape associated with the 1<sup>st</sup> natural frequency:  
 $f_{03} = 25.5 \text{ Hz}$

Mode shape associated with the 1<sup>st</sup> natural frequency:  
 $f_{04} = 25.7 \text{ Hz}$



Mode shape associated with the 1st natural frequency:  $f_{05} = 30.2 \text{ Hz}$

Fig. 3 Footbridge vibration modes



**Fig. 4** Time History Graph

Figure 4 illustrates the mode shapes corresponding to the first five natural frequencies of the pedestrian footbridge with span equal to 33m, it could be clearly observed that the maximum displacement occurs in mode shape 4 at 0.097m and at 4.7%.

#### 4.2. Time History Analysis Result

The present analysis proceeds with the evaluation of the footbridge's performance in terms of vibration serviceability effect due to human activities. In this case, dynamic forces induced by pedestrian walking.

The footbridge peak accelerations was determined based on the developed finite element model (FEM) as revealed by the time history analysis graph in figure 4.

From the time history graph, it is shown that the maximum acceleration of the footbridge is  $0.56\text{m/s}^2$  which is greater than the eurocode standard specification of  $0.5\text{m/s}^2$  for maximum comfort (i.e. vertical vibration). This therefore implies that the footbridge acceleration as analyzed is not satisfactory and so some measures need to be taken in order to remediate this effect and control the vibrations. To do this, there are different ways to control vibration on a footbridge deck. Some standard method could be by modifying the stiffness, changing of the structural mass (i.e. changing of some member sizes) therefore changing the entire structural layout. Some standard method may be done by introducing dampers into the structures.

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#### 5. CONCLUSION

Pedestrian bridges are very often lively structures prone to human and used vibrations, necessitating the vibration serviceability assessment in design stage. The Eurocode enable the designer to check the vibration serviceability of the footbridge based on a prediction of the maximum acceleration levels. In this research project, the leventis Footbridge Aba road have been studied. The finite element model of the 33m span footbridge was developed to simulate the physical & dynamic behaviour of the structure and to predict the response under human induced loading.

In design stage, uncertainty with regard to the predicted dynamic properties of the footbridge is inevitable. The footbridge result were obtained through a simple modelling scheme using Midas FEA. Two distinct analysis were carried out namely; Eigenvalue analysis, which reveals the natural frequencies and the vibration anode shapes and also the time history analysis which reveals the peak (maximum) acceleration of the structure with respect to time.

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Having studied the leventis footbridge Aba Road, Port Harcourt, results obtained from Eigenvalue and Time history analysis shows that the peak acceleration exceeds the eurocode specification for maximum comfort. It is therefore pertinent to recommend ways to avoid/reduce the effect of vibration viz:

- ❖ Modify stiffness by changing structural arrangement or elements.
- ❖ Modify the mass of the structure by introducing extra mass.
- ❖ Modify the damping properties of the structure using advanced technique (dampers, Time Division Multiplexing (TDM) etc.

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