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Non local Infinitely Solid Cylinder Composed on Two Different Thermoelastic Materials

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ABSTRACT

In this work, we consider a two-dimensional problem of an infinitely long solid cylinder consisting of two different homogeneous and isotropic thermoelastic materials within the context of the fractional order theory of thermoelasticity. The lateral surface of the cylinder is taken to be traction free and is subjected to a known temperature distribution which is a function of time t and z. There are no body forces or heat sources affecting the medium. Laplace and exponential Fourier transform techniques are used to solving the problem. The inverse Laplace and exponential Fourier transforms are obtained using a numerical technique.

The predictions of the fractional order theory are discussed and compared with those for the generalized theory of thermoelasticity. We also study the effect of the fractional derivative parameters of the two media on the behavior of the solution. Numerical results are computed and represented graphically for the temperature, displacement and stress distributions.

Key words: Fractional calculus; Solid Cylinder; Different Materials; Laplace transform; Fourier transforms

1. INTRODUCTION

Lord and Shulman [1] introduced the theory of generalized thermoelasticity with one relaxation time by using the Maxwell-Cattaneo law of heat conduction instead of the conventional Fourier's law. The heat equation associated with this theory is hyperbolic and hence eliminates the paradox of infinite speeds of propagation inherent in both the uncoupled and the coupled theories of thermoelasticity. The uniqueness of solution for this theory was proved under different conditions by Ignaczak [2], Sherief and Dhaliwal [3] and by Sherief [4]. Exact solution for a problem of a spherical cavity was obtained by Sherief and Saleh [5]. Some problems for a penny-shaped crack and a mode I crack were solved by Sherief and El-Maghraby [6-7]. This theory was extended by Sherief et al. [8] to micropolar media. Anwar and Sherief [9] studied A Problem in Generalized Thermoelasticity for an Infinitely Long Annular Cylinder Composed of Two Different Materials.

Fractional calculus has been used successfully to modify many existing models of physical processes. Caputo and Mainardi [10-11] and Caputo [12] found good agreement with experimental results when using fractional derivatives for a description of viscoelastic materials and established the connection between fractional derivatives and the theory of linear viscoelasticity.

The solution obtained by using ordinary derivatives predicts an instantaneous response while that obtained by using fractional derivatives predicts a retarded response that depends on the history of the applied causes. This is more in accord with physical observations [13].

The general space-time-fractional heat conduction equation in the one-dimensional case has been formulated by Gorenflo et al [14]. Povstenko [15] made a review of thermoelasticity that uses fractional heat conduction equation. The theory of thermal stresses based on the heat conduction equation with the Caputo time-fractional derivative is used by Povstenko [16] to investigate thermal stresses in an infinite body with a circular cylindrical hole. Povstenko proposed and investigated new models that use fractional derivative in [17-18].

The fractional order theory of thermoelasticity was derived by Sherief et al. [19]. It is a generalization of both the coupled and the generalized theories of thermoelasticity. Sherief and Abd El Latief [20, 21] have solved a 1D problems for a half space and for spherical cavity in this theory, solved a 2D problem of half- space [22], studied the effect of the fractional derivative parameter on fractional thermoelastic material with variable thermal conductivity [23] and applied this theory to a 1D problem for a half-space overlaid by a thick layer of a different materials [24].

Recently, Hamza et al. [25] derive a new theory of thermoelasticity associated with two relaxation times using the methodology of fractional calculus, Hamza et al. [26] have solved 1D problems in the context of this theory, derive a new mathematical model of Maxwell's equations in an electromagnetic field using the physical principles of fractional calculus[27]. Some contribution works that use fractional calculus can be found in [28-30].

In this work, we solve a 2D problem of an infinitely long solid cylinder consisting of two different homogeneous and isotropic thermo elastic materials within the context of the fractional order theory of thermoelasticity [19]. The lateral surface of the cylinder is taken to be traction free and is subjected to a known temperature distribution. The solution is obtained for different values of the fraction parameters of the two media. The fractional parameters effects on the media in radial and axial directions are discussed.

2. FORMULATION OF THE PROBLEM

We consider a two-dimensional problem of an infinitely long solid cylinder consisting of two different homogeneous and isotropic thermo elastic materials. The inner layer occupies the region $0 \le r \le a$, $-\infty < z < \infty$ and the outer layer occupies the region $a \le r \le b$, $-\infty < z < \infty$, where (r, ϕ, z) are cylindrical polar coordinates with unit vectors $\underline{e}_r$, $\underline{e}_\phi$ and $\underline{e}_z$. The lateral surface of the cylinder is taken to be traction free and is subjected to a known temperature distribution which is a function of time t and z. There are no body forces or heat sources affecting the medium.

Due to the physical of the problem, all considered quantities will depend on r, z and t only.

The displacement vector $\underline{\mathbf{u}}_i$ takes the form

$$\underline{\mathbf{u}}_i = u_i(r,z,t)\underline{e}_r + w_i(r,z,t)\underline{e}_z, \; i = 1, 2. \tag{1}$$

where the suffix 1 refers to the inner medium and the suffix 2 refers to the outer medium.

The cubical dilatation e_i in medium i is thus given by

$$e_i = \frac{\partial u_i}{\partial r} + \frac{u_i}{r} + \frac{\partial w_i}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(ru_i) + \frac{\partial w_i}{\partial z}. \tag{2}$$

The equation of motion in vector form can be written as

$$\mu_i \nabla^2 \underline{\mathrm{u}}_i + (\lambda_i + \mu_i)\,\mathrm{grad\,div}\,\underline{\mathrm{u}}_i - \gamma_i\,\mathrm{grad}T_i = \rho_i \frac{\partial^2 \underline{\mathrm{u}}_i}{\partial t^2}. \tag{3}$$

where λ_i, μ_i are Lamé's constants, ρ_i is the density, T_i is the absolute temperature and γ_i is a material constant given by $(3\lambda_i + 2\mu_i)\alpha_{ti}$ where α_{ti} is the coefficient of linear thermal expansion. ∇^2 is Laplace's operator in cylindrical polar coordinates, given by

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\left(\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2}.$$

Applying the divergence operator to both sides of equation (3), we obtain

$$(\lambda_i + 2\mu_i)\nabla^2 e_i - \gamma_i \nabla^2 T_i = \rho_i \frac{\partial^2 e_i}{\partial t^2}. \tag{4}$$

The time fraction heat conduction equations [19] for both media are given by

$$k_i \nabla^2 T_i = \rho_i c_{E_i}\left(1 + \tau_i \frac{\partial^{\alpha_i}}{\partial t^{\alpha_i}}\right)\frac{\partial T_i}{\partial t} + \gamma_i T_0 \left(1 + \tau_i \frac{\partial^{\alpha_i}}{\partial t^{\alpha_i}}\right)\frac{\partial e_i}{\partial t}, \; i = 1, 2. \tag{5}$$

where k_i is the thermal conductivity, c_i is the specific heat at constant strain, T_0 is a reference temperature assumed to be such that $|T_i - T_0| << 1$, τ_i is the relaxation time and α_i is the fraction order parameters for the two media, satisfy $0 \le \alpha_i \le 1$ in the Caputo sense.

The constitutive relations have the form:

$$\sigma_{rr_i} = 2\mu_i \frac{\partial u_i}{\partial r} + \lambda_i e - \gamma_i (T_i - T_0), \tag{6a}$$

$$\sigma_{\phi\phi_i} = 2\mu_i \frac{u_i}{r} + \lambda_i e - \gamma_i (T_i - T_0), \tag{6b}$$

$$\sigma_{zz_i} = 2\mu_i \frac{\partial w_i}{\partial z} + \lambda_i e - \gamma_i (T_i - T_0), \tag{6c}$$

$$\sigma_{rz_i} = \mu_i \left(\frac{\partial w_i}{\partial r} + \frac{\partial u_i}{\partial z} \right), \tag{6d}$$

$$\sigma_{r\phi_i} = \sigma_{\phi z_i} = 0. \tag{6e}$$

We also have the modified Fourier's law of heat conduction [19], namely

$$q_{ri} + \tau_i \frac{\partial^{\alpha_i} q_{ri}}{\partial t^{\alpha_i}} = -k_i \frac{\partial T_i}{\partial r}, \tag{7a}$$

$$q_{zi} + \tau_i \frac{\partial^{\alpha_i} q_{zi}}{\partial t^{\alpha_i}} = -k_i \frac{\partial T_i}{\partial z}, \tag{7b}$$

where q_{ri} and q_{zi} are the heat flux components in the radial and z-direction respectively.

Let us introduce the following non-dimensional variables:

$$(r^*, z^*, u_i^*, w_i^*) = c_1\eta_1 (r, z, u_i, w_i), \; t^* = c_1^2 \eta_1 t, \; \tau_i^* = c_1^{2\alpha_i} \eta_1^{\alpha_i} \tau_i$$

$$\sigma_{ij}^* = \frac{\sigma_{ij}}{(\lambda_1 + 2\mu_1)}, \theta_i^* = \frac{\gamma_1 (T_i - T_0)}{(\lambda_1 + 2\mu_1)}, \; (q_{ri}^*, q_{zi}^*) = \frac{\gamma_1}{k_1 c_1 \eta_1 (\lambda_1 + 2\mu_1)} (q_{ri}, q_{zi})$$

where $c_1 = \sqrt{\dfrac{\lambda_1 + 2\mu_1}{\rho_1}}, \eta_1 = \dfrac{\rho_1 c_{E1}}{k_1}$.

Using the above non-dimensional variables, (dropping the asterisks for convenience), The governing equations (3)-(7a) in non-dimensional form become

$$\nabla^2 \underline{\mathbf{u}}_i + (\beta_i^2 - 1)\, grad\, e_i - \beta_i^2 \zeta_i\, grad\, \theta_i = \beta_i^2 V_i \frac{\partial^2 \underline{\mathbf{u}}_i}{\partial t^2}, \tag{8}$$

$$\nabla^2 e_i - \xi_i \nabla^2 \theta_i = V_i \frac{\partial^2 e_i}{\partial t^2}, \tag{9}$$

$$q_{ri} + \tau_i \frac{\partial^{\alpha_i} q_{ri}}{\partial t^{\alpha_i}} = -\chi_i \frac{\partial \theta_i}{\partial r}, \tag{10a}$$

$$q_{zi} + \tau_i \frac{\partial^{\alpha_i} q_{zi}}{\partial t^{\alpha_i}} = -\chi_i \frac{\partial \theta_i}{\partial z}, \tag{10b}$$

$$\nabla^2 \theta_i = \upsilon_i \left(1 + \tau_i \frac{\partial^{\alpha_i}}{\partial t^{\alpha_i}} \right) \frac{\partial \theta_i}{\partial t} + \varepsilon_i \left(1 + \tau_i \frac{\partial^{\alpha_i}}{\partial t^{\alpha_i}} \right) \frac{\partial e_i}{\partial t}, \tag{11}$$

$$\sigma_{rr_i} = \omega_i \frac{\partial u_i}{\partial r} + \psi_i e_i - \delta_i \theta_i, \tag{12a}$$

$$\sigma_{\phi\phi_i} = \omega_i \frac{u_i}{r} + \psi_i e_i - \delta_i \theta_i, \tag{12b}$$

$$\sigma_{zz_i} = \omega_i \frac{\partial w_i}{\partial z} + \psi_i e_i - \delta_i \theta_i, \tag{12c}$$

$$\sigma_{rzi} = \omega_i\left(\frac{\partial w_i}{\partial r} + \frac{\partial u_i}{\partial z}\right), \tag{12d}$$

$$\sigma_{r\phi i} = \sigma_{\phi zi} = 0, \tag{12e}$$

where $\delta_i = \dfrac{\gamma_i}{\gamma_1}$, $\beta_i = \dfrac{\lambda_i + \mu_i}{\mu_i}$, $V_i = \dfrac{\rho_i(\lambda_1 + 2\mu_1)}{\rho_1(\lambda_i + 2\mu_i)}$, $\zeta_i = \dfrac{\gamma_i(\lambda_1 + 2\mu_1)}{\gamma_1(\lambda_i + 2\mu_i)}$,

$$\varepsilon_i = \frac{\gamma_i\,\gamma_1 T_0}{k_i \eta_1(\lambda_1 + 2\mu_1)},\ \upsilon_i = \frac{\eta_i}{\eta_1},\ \omega_i = \left(\frac{2\mu_i}{\lambda_1 + 2\mu_1}\right),\ \psi_i = \left(\frac{\lambda_i}{\lambda_1 + 2\mu_1}\right),\ \chi_i = \frac{k_i}{k_1}$$

The boundary conditions of the problem are assumed to be as follows:

$$\sigma_{rr} = 0\ ,\quad \sigma_{rz} = 0\ ,\ \theta = f(t,z)\ ,\ \text{at } r = b \tag{13}$$

where b is the radius of the lateral cylinder.

The continuity conditions of the problem are given by

$$\theta_1 = \theta_2\ ,\ u_1 = u_2\ ,\ w_1 = w_2\ ,\sigma_{rr1} = \sigma_{rr2}, \sigma_{rz1} = \sigma_{rz2},\ q_{r1} = q_{r2},\ \text{at } r = a. \tag{14}$$

where a is the radius of the inner cylinder.

3. SOLUTION IN THE TRANSFORMED DOMAIN

Applying the Laplace transform with parameter s defined by the relation

$$\overline{f}(r,z,s) = \mathrm{L}[f(r,z,t)] = \int_0^\infty e^{-st}\ f(r,z,t)\,dt.$$

to both sides of equations (8-11), we obtain

$$\nabla^2 \underline{\overline{\mathrm{u}}}_i + (\beta_i^2 - 1)\ \mathrm{grad}\,\overline{e}_i - \beta_i^2\zeta_i\ \mathrm{grad}\,\overline{\theta}_i = \beta_i^2 V_i s^2\,\underline{\overline{\mathrm{u}}}_i, \tag{15}$$

$$\xi_i \nabla^2 \overline{\theta}_i = (\nabla^2 - V_i s^2)\,\overline{e}_i, \tag{16}$$

$$\overline{q}_{ri}(1 + \tau_i s^{\alpha_i}) = -\chi_i\,\frac{\partial \overline{\theta}_i}{\partial r}, \tag{17}$$

$$\left[\nabla^2 - \upsilon_i s\left(1 + \tau_i s^{\alpha_i}\right)\right]\overline{\theta}_i = \varepsilon_i s\left(1 + \tau_i s^{\alpha_i}\right)\overline{e}_i. \tag{18}$$

Eliminating $\overline{e}_i$ between equations (16) and (18), we get

$$\left\{\nabla^4 - \left[(s\upsilon_i + \xi_i\varepsilon_i s)\left(1 + \tau_i s^{\alpha_i}\right) + s^2 V_i\right]\nabla^2 + V_i\upsilon_i s^3\left(1 + \tau_i s^{\alpha_i}\right)\right\}\overline{\theta}_i = 0, \tag{19}$$

The above equation can be factorized as

$$\left(\nabla^2 - k_{i1}^2\right)\left(\nabla^2 - k_{i2}^2\right)\overline{\theta}_i = 0 \tag{20}$$

where k_{ij}, i= 1, 2, j = 1,2, are the roots of the characteristic equation,

$$\left\{k^4 - \left[(s\upsilon_i + \xi_i\varepsilon_i s)\left(1 + \tau_i s^{\alpha_i}\right) + s^2 V_i\right]k^2 + V_i\upsilon_i s^3\left(1 + \tau_i s^{\alpha_i}\right)\right\}\overline{\theta}_i = 0 \tag{21}$$

The solution of equation (20) can be written in the form

$\overline{\theta}_i = \overline{\theta}_{i1} + \overline{\theta}_{i2}$ where θ_{ij} is the solution of the equation.

$$\left(\nabla^2 - k_{ij}^2\right)\overline{\theta}_{ij} = 0\ ,\qquad i,j = 1,2, \tag{22}$$

Similarly eliminating $\overline{\theta}$ between equations (16) and (18), we get

$$\left\{\nabla^4 - \left[(s\upsilon_i + \xi_i\varepsilon_i s)\left(1 + \tau_i s^{\alpha_i}\right) + s^2 V_i\right]\nabla^2 + V_i\upsilon_i s^3\left(1 + \tau_i s^{\alpha_i}\right)\right\}\overline{e}_i = 0. \tag{23}$$

Applying Fourier transform with parameter p defined by the relation,

$f^*(r,p,t) = \mathrm{F}[f(r,z,t)] = \int_{-\infty}^{\infty} e^{ipz} f(r,z,t)\,dz$, where $i = \sqrt{-1}$.

to both sides of equation (22), we obtain

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \omega_{ij}^2\right)\bar{\theta}_{ij}^* = 0. \quad (24)$$

Where, $\omega_{ij}^2 = p^2 + k_{ij}^2$

Equation (24) is the modified Bessel differential equations whose solution is

$$\bar{\theta}_i^* = \sum_{j=1}^{2} A_{ij}\,(k_{ij}^{\ 2} - V_i\, s^2)\, I_0(\omega_{ij} r) + B_{ij}\,(k_{ij}^{\ 2} - V_i\, s^2)\, K_0(\omega_{ij} r). \quad (25)$$

where I_0 and K_0 are the modified Bessel functions of the first and second kinds ,respectively of order zero and A_{ij} , B_{ij} are parameters depending on *s* and *p*.

The function $K_0\,(q_{ij}\,r)$ is not bounded at the inner region. The solution is given by

$$\bar{\theta}_1^* = \sum_{j=1}^{2} A_{1j}\,(k_{1j}^{\ 2} - s^2)\, I_0(\omega_{1j} r)\ , \quad (27)$$

$$\bar{\theta}_2^* = \sum_{j=1}^{2}\ (k_{2j}^2 - V_2 s^2)\left[A_{2j}\, I_0(\omega_{2j} r) + B_{2j}\, K_0(\omega_{2j} r)\right]. \quad (28)$$

In a similar manner, we obtain

$$\bar{e}_1^* = \sum_{j=1}^{2} A_{1j}\, k_{1j}^{\ 2}\, I_0(\omega_{1j} r)\ , \quad (29)$$

$$\bar{e}_2^* = \xi_2 \sum_{j=1}^{2}\ k_{2j}^2\left[A_{2j}\, I_0(\omega_{2j} r) + B_{2j}\, K_0(\omega_{2j} r)\right]. \quad (30)$$

Substituting from equations (27-28) into equation (17), we get the heat flux components in the form

$$\bar{q}_{r1}^* = \frac{1}{1+\tau_1 s^{\alpha_1}} \sum_{j=1}^{2} A_{1j}\, \omega_{1j}\,(k_{1j}^{\ 2} - s^2)\, I_1(\omega_{1j} r) \quad (31)$$

$$\bar{q}_{r2}^* = \frac{\chi_2}{1+\tau_2 s^{\alpha_2}} \sum_{j=1}^{2}\ \omega_{2j}\,(k_{2j}^2 - V_2 s^2)\left[A_{2j}\, I_1(\omega_{2j} r) - B_{2j}\, K_1(\omega_{2j} r)\right] \quad (32)$$

Using the vector identity, $\mathrm{curl\,curl}\,(\underline{\mathbf{u}}) = \mathrm{grad\ div}\,(\underline{\mathbf{u}}) - \nabla^2(\underline{\mathbf{u}})$ then equation (8) takes the form

$$\beta_i^2\ grad\, e_i - curl\, curl\, \underline{\mathbf{u}}_i - \beta_i^2 \zeta_i\, grad\, \theta_i = \beta_i^2 V_i \frac{\partial^2 \underline{\mathbf{u}}_i}{\partial t^2}. \quad (33)$$

The second term of the above equation upon equation (1) has the form

$$curl\, curl\, \underline{\mathbf{u}}_i = \left(\frac{\partial^2 w_i}{\partial r\,\partial z} - \frac{\partial^2 u_i}{\partial z^2}\right)\underline{\mathbf{e}}_r + \left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_i}{\partial z}\right) - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w_i}{\partial r}\right)\right)\underline{\mathbf{e}}_z. \quad (34)$$

Substituting from equation (34) into equation (33) and equating components of $\underline{e}_r$ and $\underline{e}_z$ on both sides, we obtain

$$\beta_i^2 \frac{\partial e_i}{\partial r} - \frac{\partial^2 w_i}{\partial r\,\partial z} + \frac{\partial^2 u_i}{\partial z^2} - \beta_i^2 \zeta_i \frac{\partial \theta_i}{\partial r} = \beta_i^2 V_i \frac{\partial u_i}{\partial t^2}, \quad (35)$$

$$\beta_i^2 \frac{\partial e_i}{\partial z} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_i}{\partial z}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w_i}{\partial r}\right) - \beta_i^2 \zeta_i \frac{\partial \theta_i}{\partial z} = \beta_i^2 V_i \frac{\partial^2 w_i}{\partial t^2}. \quad (36)$$

Differentiating equation (2) with respect to *r*, we obtain

$$\frac{\partial^2 w_i}{\partial z \partial r} = \frac{\partial e_i}{\partial r} - \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r u_i)\right]. \tag{37}$$

Substituting from equation (37) into equation (35), we get

$$\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r)\right) + \frac{\partial^2}{\partial z^2} - \beta_i^2 V_i \frac{\partial^2}{\partial t^2}\right] u_i = \frac{\partial}{\partial r}\Big((1-\beta_i^2) e_i + \beta_i^2 \zeta_i \theta_i\Big). \tag{38}$$

Assuming that $u_i = \dfrac{\partial f_i}{\partial r}$ equation (38) reduces to

$$\frac{\partial}{\partial r}\left[\nabla^2 - \beta_i^2 V_i \frac{\partial^2}{\partial t^2}\right] f_i = \frac{\partial}{\partial r}\Big((1-\beta_i^2) e_i + \beta_i^2 \zeta_i \theta_i\Big). \tag{39}$$

The above equation is obtained by using the relation

$$\left(\nabla^2 - \frac{1}{r^2}\right)\frac{\partial f}{\partial r} = \frac{\partial}{\partial r}(\nabla^2 f) \tag{40}$$

Integrating both sides of equation (39) with respect to *r*, we get

$$\left[\nabla^2 - \beta_i^2 V_i \frac{\partial^2}{\partial t^2}\right] f_i = \Big((1-\beta_i^2) e_i + \beta_i^2 \zeta_i \theta_i\Big) \tag{41}$$

Applying Laplace and Fourier transforms to both sides of equation (41), we obtain

$$\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) - m_i^2\right)\overline{f}_i^* = \left((1-\beta_i^2)\,\overline{e}^*{}_i + \beta_i^2 \zeta_i\,\overline{\theta}^*{}_i\right). \tag{42}$$

where $m_i^2 = \beta_i^2 V_i s^2 + p^2$

Substituting from equations (27-30) into the right-hand side of equation (42), we obtain

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - m_1^2\right)\overline{f}^*{}_1 = \sum_{j=1}^{2} A_{1j}\,(k_{1j}{}^2 - \beta_1^2 s^2)\, I_0(\omega_{1j} r) \tag{43}$$

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - m_2^2\right)\overline{f}^*{}_2 = \zeta_2 \sum_{j=1}^{2} \Big[\, A_{2j}\, I_0(\omega_{2j} r)(k_{2j}^2 - \beta_2^2 V_2 s^2) + B_{2j}\, K_0(\omega_{2j} r)(k_{2j}^2 - \beta_2^2 V_2 s^2)\,\Big] \tag{44}$$

The solutions of equations (43) and (44) take the form

$$\overline{f}^*{}_1 = C_1 I_0(m_1 r) + A_{11} I_0(\omega_{11} r) + A_{12} I_0(\omega_{12} r) \tag{45}$$

$$\overline{f}^*{}_2 = C_2 I_0(m_2 r) + D_2 K_0(m_2 r) + \sum_{j=1}^{2} \xi_2 (A_{2j} I_0(\omega_{2j} r) + B_{2j} K_0(\omega_{2j} r)) \tag{46}$$

Thus the solution of equation (38) takes the form

$$\overline{u}^*{}_1 = C_1 m_1 I_1(m_1 r) + \sum_{j=1}^{2} A_{1j} q_{1j} I_1(\omega_{1j} r) \tag{47}$$

$$\overline{u}^*{}_2 = C_2 m_2 I_1(m_2 r) - D_2 m_2 K_1(m_2 r) + \sum_{j=1}^{2} \xi_2 (A_{2j} q_{2j} I_1(\omega_{2j} r) - B_{2j} q_{2j} K_1(\omega_{2j} r)) \tag{48}$$

In order to find the displacement components w_i applying Laplace and Fourier transforms to both sides of equation (2)

$$\overline{w}_i^* = \frac{1}{I\,p}\left[\overline{e}_i^* - \frac{1}{r}\frac{\partial}{\partial r}\left(r\,\overline{u}_i^*\right)\right],\ I = \sqrt{-1}. \tag{49}$$

Using the equations (29- 30) and (47-48), we obtain

$$\overline{w}^{*}_{1} = \frac{I}{p}\left[C_1 m_1^2 I_0(m_1 r) + p^2 \sum_{j=1}^{2} A_{1j} I_0(\omega_{1j} r) \right] \tag{50}$$

$$\overline{w}^{*}_{2} = \frac{I}{p}\left\{ m_2^2 \left(C_2 I_0(m_2 r) + D_2 K_0(m_2 r) \right) + \zeta_2 p^2 \sum_{j=1}^{2} \left[A_{2j} I_0(\omega_{2j} r) + B_{2j} K_0(\omega_{2j} r) \right] \right\} \tag{51}$$

Applying Laplace and Fourier transforms into the equations (12), we obtain

$$\overline{\sigma}^{*}_{rri} = \omega_i \frac{\partial \overline{u}^{*}_i}{\partial r} + \psi_i \overline{e}^{*}_i - \delta_i \overline{\theta}^{*}_i \ , \tag{52a}$$

$$\overline{\sigma}^{*}_{\phi\phi i} = \omega_i \frac{\overline{u}^{*}_i}{r} + \psi_i \overline{e}^{*}_i - \delta_i \overline{\theta}^{*}_i \ , \tag{52b}$$

$$\overline{\sigma}^{*}_{zzi} = \omega_i \frac{\partial \overline{w}^{*}_i}{\partial z} + \psi_i \overline{e}^{*}_i - \delta_i \overline{\theta}^{*}_i \ , \tag{52c}$$

$$\overline{\sigma}^{*}_{rzi} = \omega_i \left(\frac{\partial \overline{w}^{*}_i}{\partial r} + I\, p \overline{u}^{*}_i \right), \tag{52d}$$

Substituting from equations (27-30), (47-48) and (50-51) into equations (52), we get

$$\overline{\sigma}^{*}_{rr1} = C_1 m_1 \left[\omega_1 m_1 I_0(m_1 r) - \frac{\omega_1}{r} I_1(m_1 r) \right] + \sum_{j=1}^{2} A_{1j} \left[\left(\omega_1 q_{1j}^2 + k_{1j}^2 (\psi_1 - 1) + s^2 \right) I_0(\omega_{1j} r) - \frac{q_{1j}\,\omega_1}{r} I_1(\omega_{1j} r) \right] \tag{53}$$

$$\begin{aligned} \overline{\sigma}^{*}_{rr2} = {} & C_2 m_2 \left[\omega_2 m_2 I_0(m_2 r) - \frac{\omega_2}{r} I_1(m_2 r) \right] + D_2 m_2 \left[\omega_2 m_2 k_0(m_2 r) + \frac{\omega_2}{r} k_1(m_2 r) \right] \\ & + \sum_{j=1}^{2} \left\{ A_{2j} \left[\left(\omega_2 \zeta_2 q_{2j}^2 + (\psi_2 \zeta_2 + \delta_2) k_{2j}^2 - \delta_2 V_2 s^2 \right) I_0(\omega_{2j} r) - \frac{\zeta_2 q_{2j} \omega_2}{r} I_1(\omega_{2j} r) \right] \right. \\ & \left. + B_{2j} \left[\left(\omega_2 \zeta_2 q_{2j}^2 + (\psi_2 \zeta_2 - \delta_2) k_{2j}^2 + \delta_2 V_2 s^2 \right) K_0(\omega_{2j} r) + \frac{\zeta_2 q_{2j} \omega_2}{r} K_1(\omega_{2j} r) \right] \right\} \end{aligned} \tag{54}$$

$$\overline{\sigma}^{*}_{rz1} = \frac{\omega_1 I}{p} \left[C_1 m_1 (m_1^2 + p^2) I_1(m_1 r) + 2p^2 \sum_{j=1}^{2} A_{1j}\, q_{1j}\, I_1(\omega_{1j} r) \right], \tag{55}$$

$$\overline{\sigma}^{*}_{rz2} = \frac{\omega_2}{2Ip} \left\{ m_2 (m_2^2 + p^2) \left[C_2 I_1(m_2 r) - D_2 K_1(m_2 r) \right] + 2\zeta_2 p^2 \sum_{j=1}^{2} q_{2j} \left[A_{2j} I_1(\omega_{2j} r) - B_{2j} K_1(\omega_{2j} r) \right] \right\} \tag{56}$$

$$\begin{aligned} \overline{\sigma}^{*}_{zz1} = {} & \omega_1 C_1 m_1^2 I_0(m_1 r) + A_{11} \left[\omega_1 \left(k_{11}^2 + \omega_{11}^2 \right) + \psi_1 k_{11}^2 - \left(k_{11}^2 - s^2 \right) \right] I_0(\omega_{11} r) \\ & + A_{12} \left[\omega_1 \left(k_{12}^2 + \omega_{12}^2 \right) + \psi_1 k_{12}^2 - \left(k_{12}^2 - s^2 \right) \right] I_0(\omega_{12} r) , \end{aligned} \tag{57}$$

$$\bar{\sigma}^{*}_{zz2} = C_2\omega_2 m_2^2 I_0(m_2 r) + D_2\omega_2 m_2^2 K_0(m_2 r)$$
$$+\sum_{j=1}^{2} \left\{A_{2j}\left[\omega_2\left(\zeta_2 k_{2j}^2 + R_{2j}\omega_{2j}^2\right) + \psi_1\zeta_2 k_{2j}^2 - \delta_2\left(k_{2j}^2 - V_2 s^2\right)\right] I_0(\omega_{2j} r)\right. . \tag{58}$$
$$\left. + B_{2j}\left[\omega_2\left(\zeta_2 k_{2j}^2 + R_{2j}\omega_{2j}^2\right) + \psi_1\zeta_2 k_{2j}^2 - \delta_2\left(k_{2j}^2 - V_2 s^2\right)\right] K_0(\omega_{2j} r)\right\}$$

Applying Laplace and Fourier transforms into the boundary conditions of the problem, we obtain

$$\bar{\sigma}^{*}_{rr2} = 0 \ , \ \bar{\sigma}^{*}_{rz2} = 0 \ , \ \bar{\theta}_2^{*} = \bar{f}^{*}(s,p) \ , \text{ at r = b.} \tag{59}$$

Applying Laplace and Fourier transforms into the continuity conditions of the problem, we obtain

$$\bar{\theta}^{*}_1 = \bar{\theta}^{*}_2 \ , \ \bar{u}^{*}_1 = \bar{u}^{*}_2 \ , \ \bar{w}^{*}_1 = \bar{w}^{*}_2 \ , \bar{\sigma}^{*}_{rr1} = \bar{\sigma}^{*}_{rr2} , \bar{\sigma}^{*}_{rz1} = \bar{\sigma}^{*}_{rz2} , \bar{q}^{*}_1 = \bar{q}^{*}_2 \ , \text{ at r = a} \tag{60}$$

Applying the boundary and continuity conditions (60) and (61), we arrive at a linear system of nine equations which contain nine parameters $A_{11}, A_{12}, A_{21}, A_{22}, B_{21}, B_{22}, C_1, C_2$ and D_2 as following

$$C_2\left[\omega_2 m_2^2 I_0(m_2 b) - \frac{\omega_2 m_2}{b} I_1(m_2 b)\right] + D_2\left[\omega_2 m_2^2 k_0(m_2 b) - \frac{\omega_2 m_2}{b} k_1(m_2 b)\right]$$
$$+\sum_{j=1}^{2}\left\{A_{2j}\left[\left(\omega_2 R_{2j}\omega_{2j}^2 + \psi_2\zeta_2 k_{2j}^2 - \delta_2\left(k_{2j}^2 - V_2 s^2\right)\right) I_0(\omega_{2j} b) - \frac{R_{2j} q_{2j}\omega_2}{b} I_1(\omega_{2j} b)\right]\right. \tag{61a}$$
$$\left. + B_{2j}\left[\left(\omega_2 R_{2j}\omega_{2j}^2 + \psi_2\zeta_2 k_{2j}^2 - \delta_2\left(k_{2j}^2 - V_2 s^2\right)\right) K_0(\omega_{2j} b) + \frac{R_{2j} q_{2j}\omega_2}{b} K_1(\omega_{2j} b)\right]\right\} = 0$$

$$m_2\beta_2^2 V_2 s^2\left[C_2 I_1(m_2 b) - D_2 K_1(m_2 b)\right] + \sum_{j=1}^{2}\omega_{2j} k_{2j}^2\left(\zeta_2 + R_{2j}\right)\left[A_{2j} I_1(\omega_{2j} b) - B_{2j} K_1(\omega_{2j} b)\right] = 0 \tag{61b}$$

$$\sum_{j=1}^{2}\ (k_{2j}^2 - V_2 s^2)\left[A_{2j}\, I_0(\omega_{2j} b) + B_{2j}\, K_0(\omega_{2j} b)\right] = 0 \tag{61c}$$

$$\sum_{j=1}^{2}\ \left\{A_{1j}\,(k_{1j}^2 - s^2) I_0(\omega_{1j} a) - (k_{2j}^2 - V_2 s^2)\left[A_{2j}\, I_0(\omega_{2j} a) + B_{2j}\, K_0(\omega_{2j} a)\right]\right\} = 0 \tag{61d}$$

$$C_1 m_1 I_1(m_1 a) + A_{11}\omega_{11} I_1(\omega_{11} a) + A_{12}\omega_{12} I_1(\omega_{12} a) - C_2 m_2 I_1(m_2 a)$$
$$+ D_2 m_2 K_1(m_2 a) - \sum_{j=1}^{2}\ R_{2j}\left(A_{2j}\omega_{2j} I_1(\omega_{2j} a) - B_{2j}\omega_{2j} K_1(\omega_{2j} a)\right) = 0 , \tag{61e}$$

$$C_1 m_1^2 I_0(m_1 a) + A_{11}\left(k_{11}^2 + \omega_{11}^2\right) I_0(\omega_{11} a) + A_{12}\left(k_{12}^2 + \omega_{12}^2\right) I_0(\omega_{12} a)$$
$$-\left\{C_2 m_2^2 I_0(m_2 a) + D_2 m_2^2 K_0(m_2 a) + \sum_{j=1}^{2}\ \left[A_{2j}\left(\zeta_2 k_{2j}^2 + R_{2j}\omega_{2j}^2\right) I_0(\omega_{2j} a)\right.\right. \tag{61f}$$
$$\left.\left. + B_{2j}\left(\zeta_2 k_{2j}^2 + R_{2j}\omega_{2j}^2\right) K_0(\omega_{2j} a))\right]\right\} = 0$$

$$C_1\left[\omega_1 m_1^2 I_0(m_1 a) - \frac{\omega_1 m_1}{a} I_1(m_1 a)\right] + A_{11}\left[\left(\omega_1\omega_{11}^2 + \psi_1 k_{11}^2 - k_{11}^2 + s^2\right) I_0(\omega_{11} a) - \frac{\omega_{11}\omega_1}{a} I_1(\omega_{11} a)\right]$$
$$+ A_{12}\left[\left(\omega_1\omega_{12}^2 + \psi_1 k_{12}^2 - k_{12}^2 + s^2\right) I_0(\omega_{12} a) - \frac{\omega_{12}\omega_1}{a} I_1(\omega_{12} a)\right]$$
$$-C_2\left[\omega_2 m_2^2 I_0(m_2 a) - \frac{\omega_2 m_2}{a} I_1(m_2 a)\right] - D_2\left[\omega_2 m_2^2 k_0(m_2 a) - \frac{\omega_2 m_2}{a} k_1(m_2 a)\right] \tag{61g}$$
$$-\sum_{j=1}^{2}\left\{A_{2j}\left[\left(\omega_2 R_{2j}\omega_{2j}^2 + \psi_2\zeta_2 k_{2j}^2 - \delta_2\left(k_{2j}^2 - V_2 s^2\right)\right) I_0(\omega_{2j} a) - \frac{R_{2j}\omega_{2j}\omega_2}{a} I_1(\omega_{2j} a)\right]\right.$$
$$\left. + B_{2j}\left[\left(\omega_2 R_{2j}\omega_{2j}^2 + \psi_2\zeta_2 k_{2j}^2 - \delta_2\left(k_{2j}^2 - V_2 s^2\right)\right) K_0(\omega_{2j} a) + \frac{R_{2j}\omega_{2j}\omega_2}{a} K_1(\omega_{2j} a)\right]\right\} = 0$$

$$\left[C_1 m_1 \beta_1^2 s^2 I_1(m_1 a) + 2A_{11}\omega_{11}k_{11}^2 I_1(\omega_{11}a) + 2A_{12}\omega_{12}k_{12}^2 I_1(\omega_{12}a)\right]$$
$$-\left\{m_2\beta_2^2 V_2 s^2\left[C_2 I_1(m_2 a) - D_2 K_1(m_2 a)\right] + \sum_{j=1}^{2}\omega_{2j}k_{2j}^2\left(\zeta_2 + R_{2j}\right)\left[A_{2j}I_1(\omega_{2j}a) - B_{2j}K_1(\omega_{2j}a)\right]\right\} = 0 \tag{61h}$$

$$A_{11}\omega_{11}(k_{11}^2 - s^2)I_1(\omega_{11}a) + A_{12}\omega_{12}(k_{12}^2 - s^2)I_1(\omega_{12}a)$$
$$-\chi_2\tau\sum_{j=1}^{2}\omega_{2j}(k_{2j}^2 - V_2 s^2)\left[A_{2j}I_1(\omega_{2j}a) - B_{2j}K_1(\omega_{2j}a)\right] = 0 \tag{61i}$$

where $\tau = \dfrac{1+\tau_1 s^{\alpha_1}}{1+\tau_2 s^{\alpha_2}}$

thus, the solution to the problem in the transformed domain is obtained by solving the above system.

4. NUMERICAL RESULTS

The Double inverse of Laplace Fourier transforms was obtained by using the inversion formula of the transforms and Romberg numerical integration technique. FORTRAN programming language was used on a personal computer. The accuracy maintained was 5 significant digits for both the numerical integration and the inversion of the Laplace transform. The numerical method outlined in [31] was used to obtain the inverse Laplace transforms for the temperature, displacement and stress distributions.

During numerical computations, the inner material was taken to be made of pure aluminium material and the outer material was taken to be made of pure zinc material. The constants of the problem were taken as shown in table1.

Table -1 The Material constants

a=1	μ_1 =3.86×10^{10}	T_0 = 293	ρ_1 =2707
b = 2	μ_2 = 3.88× 10^{10}	τ_1 = 0.02	ρ_2= 7144
c_1=896	k_1 =204	τ_2 = 0.005	h = 0.2
c_2 = 384.3	k_2 = 112.2	λ_1 = 4.7×10^{10}	λ_2 = 9.07×10^{10}
α_{t1}=8.418d-5	α_{t2}=4.106d-5		

The computations were carried out for the function

$$\theta = f(r,z,t) = \begin{cases} 1 & ,if\ -h \le z \le h \\ 0 & ,otherwise \end{cases}$$, at r = b.

where h is a width of temperature distribution, applying Laplace and Fourier transforms,
we obtain

$$\bar{\theta}^*(b,p,s) = \frac{2\sin ph}{ps}.$$

The investigation of the effect of the outer cylinder whose fractional derivative parameter α_2 at z = 0 in the radial direction has been carried out in the preceding discussions.

The computations were performed for a wide range of ($0 \le r \le 2$), different values of $\alpha_2 = \{0.5, 0.999, 1\}$, one value of time, namely $t = 0.13$ and $\alpha_1 = 1$. This enables us to represent the typical numerical results in Figures (1) - (3), for the temperature θ, displacement *u* and the thermal stresses σ_{rr}, respectively. These figures show that for $\alpha_2 = 0.999, 1$ the waves not pass to the inner media and it will vanish in the outer media whose fractional parameter α_1 = 1. This result is in agreement with the generalized theory of thermoelasticity that the waves have finite speed. While $\alpha_2 = 0.5$ the waves will pass to the inner media since it has infinite speed. This result is consistent with [24].

The investigation of the effect of the outer cylinder whose fractional derivative parameter α_2 inside the outer cylinder r = 1.5 in the direction parallel to the axis of the cylinders has been carried out in the preceding discussions.

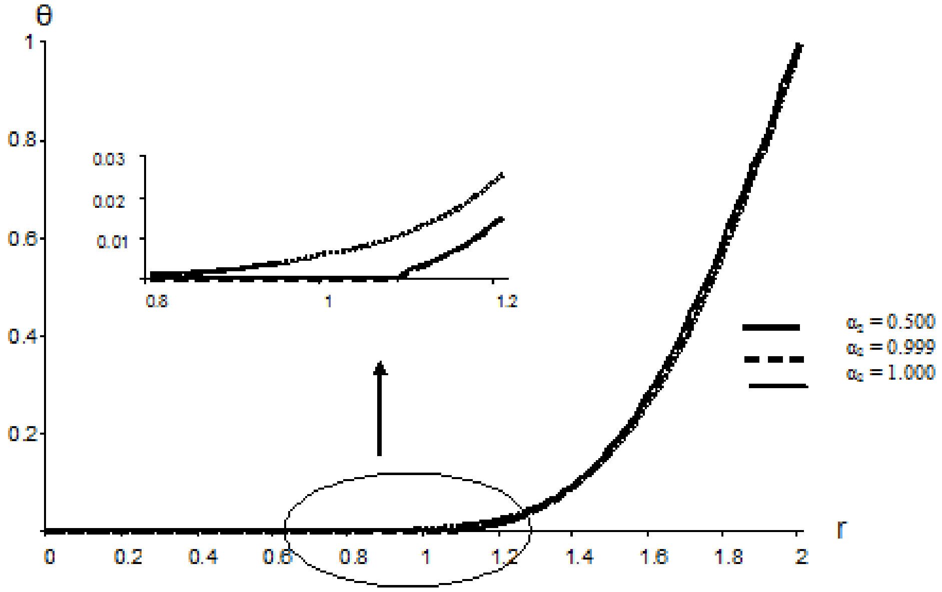

Fig. 1 α_2 radial effect on the temperature distribution for α_1=1 and t = 0.13

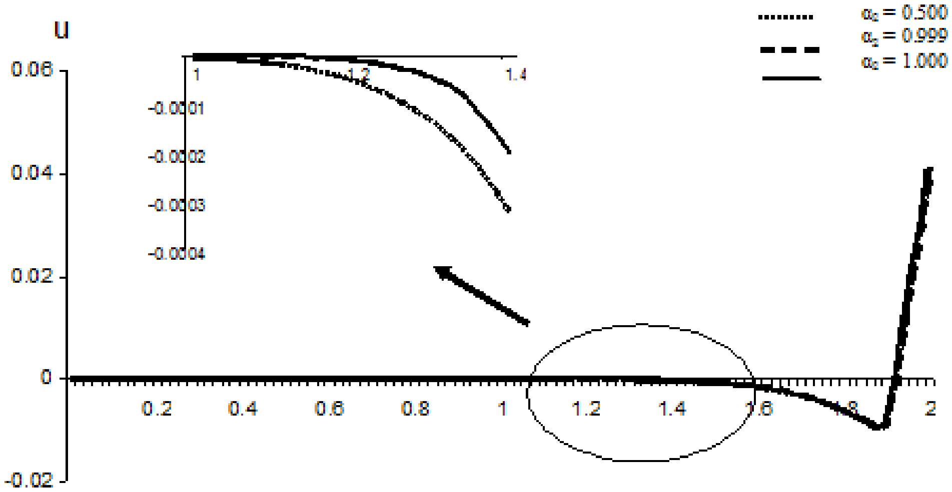

Fig. 2 α_2 radial effect on the displacement distribution for α_1=1 and t = 0.13

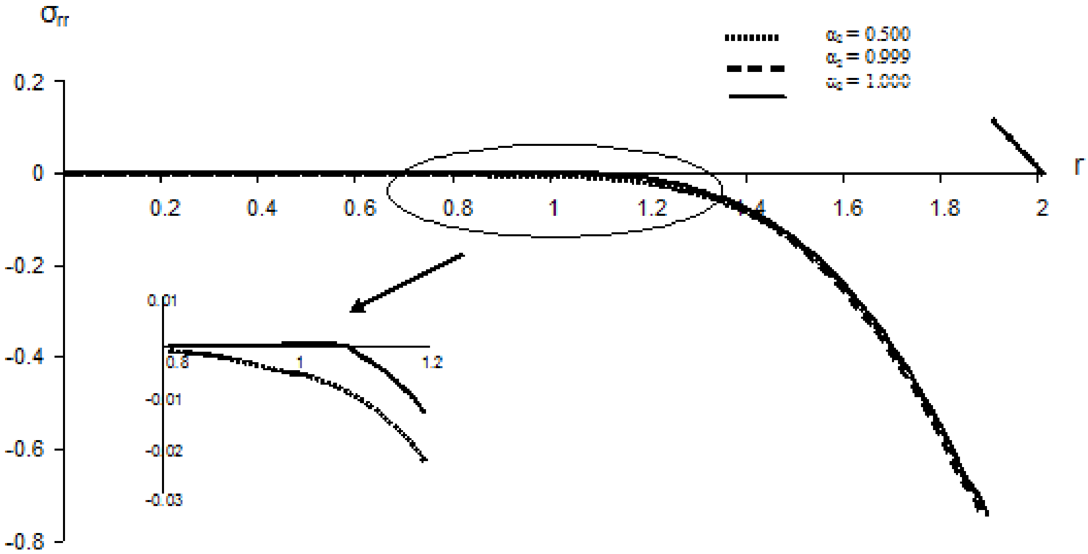

Fig. 3 α_2 radial effect on the stresses distribution for α_1=1 and t = 0.13

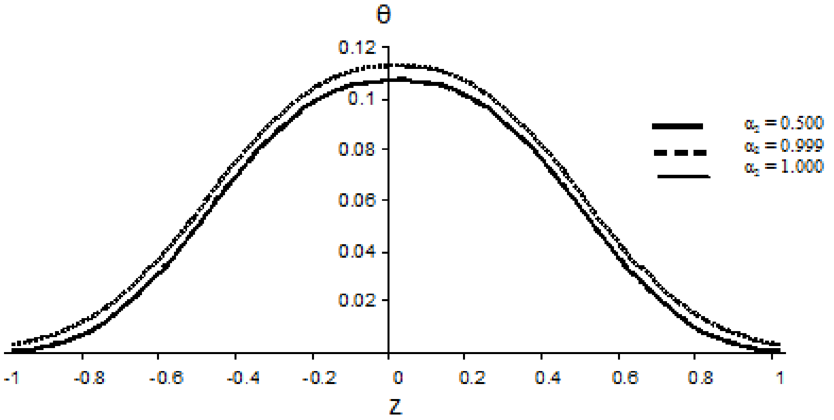

Fig. 4 α_2 axial effect on the temperature Distribution for α_1=1 and t = 0.1

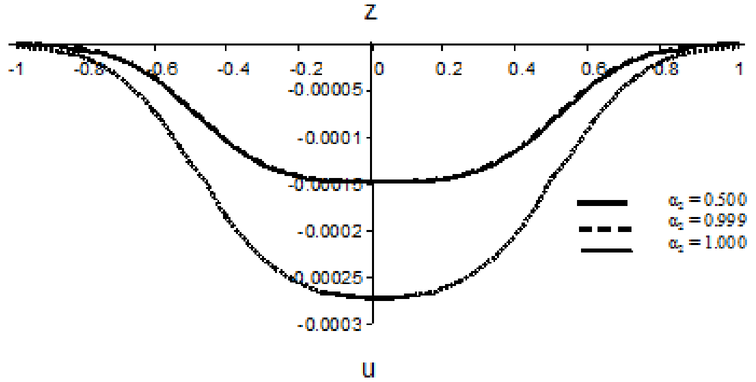

Fig. 5 α_2 axial effect on the displacement distribution for α_1=1 and t = 0.1

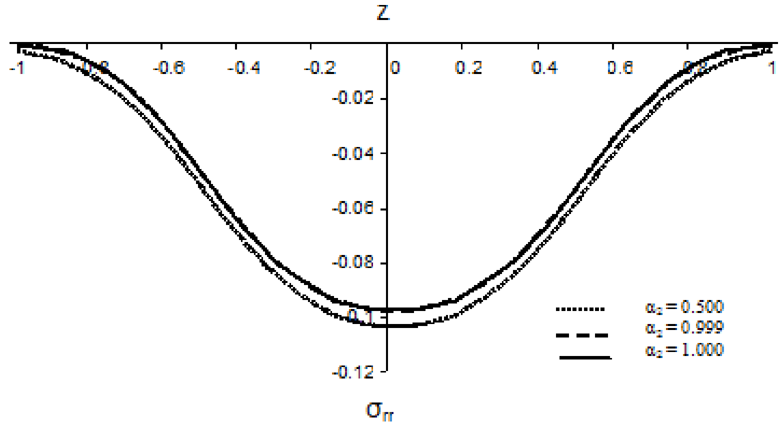

Fig. 6 α_2 axial effect on the stresses distribution for α_1=1 and t = 0.1

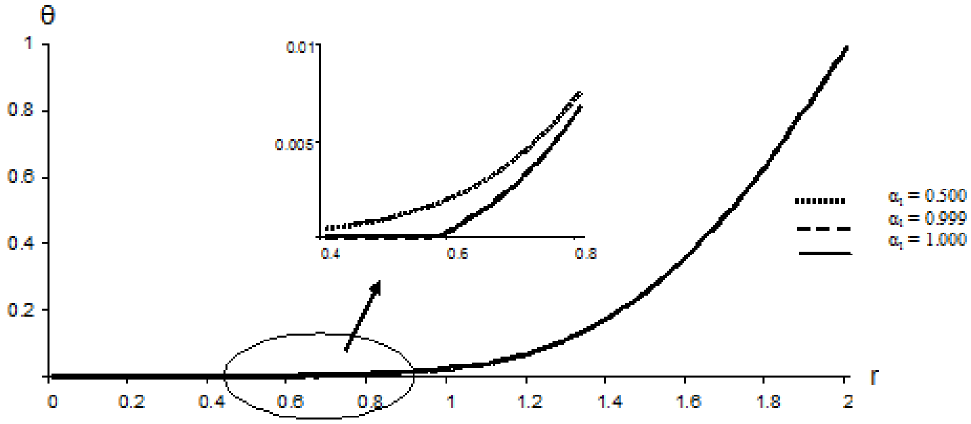

Fig. 7 α_1 radial effect on the temperature distribution for α_2=1 and t = 0.2

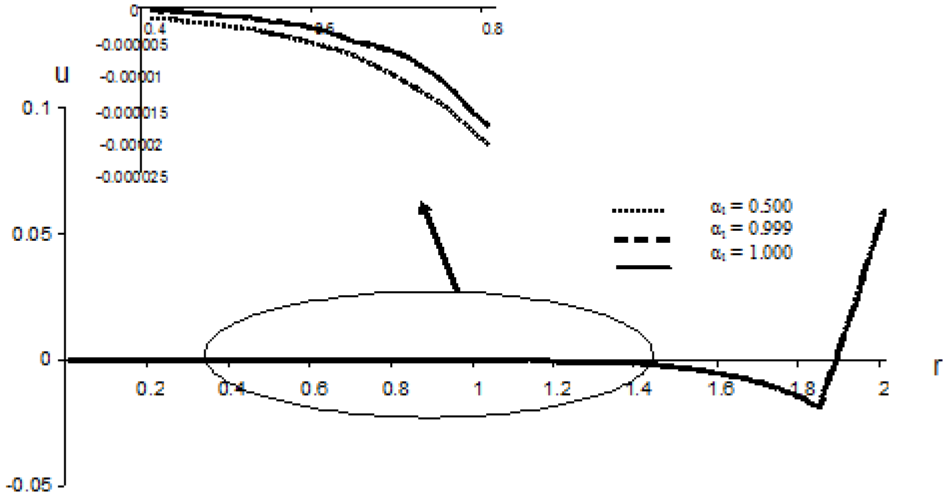

Fig. 8 α_1 radial effect on the displacement distribution for α_2=1 and t = 0.2

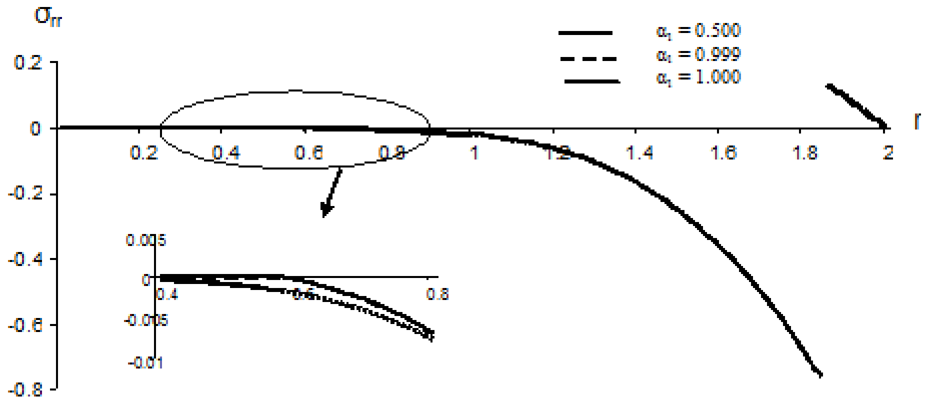

Fig. 9 α_1 radial effect on the stresses Distribution for α_2=1 and t = 0.2

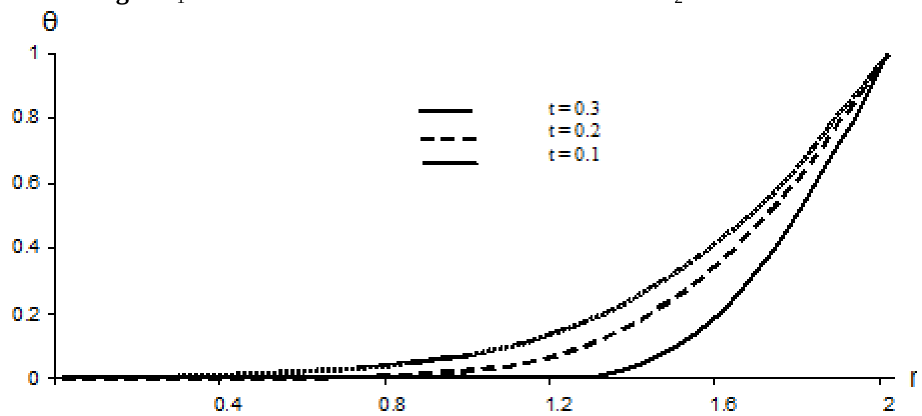

Fig. 10 α_1 radial effect on the temperature distribution for α_1= 1 and α_2= 1

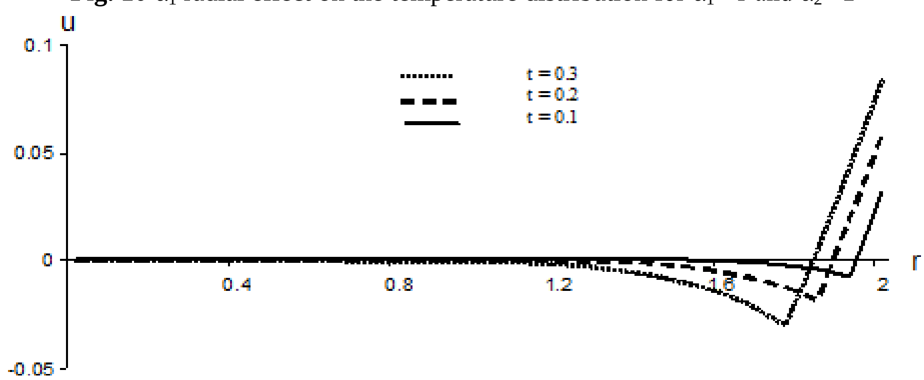

Fig. 11 Displacement Distribution for α_1= 1 and α_2= 1

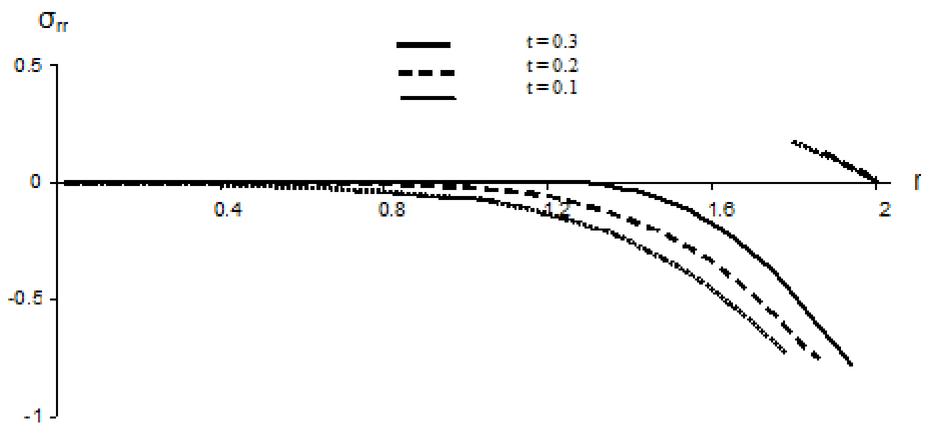

Fig. 12 Stresses Distribution for $\alpha_1 = 1$ and $\alpha_2 = 1$

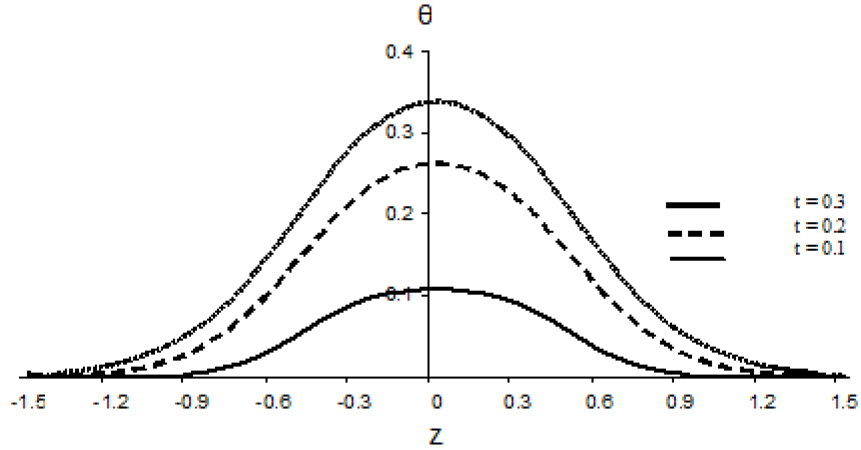

Fig. 13 Temperature Distribution for $\alpha_1 = 1$ and $\alpha_2 = 1$

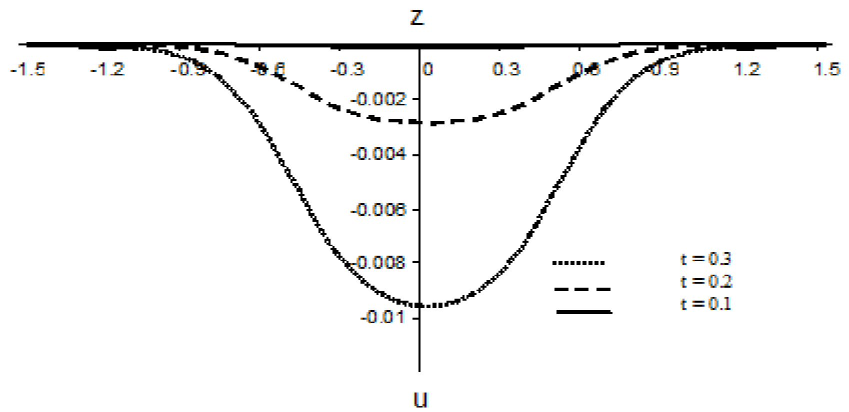

Fig. 14 Displacement Distribution for $\alpha_1 = 1$ and $\alpha_2 = 1$

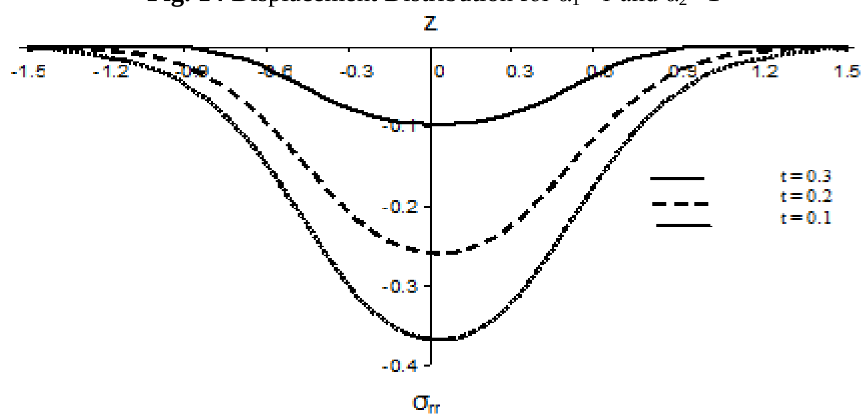

Fig. 15 Stresses Distribution for $\alpha_1 = 1$ and $\alpha_2 = 1$

Figures (4), (5) and (6), respectively, exhibit the temperature, displacement and stress distributions for a wide range of ($-1 \le z \le 1$), α_2 = {0, 0.999, 1} when t = 0.1, α_1 = 1 at r=1.8 parallel to z-axis. Obviously, the effects of the fractional parameter α_2 on the field profiles in the axis direction are noticeable than the radial direction. Also for α_2 =0.999, 1 the waves vanish rapidly than when α_2 = 0.5. Thus the wave has the finite speed for α_2 =0.999, 1 while for α_2 = 0.5 it has infinite.

The investigation of the effect of the inner cylinder whose fractional derivative parameter α_1 at z = 0 in the radial direction has been carried out as follow.

Figures (7), (8) and (9), respectively, depict the temperature, displacement and stress distributions for α_1 = {0, 0.999, 1}, α_2 = 1 when the time is large enough to penetrate to the inside media, namely, t = 0.2. These figures show the effects of the inner media fractional parameter α_1. We notice that for α_1 = 0.999, 1 the waves have finite speed while for α_1 = 0.5 the wave has infinite speed.

Finally, the computation was carried out for different values of time, namely, t={0.1, 0.2, 0.4}. The temperature, displacement and stress profiles are shown at z = 0 and r =1.5 in figures (10-12) and (13-15), on respectively. These figures show that as the time increase as the waves penetrates the media to a larger distance.

5. CONCLUSION

When the fractional time parameter is close to one, then the solution seems to behave like the generalized theory of thermoelasticity. This result indicates that the fractional model of thermoelasticity may preserve the advantage of the generalized theory of thermoelasticity that the velocity of waves is finite.

The fractional parameters have a noticeable effect in the axial direction of the cylinders more than the radial direction.

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