



Bispectrum and Bicoherence Analysis using HOSA (Higher Order Spectral Analysis) –A Review

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ABSTRACT

For many years correlation and power spectrum have been principal tools for digital signal processing applications. The information enclosed in the power spectrum is essentially that of the autocorrelation sequence; which is adequate for comprehensive statistical descriptions of Gaussian signals of known means. However, there are practical conditions where one needs to look beyond autocorrelation of a signal to extract information regarding deviation from Gaussianity and the presence of phase relations. Higher order spectra, also known as polyspectra, are spectral representations of higher order statistics, i.e. moments and cumulants of third order and beyond. HOS (higher order statistics or higher order spectra) can detect deviations from linearity, stationarity or Gaussianity in the signal. Signals which are non-linear, non-stationary and non-Gaussian in nature are therefore more advantageous to analyze with HOS compared to the use of second-order correlations and power spectra. In this paper, a review on the HOS techniques was made mainly including the Bispectrum and Bicoherence analysis and their application in various fields.

Key words: Correlation, Power Spectrum, Autocorrelation, HOS, Linearity, Stationarity, Gaussianity, Polyspectra, Bispectrum, Bicoherence analysis

1. INTRODUCTION

Higher-order spectra, defined in terms of the higher-order moments or cumulants of a signal. The Higher-Order Spectral Analysis (HOSA) Toolbox in Matlab provides complete higher-order spectral analysis capabilities for signal processing applications. The toolbox is an excellent resource for the advanced researcher and the practicing engineer, as well as the novice student who wants to learn about concepts and algorithms in statistical signal processing. The HOSA Toolbox is a collection of M-files that implement a variety of advanced signal processing algorithms for spectral estimation, polyspectral estimation, and computation of time-frequency distributions, with applications such as parametric and nonparametric blind system identification, time delay estimation, harmonic retrieval, direction of arrival estimation, parameter estimation of Volterra (nonlinear) models, and adaptive linear prediction. Other potential applications include acoustics, biomedicine, econometrics, exploration seismology, nondestructive testing, oceanography, plasma physics, radar, sonar, speech etc. [1]

The study in [2] is to select a set of higher order spectral features for emotion/stress recognition system. 50 Bispectral (28 features) and Bicoherence (22 features) based higher order spectral features were extracted from speech signal and its glottal waveform. These features were combined with Inter-Speech features to further improve the recognition rates. There is more information in a stochastic non-Gaussian or deterministic signal than conveyed by its auto-correlation or spectrum. Signal processing algorithms based on higher order spectra are now available for use in commercial and military applications. The emergence of low cost, very high speed hardware chips and the ever growing availability of fast computers now demand that we extract more information than we have been doing in the past from signals, so that better decisions can be made. All of the new algorithms that have been developed using higher-order spectra are application driven. That is why higher order spectra (HOS) are an attractive scheme for the above mentioned field. [3]

Electroencephalogram (EEG) is a reliable reflection of many physiological factors modulating the brain. Bispectrum is very useful for analyzing non-Gaussian signals such as EEG, and detecting the quadratic phase coupling between distinct frequency components in EEG signals. The aim of the study in [4] was to test the existence of nonlinear phase coupling within the EEG signals in a certain psycho-physiological state; meditation. Eleven meditators and four non-meditators

were asked to do meditation by listening to the guidance of the master, and 10 subjects were asked to do meditation by themselves. Bispectrum estimation was applied to analyze EEG signals, before and during meditation. EEG signals were recorded using 16-channel PowerLab. ANOVA test was used to establish significant changes in Bispectrum parameters, during two different states (before and during meditation).

In this paper [5], bispectral analysis is applied on the widespread diffuse cross-frequency interactive effects. The Event Related Potentials (ERPs) research method was used in this and it could collect the widespread diffuse cross-frequency from Mild Cognitive Impairment (MCI) patients' brain wave. The brain wave data were collected from 12 MCI subjects, 12 healthy elderly, and 12 healthy young. The findings showed that the decreased interhemispheric coherence of 8.8Hz for MCI compared with healthy elderly in the central-parietal cortex to respective surrounding sites and each MCI subject showed significantly widespread diffuse pattern of cross-frequency interactions in comparison with the healthy controls in the left central-parietal and right frontal.

Use of the EEG signal is preferred over facial expression, as people cannot control the EEG signal generated by their brain; the EEG ensures a stronger reliability in the psychological signal. However, because of its uniqueness between individuals and its vulnerability to noise, use of EEG signals can be rather complicated. In this [6], a methodology was proposed to conduct EEG based emotion recognition by using a filtered bispectrum as the feature extraction subsystem and an artificial neural network (ANN) as the classifier. The bispectrum is theoretically superior to the power spectrum because it can identify phase coupling between the nonlinear process components of the EEG signal. In the feature extraction process, to extract the information contained in the bispectrum matrices, a 3D pyramid filter is used for sampling and quantifying the bispectrum value. Experiment results show that the mean percentage of the bispectrum value from 5×5 nonoverlapped 3D pyramid filters produces the highest recognition rate. The extracted bispectrum values of an EEG signal using 3D filtering as a feature extraction method is suitable for use in an EEG-based emotion recognition system.

Article [7] aims to detect obstructive sleep apnea (OSA) utilizing exclusively electrocardiogram (ECG) recordings during sleep and present a minute by-minute signal processing technique. A wide range of features based on heart rate variability (HRV) and ECG-derived respiratory (EDR) signals are considered. The novelty arises from employing bispectral analysis to the HRV and EDR signals in order to illustrate quadratic phase-coupling that can be observed among signal components with different frequencies. From this perspective, a new feature set based on a higher order spectrum of HRV and EDR signals is introduced and it is utilized to extract information regarding their non-linearity and non-Gaussianity. Physiological signals are non-stationary, non-linear and chaotic in nature. Linear and power spectral frequency methods are not very effective in the diagnosis of bio-signals. Various experiments reported that the 2D plots of bispectrum and bicoherence of various pathological signals is unique and different HOS parameters have been used to differentiate and classify them. These features were reported to have performance superior than their power spectrum counterpart. Paper [8] discusses these HOS features and its applications on various physiological signals.

Occipital EEG activity is known to be different from the frontal EEG during wakefulness and anaesthesia. However, less is known about occipital non-linear dynamics analyzed by EEG-bicoherence, which can reflect the oscillatory features that are dependent on thalamocortical modulation. Forty patients were anesthetized using sevoflurane (1% or 3%) combined with remifentanyl. Frontal and occipital EEGs were simultaneously collected, and bicoherence was analyzed before and after induction of anesthesia. The occipital bicoherence spectrum in the perianesthesia period is quite different from the frontal bicoherence spectrum, which is not usually shown in the power spectrum [9].

In the analysis of data from nonlinear systems both the bispectrum and the bicoherence have emerged as useful tools. Both are frequently used to detect the influence of a nonlinear system on the joint probability distribution of the system input. This work significantly generalizes by providing an analytical expression for the bispectrum of the response of quadratically nonlinear systems subject to stationary, jointly non-Gaussian inputs possessing arbitrary auto-correlation function. The expression is then used to determine the optimal input probability density function for detecting a quadratic nonlinearity in a second-order system. It is also shown how the expression can be used to design an optimal nonlinear filter for detecting deviations from normality in the probability density of a signal [10]. HOSA techniques are used for stator current analysis under stable conditions by means of power frequency spectrum density (PSD); and Multiple Signal Classification (MUSIC) and bispectrum are used under dynamics conditions. Therefore, it is possible to improve the accuracy and efficiency of technique. Experimental results validate the analysis and demonstrate that HOSA can be applied to detect and identify short circuit failures in Permanent Magnet Synchronous Motor [11].

From the works stated in the literature, the techniques of HOS are applied to various fields which mainly include biomedical, military, electrical fields etc. This paper gives the brief explanation of HOS which involves mainly the Bispectrum and Bicoherence analysis.

2. HIGHER ORDER SPECTRA

Fig. 1 demonstrates the various higher-order spectra for a discrete-time signal. Although higher-order statistics and spectra of a signal defined in terms of moments and cumulants, moments and moment spectra are very useful in the analysis of deterministic signals (transient and periodic) whereas cumulants and cumulants spectra are more useful in the analysis of stochastic signals. The two spectra are identical for three order cumulants (the bispectrum). Unlike the power

spectrum which is real-valued, higher order spectra is a complex valued and have both magnitude and phase. The phase of the bispectrum is referred as biphas and that of the trispectrum as the triphase [3].

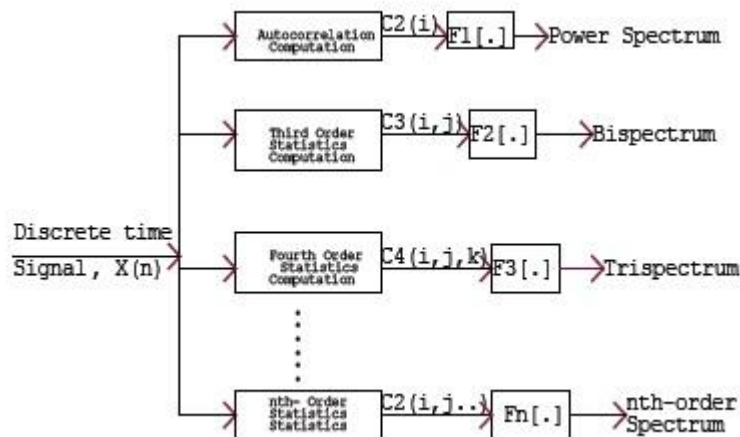


Fig. 1 The various higher order spectra for a deterministic signal. F [.] denotes n-dimensional Fourier Transform (Source: Sanullah)

3. HOSA Techniques

3.1 Power Spectrum

Techniques for estimating the conventional power spectrum fall into three broad categories: the nonparametric or conventional methods, the parametric or model-based methods, and the criterion-based methods. The first category includes two classes: the direct methods, which are based on the FT of the observed data; and indirect methods, which are based on computing the FT of the estimated autocorrelation sequence of the data. The class of parametric methods includes algorithms such as MA, AR, and ARMA modeling, and Eigen-space based methods such as MUSIC, Min-Norm, etc., which are appropriate for harmonic models. Criterion-based methods include Burg’s Maximum Entropy algorithm and Capon’s Maximum-Likelihood algorithm. The conventional estimators are easy to understand and easy to implement, but are limited by their resolving power (the ability to separate two closely spaced harmonics), particularly when the number of samples is small. For random signals, these estimators typically require long observation intervals in order to achieve acceptably low values for the variances of the estimate [1].

3.2 Polyspectra and Cross-polyspectra

Estimators of polyspectra are natural extensions of estimators of the power spectrum, with some important differences in the smoothing requirements. Hence, it will be useful to review power spectrum estimation techniques first [1].

3.3 Bicoherence

Higher order spectra are functions of two or more component frequencies unlike the power spectrum which is a function of a single frequency. Numerically computed estimates of higher order spectra may have non-zero values, they may or may not be statistically significant. Statistical significance depends on the number of degrees of freedom in the estimate. Of particular interest in the analysis of phase coupling between Fourier components is the value of the magnitude of the higher order spectrum independent of the powers at the component frequencies. This can be achieved by normalizing the magnitude with powers at the component frequencies. Since non-linear interactions result in the generation of phase-coupled power at sum and difference frequencies, the normalized spectra are also useful in detection and characterization of nonlinearity in systems. A normalized higher-order spectrum or nth-order coherency index is a function that combines the cumulants spectrum of order n with the power spectrum (n=2) of a signal. For a discrete-time signal, the 3rd and 4th order coherence are respectively defined by

$$B_x(\text{Bicoherence})(f_1, f_2) = \frac{B_x}{\sqrt{P_x(f_1)P_x(f_2)P_x(f_1 + f_2)}} \tag{1}$$

$$T_x(\text{tricoherence})(f_1, f_2, f_3) = \frac{T_x}{P_x(f_1)P_x(f_2)P_x(f_3)P_x(f_1 + f_2 + f_3)} \tag{2}$$

These functions are very useful in the detection and characterization of nonlinearities in time series and in discriminating linear processes from nonlinear ones. A signal is said to be a linear non-Gaussian process of order n if the magnitude of the nth-order coherence, is constant over all frequencies; otherwise, the signal is said to be a non-linear process [13]. They are also useful in detecting phase coupling between Fourier components. It has been shown that consistent estimates of the power spectrum and the bispectrum lead to consistent estimates of the bicoherence. Routines bicoher and bicoherx may be used to estimate the auto bicoherence and the cross-bicoherence [1].

3.4 Bispectrum

The relation between frequency components is significant and the bispectrum gives clear information about them whereas the power spectrum does not give any information about the frequency components. To examine the nonlinear signals one of the best methods is Higher Order Statistics, as it encloses the relations between phase components. The bispectrum employs the third order cumulants and it shows the information which is not presented by the spectral domain. The bispectrum $B(f_1, f_2)$ of a non-Gaussian signal, $x(t)$, is a two-dimensional Fourier transform of the third order cumulants $C(m, n)$ defined as:

$$C(m, n) = E[x(k)x(k+m)x(k+n)] \quad (3)$$

Where E is the Expectation function

The bispectrum formula related to (1) is written as:

$$B(f_1, f_2) = E[X(f_1)X(f_2)X^*(f_1 + f_2)] \quad (4)$$

Where $X(f)$ is the Fourier transform of $x(t)$ and $*$ represents conjugate complex. As seen from (2), bispectrum contains the information about the relation of phase between the frequency components at f_1 , f_2 and $f_1 + f_2$. The definition of phase coupled between f_1 and f_2 is: if there is an existence of a third component at a frequency of $f_1 + f_2$, phase differs from the sum of phases of the f_1 and f_2 by a constant at all times. The frequency at $f_1 + f_2$ is generated because of the non-linearity in the system. This produces a non-zero value of the bispectrum. It means, if the components at f_1 , f_2 and $f_1 + f_2$ have statistically independent random phases, the bispectrum shows a zero value. [12].

3.4.1 Bispectrum estimation using the direct (FFT-based) method

Bispectrum can be estimated through several techniques such as biased, parametric, direct and indirect methods. In this paper we use the direct method which involves Fast Fourier Transform (FFT) of the third order cumulants, to compute the bispectrum of the STHX process signal. In Bispectrum estimation using the direct (FFT-based) method, the data is segmented into probable overlapping records. The mean is removed from each record and the FFT is calculated. The Bispectrum of the k th record is calculated as

$$B_k(m, n) = x_k(m).x_k(n).x_k^*(m+n) \quad (5)$$

where x_k denotes the FFT of the k^{th} record. The Bispectrum estimates are averaged across records and an optional frequency domain smoother is applied.

3.4.2 Bispectrum estimation using the parametric method

In Bispectrum estimation using the parametric method, AR parameters should be estimated. The AR order corresponds with the index at which the singular values show the maximum drop.

3.4.3 Bispectrum estimation using biased method

Let $y(n)$, $n = 1 \dots N$ denotes the time series. Let M denote the number of samples per segment. Let O denote the percentage overlap between segments. Let $M_1 = M - M * O/100$. Then, the time series, y , is segmented into K records of M samples each, where $K = (N - M * O/100)/M$. The k^{th} record or segment of X consists of the samples

$$y_k(i) = y(i + (k-1)*M_1), \quad i=1, \dots, M, k = 1, \dots, K \quad (6)$$

The sample mean is removed from the k^{th} record, and sample estimates of the cumulants are computed [13].

4. CONCLUSION

Linear and power spectral frequency methods are not very effective in the diagnosis of signals. Higher order spectral analysis has the ability to detect non-linearity, deviations from Gaussianity and the phase relationships between harmonic components. Various experiments reported that the HOS analysis (i.e. the 2D plots of bispectrum and bicoherence) of various signals is unique and different HOS parameters have been used to differentiate and classify them. These features were reported to have performance superior than their power spectrum counterpart. Still pertinent research has been carried out using HOSA in various fields to find out their relevant parameters and it is a big boon to researchers doing research in statistical signal processing.

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