European Journal of Advances in Engineering and Technology, 2017, 4 (4): 287-294



Research Article

ISSN: 2394 - 658X

Effects of Heat Absorption and Porosity of the Medium on MHD Flow past a Vertical Plate

US Rajput and Gaurav Kumar

Department of Mathematics and Astronomy, University of Lucknow, UP, India rajputgauravlu@gmail.com

ABSTRACT

Effects of heat absorption and porosity of the medium on unsteady MHD flow past a moving vertical plate with variable wall temperature and mass diffusion in the presence of Hall current is studied here. Earlier we [5] have studied chemical reaction effect on unsteady MHD flow past an impulsively started oscillating inclined plate with variable temperature and mass diffusion in the presence of Hall current. We obtained the results which were in agreement with the desired flow phenomenon. To study further, we are changing the model by considering heat absorbing fluid, and changing the geometry of the model. Here in this paper we are taking the plate positioned vertically upward. Further, medium of the flow is taken as porous. The governing equations involved in the flow model are solved by the Laplace-transform technique. The results obtained have been analyzed with the help of graphs drawn for different parameters. The numerical values obtained for the drag at boundary have been tabulated. Here too, the results are found to be in agreement with the actual flow.

Key words: MHD flow, heat absorbing fluid, mass diffusion, Hall current

INTRODUCTION

The unsteady flow under the action of strong magnetic field plays a decisive role in different areas of science and technology. MHD effect on heat transfer over stretching sheet embedded in porous medium with variable viscosity, viscous dissipation and heat source or sink was analyzed by Dessie and Naikoti [1]. They solved flow model by using Runge-Kutta fourth order numerical method, and observed that if heat sink parameter is increased then the thermal boundary layer increases, whereas the thermal boundary layer decreases with heat source. Hall effect on unsteady MHD natural convection flow of a heat absorbing fluid past an accelerated moving vertical plate with ramped wall temperature was investigated by Seth et al [7]. The model considered by Seth et al [7] was solved by Laplace transform technique. In their study they observed that velocity and temperature of fluid near the vertical plate decreases when heat absorption parameter increases. Reddy et al [6] have presented radiation and mass transfer effects on nonlinear MHD boundary layer flow of liquid metal over a porous stretching surface embedded in porous medium with heat generation. Hossain et al [10] have developed MHD free convection and mass transfer flow through a vertical oscillatory porous plate with Hall, ion-slip currents and heat source in a rotating system. Hossain et al [10] have used explicit finite difference method to solve the coupled non-linear partial differential equations. They examined that the shear stress and Sherwood numbers are decreased with the increase in heat source parameter, and Nusselt number is increased with increase in heat source. Mythreye et al [3] have proposed chemical reaction on unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. They solved the governing equations by using perturbation technique, and explained the presence of heat absorption effect caused reductions in the fluid temperature which resulted in decrease in the fluid velocity. Similar study was done by Seth et al [8], Shehzad et al [9] and Ibrahim and Suneetha [2]. Chemical reaction effect on unsteady MHD flow past an impulsively started oscillating inclined plate with variable temperature and mass diffusion in the presence of Hall current was investigated by us [5]. Further we [4] have worked on effects of Hall current and chemical reaction on MHD flow through porous medium past an oscillating inclined plate with variable temperature and mass diffusion. The main purpose of the present investigation is to study the effects of heat absorption and porosity of the medium on unsteady MHD flow past a moving vertical plate with variable wall temperature and mass

diffusion in the presence of Hall current. The models have been solved using the Laplace transforms technique. The results are shown with the help of graphs and table.

MATHEMATICAL ANALYSIS

The unsteady flow of an electrically conducting, incompressible, viscous fluid past through porous medium in a vertical plate has been considered. The x axis is taken in the direction of the motion and z normal to it. A transverse magnetic field B_0 of uniform strength is applied on the flow. The magnetic Reynolds number is considered to be small so that the induced magnetic field is neglected. Initially it has been considered that the plate as well as the fluid is at the same temperature T_{∞} . The species concentration in the fluid is taken as C_{∞} . At time t > 0, the plate starts moving with a velocity u_0 in its own plane, and temperature of the plate is raised to T_w . The concentration C near the plate is raised linearly with respect to time. The governing equations are as under:

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial z^2} + g\beta \left(T - T_{\infty}\right) + g\beta^* \left(C - C_{\infty}\right) - \frac{\sigma B_0^2 \left(u + mv\right)}{\rho (1 + m^2)} - \frac{\upsilon u}{K}$$
(1)

$$\frac{\partial v}{\partial t} = v \frac{\partial^2 v}{\partial z^2} + \frac{\sigma B_0^2 (mu - v)}{\rho (1 + m^2)} - \frac{v v}{K}$$
(2)

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial z^2}$$
(3)

$$\rho C_{p} \frac{\partial T}{\partial t} = k \frac{\partial^{2} T}{\partial z^{2}} - Q(T - T_{\infty})$$
(4)

The initial and boundary conditions are

$$t \le 0: u = 0, v = 0, T = T_{\infty}, C = C_{\infty}, \text{ for every } z.$$

$$t > 0: u = u_0, v = 0, T = T + (T_w - T_{\infty})A, C = C_{\infty} + (C_w - C_{\infty})A, \text{ at } z=0.$$

$$u \to 0, v \to 0, T \to T_{\infty}, C \to C_{\infty} \text{ as } z \to \infty, \text{ where } A = \frac{u_0^2 t}{v}.$$
(5)

The following non-dimensional quantities are introduced to transform equations (1), (2), (3) and (4) into dimensionless form:

$$\bar{z} = \frac{zu_0}{v}, \ \bar{u} = \frac{u}{u_0}, \ \bar{v} = \frac{v}{u_0}, \ \theta = \frac{(T - T_{\infty})}{(T_w - T_{\infty})}, \ S_c = \frac{v}{D}, \ \mu = \rho v, \ P_r = \frac{\mu C_p}{k}, \ H = \frac{Qv}{u_0^2 \rho C_p}$$

$$G_r = \frac{g\beta v(T_w - T_{\infty})}{u_0^3}, \ M = \frac{\sigma B_0^2 v}{\rho u_0^2}, \ G_m = \frac{g\beta^* v(C_w - C_{\infty})}{u_0^3}, \ \overline{C} = \frac{(C - C_{\infty})}{(C_w - C_{\infty})}, \ \overline{t} = \frac{tu_0^2}{v}.$$

$$(6)$$

The dimension less flow model becomes

$$\frac{\partial \overline{u}}{\partial \overline{t}} = \frac{\partial^2 \overline{u}}{\partial \overline{z}^2} + G_r \theta + G_m \overline{C} - \frac{M(\overline{u} + m\overline{v})}{(1 + m^2)} - \frac{1}{\overline{K}} \overline{u}$$
(7)

$$\frac{\partial \overline{v}}{\partial \overline{t}} = \frac{\partial^2 \overline{u}}{\partial \overline{z}^2} + \frac{M(m\overline{u} - \overline{v})}{(1 + m^2)} - \frac{1}{\overline{K}} \overline{v}$$
(8)

$$\frac{\partial \overline{C}}{\partial \overline{z}} = \frac{1}{n} \frac{\partial^2 \overline{C}}{\partial \overline{z}^2}$$
(9)

$$\frac{\partial t}{\partial \theta} = \int \frac{\partial^2 \theta}{\partial t^2} dt = 0$$

$$\frac{\partial \theta}{\partial \bar{t}} = \frac{1}{P_r} \frac{\partial \theta}{\partial \bar{z}^2} - H\theta \tag{10}$$

The corresponding boundary conditions become

$$t \le 0: \overline{u} = 0, \ \overline{v} = 0, \ \theta = 0, \ C = 0, \ \text{for every } \overline{z} \\ \overline{t} > 0: \overline{u} = 1, \ \overline{v} = 0, \ \theta = \overline{t}, \ \overline{C} = \overline{t}, \ \text{at} \ \overline{z} = 0. \\ \overline{u} \to 0, \ \overline{v} \to 0, \ \theta \to 0, \ \overline{C} \to 0, \ \text{as} \ \overline{z} \to \infty.$$

$$(11)$$

Dropping bars in the above equations, we get

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial z^2} + G_r \theta + G_m C - \frac{M(u+mv)}{(1+m^2)} - \frac{1}{K} u$$
(12)

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial z^2} + \frac{M(mu - v)}{(1 + m^2)} - \frac{1}{K}v$$
(13)

$$\frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial z^2}$$
(14)

$$\frac{\partial\theta}{\partial t} = \frac{1}{P_{e}} \frac{\partial^{2}\theta}{\partial z^{2}} - H\theta$$
⁽¹⁵⁾

The boundary conditions are

$$t \le 0: u = 0, v = 0, \theta = 0, C = 0, \quad \text{for every } z.$$

$$t > 0: u = 1, v = 0, \ \theta = t, \ C = t, \quad \text{at} \quad z = 0.$$

$$u \to 0, \ v \to 0, \ \theta \to 0, \ C \to 0, \quad \text{as} \ z \to \infty.$$
(16)

Combining equations (12) and (13), the model becomes

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial z^2} + G_r \theta + G_m C - qa$$
(17)

$$\frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial z^2}$$
(18)

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial z^2} - H\theta \tag{19}$$

Here q = u + iv and $a = \frac{M(1 - im)}{1 + m^2} + \frac{1}{K}$.

Finally, the boundary conditions become:

$$t \le 0: q = 0, \theta = 0, C = 0, \text{ for all } z.$$

$$t > 0: q = 1, \theta = t, C = t, \text{ at } z=0.$$

$$q \to 0, \theta \to 0, C \to 0, \text{ as } z \to \infty.$$

$$(20)$$

The dimensionless governing equations (17) to (20), subject to the boundary conditions (20), are solved by the usual Laplace - transform technique. The solution obtained is as under:

$$C = t \left\{ (1 + \frac{z^2 S_c}{2t}) erfc[\frac{\sqrt{S_c}}{2\sqrt{t}}] - \frac{z\sqrt{S_c}}{\sqrt{\pi\sqrt{t}}} e^{-\frac{z^2}{4t}} S_c \right\},$$

$$\theta = \frac{1}{4Hi} e^{-zi\sqrt{HP_r}} \left\{ -2i\sqrt{Ht}(A_1 - e^{-zi\sqrt{HP_r}}A_2 - 2) + z\sqrt{P_r}(A_1 + e^{-zi\sqrt{HP_r}}A_2 - 2) \right\},$$

$$q = \frac{1}{2} e^{-\sqrt{az}} A_{33} + \frac{G_r}{4(a + HP_r)^2} [(at + P_r + HP_r t - 1)(e^{-\sqrt{az}}A_3 - e^{-\sqrt{HP_r}zi}A_{11}) + e^{-\sqrt{az}}A_4(\sqrt{a} + \frac{HP_r}{\sqrt{a}}) + A_{14}(1 - P_r)(A_5 - A_{12}) - \frac{1}{2i\sqrt{H}} ze^{-\sqrt{HP_r}zi}A_{10}\sqrt{P_r}(HP_r + a)] + \frac{G_m Cos\alpha}{4a^2} [2A_6 e^{-\sqrt{az}}(1 - at) + e^{-\sqrt{az}}(z\sqrt{a}A_8 + 2A_9S_c) + 2A_{15}A_7(1 - S_c)] - \frac{G_m}{2a^2\sqrt{\pi}} [2az\sqrt{tS_c}e^{-\frac{z^2S_c}{4t}} + A_{16}\sqrt{\pi}(az^2S_c + 2at + 2S_c - 2) + A_{13}A_{15}\sqrt{\pi}(1 - S_c)]$$

The symbols involved in the above equations are mentioned in the appendix.

Skin Friction

The dimensionless skin friction at the plate z=0 is obtained by

$$\left(\frac{dq}{dz}\right)_{z=0} = \tau_x + i\tau_y \,.$$

The numerical values of τ_x and τ_y , for different parameters are given in table-1.

RESULT AND DISCUSSIONS

In this present paper we have studied the effects of heat absorption and permeability of porous medium on the flow. The behaviour of other parameters like magnetic field, Hall current and thermal buoyancy is almost similar the earlier model studied by us [5]. The analytical results are shown graphically in Figs1 to7. The numerical values of skinfriction τ_x and τ_y are presented in Table-1. From Figs1 and 2, it is observed that primary and secondary velocities increase with permeability parameter (*K*). This result is due to the fact that increases in the value of (*K*) results in reducing the drag force, and hence increasing the fluid velocity.





Effect of heat absorption fluid flow behaviour is shown by Figs3 and 4. It is seen here that when heat absorption parameter (H) increases, primary velocity u increases throughout the boundary layer region; but secondary velocity v decreases near the surface of the plate. This implies that heat absorption tends to accelerate primary velocity, whereas it retards secondary velocity in the boundary layer region. Further, it is observed that the temperature decreases when Prandtl number and heat absorption parameter are increased (Figs5 and 6). However, from Fig. 7 it is observed that the temperature increases with time. This is due the reason heat is transported to the system continuously.



Table -1 Skin Friction for Different Parameter

М	М	Pr	Sc	Gm	Gr	Н	K	t	$ au_x$	$ au_y$
3	1.0	0.71	2.01	100	010	05	0.2	0.5	11.5145144	4.15812410
5	1.0	0.71	2.01	100	010	05	0.2	0.5	9.12045440	3.42149226
2	1.0	7.01	2.01	100	010	05	0.2	0.5	7.25045566	0.39490278
2	1.0	0.71	2.01	100	010	05	0.2	0.5	14.0808692	4.95645366
2	5.0	0.71	2.01	100	010	05	0.2	0.5	25.0956227	7.61259653
2	1.0	0.71	3.00	100	010	05	0.2	0.5	13.1748302	4.89180197
2	1.0	0.71	4.00	100	010	05	0.2	0.5	12.6029421	4.88279034
2	1.0	0.71	2.01	010	010	05	0.2	0.5	5.96165311	4.74283994
2	1.0	0.71	2.01	050	010	05	0.2	0.5	9.57019360	4.83777937
2	1.0	0.71	2.01	100	050	05	0.2	0.5	44.1654773	23.0300561
2	1.0	0.71	2.01	100	100	05	0.2	0.5	81.7712374	45.6220591
2	1.0	0.71	2.01	100	010	10	0.2	0.5	4.55750664	35.1068783
2	1.0	0.71	2.01	100	010	05	0.5	0.5	-8.0765170	-12.3614290
2	1.0	0.71	2.01	100	010	05	0.2	0.4	10.1257785	3.78312807
2	1.0	0.71	2.01	100	010	05	0.2	0.6	18.1080174	5.97775830

The values of skin friction are given in table 1. The value of τ_x increases with the increase in G_{mb} G_r and t. But τ_x decreases with M, P_r , S_c , m, H and K. Similar effects are observed with τ_y .

CONCLUSION

In this paper a theoretical analysis has been done to study effects of heat absorption and porosity of the medium on unsteady MHD flow past a moving vertical plate with variable wall temperature and mass diffusion in the presence of Hall current. It is observed that the primary velocity increases with increasing the values of heat absorption and permeability of the porous medium. The effect is similar on the secondary velocity except the case of heat absorption. That is the secondary velocity decreases when heat absorption parameter is increased.

Nomenclature

- D Mass diffusion
- ρ The fluid density
- σ Electrical conductivity
- μ The magnetic permeability
- *T* Temperature of the fluid
- *K* the permeability parameter
- M The magnetic Field parame-
- *m* The Hall current parameter
- *P_r Prandtl number*
- t Time
- Sc Schmidt number
- v The kinematic viscosity

- μ The coefficient of viscosity
- *G_m* Mass Grashof number
- *G_r* Thermal Grashof number
- *k* The thermal conductivity
- T_{∞} The temperature of the fluid
- T_w Temperature of the plate
- *C* Species concentration in the fluid
- \overline{C} The dimensionless concentration
- C_P Specific heat at constant pressure
- *C_w* Species concentration at the plate
- C_{∞} The concentration in the fluid

- Volumetric coefficient of thermal expansion
- β^* Volumetric coefficient of concentration expansion
- u,v Velocity of the fluid in x & y- direction
- $\overline{u}, \quad Dimensionless \ velocity \ in \ x \ \& \ y- \\ \overline{v} \quad direction$
- θ The dimensionless temperature
- g Gravity acceleration

ß

H heat absorption parameter

Appendix

$$\begin{split} &A_{1} = erfc\bigg[\frac{2Hit - z\sqrt{P_{r}}}{2\sqrt{t}}\bigg], \qquad A_{2} = erfc\bigg[\frac{2Hit + z\sqrt{P_{r}}}{2\sqrt{t}}\bigg], \qquad A_{3} = (-1 - A_{17} + e^{2\sqrt{a_{2}}}(A_{18} - 1)), \\ &A_{4} = (1 + A_{17} + e^{2\sqrt{a_{2}}}(A_{18} - 1)), \qquad A_{5} = (-1 + A_{19} + \exp(2z\sqrt{\frac{(a + H)P_{r}}{P_{r} - 1}})(A_{20} - 1)), \\ &A_{6} = (1 + A_{21} + \exp(2\sqrt{az})(1 - A_{22})), \qquad A_{7} = (-1 + A_{23} + \exp(2z\sqrt{\frac{aS_{r}}{S_{r} - 1}})(A_{24} - 1)), \\ &A_{8} = (1 + A_{21} + \exp(2\sqrt{az})(A_{22} - 1)), \qquad A_{9} = (-1 - A_{21} + \exp(2\sqrt{az})(A_{22} - 1)), \\ &A_{10} = (1 + A_{25} + \exp(2Hiz\sqrt{P_{r}})(A_{26} - 1)), \qquad A_{11} = (1 - A_{25} + \exp(2Hiz\sqrt{P_{r}})(A_{26} - 1)), \\ &A_{10} = (1 + A_{25} + \exp(2Z\sqrt{\frac{(a + H)P_{r}}{P_{r} - 1}})(A_{28} - 1)), \qquad A_{13} = (-1 - A_{29} + \exp(2Z\sqrt{\frac{aS_{r}}{S_{r} - 1}})(A_{29} - 1)), \\ &A_{12} = (-1 - A_{27} + \exp(2Z\sqrt{\frac{(a + H)P_{r}}{P_{r} - 1}})(A_{28} - 1)), \qquad A_{13} = (-1 - A_{29} + \exp(2Z\sqrt{\frac{aS_{r}}{S_{r} - 1}})(A_{29} - 1)), \\ &A_{14} = \exp(\frac{at}{P_{r} - 1} - z\sqrt{\frac{(a + H)P_{r}}{P_{r} - 1}} + \frac{HP_{r}}{P_{r} - 1}), \qquad A_{15} = \exp(\frac{at}{S_{r} - 1} - z\sqrt{\frac{aS_{r}}{S_{r} - 1}}), \\ &A_{16} = (-1 + erf(\frac{z\sqrt{S_{r}}}{2\sqrt{t}})), \qquad A_{17} = erf(\sqrt{at} - \frac{z}{2\sqrt{t}}), \qquad A_{18} = erf(\sqrt{at} + \frac{z}{2\sqrt{t}}), \\ &A_{19} = erf(\frac{z}{2\sqrt{t}} - \sqrt{\frac{(a + H)P_{r}}{P_{r} - 1}}), \qquad A_{25} = erf(\frac{z}{2\sqrt{t}} + \sqrt{\frac{(a + H)P_{r}}{P_{r} - 1}}), \\ &A_{24} = erf(\frac{z}{2\sqrt{t}} - \sqrt{\frac{aS_{r}}{S_{r} - 1}}), \qquad A_{25} = erf(Hi\sqrt{t} - \frac{z\sqrt{P_{r}}}{2\sqrt{t}}), \qquad A_{26} = erf(Hi\sqrt{t} + \frac{z\sqrt{P_{r}}}{2\sqrt{t}}), \\ &A_{27} = erf(\sqrt{t}\sqrt{\frac{aH_{r}}{P_{r} - 1}} - \frac{z\sqrt{P_{r}}}{2\sqrt{t}}), \qquad A_{28} = erf(\sqrt{\frac{(a + H)t}{P_{r} - 1}} + \frac{z\sqrt{P_{r}}}{2\sqrt{t}}), \qquad A_{29} = erf(\frac{1}{2\sqrt{t}}(\sqrt{\frac{aH_{r}}{S_{r} - 1}} - z\sqrt{S_{r}})), \\ &A_{30} = erf(\frac{2\sqrt{at} - z}{2\sqrt{t}}), \qquad A_{32} = erf(\frac{2\sqrt{at} + z}{2\sqrt{t}}), \qquad A_{33} = (A_{31} + A_{32}), \\ &A_{30} = erf(\frac{1}{2\sqrt{t}}(\sqrt{\frac{aH_{r}}{S_{r} - 1}} + z\sqrt{S_{r}})), \qquad A_{31} = erf(\frac{2\sqrt{at} - z}{2\sqrt{t}}), \qquad A_{32} = erf(\frac{2\sqrt{at} + z}{2\sqrt{t}}), \qquad A_{33} = (A_{31} + A_{32}), \end{aligned}$$

REFERENCES

[1] H Dessie and K Naikoti, MHD Effects on Heat Transfer Over Stretching Sheet Embedded in Porous Medium with Variable Viscosity, *Ain Shams Engineering Journal (Elsevier)*, **2014**, 5,967–977.

[2] SM Ibrahim and K Suneetha, Heat Source and Chemical Effects on MHD Convection Flow Embedded in a Porous Medium with Soret, Viscous and Joules Dissipation, *Ain Shams Engineering Journal(Elsevier)*, **2016**, 7(2), 811–818.

[3] A Mythreye, JP Pramoda and KS Balamurugan, Chemical Reaction on Unsteady MHD Convective Heat and Mass Transfer Past a Semi-Infinite Vertical Permeable Moving Plate with Heat Absorption, *Procedia Engineering* (*Elsevier*), **2015**,127,613 – 620.

[4] US Rajput and Gaurav Kumar, Effects of Hall Current and Chemical Reaction on MHD Flow Through Porous Medium Past an Oscillating Inclined Plate with Variable Temperature and Mass Diffusion, *European Journal of Advances in Engineering and Technology*, **2017**, 4 (1), 56-63.

[5] US Rajput and Gaurav Kumar, Chemical Reaction Effect on Unsteady MHD Flow Past an Impulsively Started Oscillating Inclined Plate with Variable Temperature and Mass Diffusion in the Presence of Hall Current, *Applied Research Journal*, **2016**, 2 (5), 244-253.

[6] GVR Reddy, KJ Reddy and R Lakshmi, Radiation and Mass Transfer Effects on Nonlinear MHD Boundary Layer Flow of Liquid Metal Over a Porous Stretching Surface Embedded in Porous Medium with Heat Generation, *WSEAS Transactions on Fluid Mechanics*, **2015**, 10, 1-12.

[7] GS Seth, R Sharma and SM Hussain, Hall Effects on Unsteady MHD Natural Convection Flow of a Heat Absorbing Fluid Past an Accelerated Moving Vertical Plate with Ramped Temperature, *Emirates Journal for Engineering Research*, **2014**, 19 (2), 19-32.

[8] GS Seth, B Kumbhakar and R Sharma, Unsteady Hydromagnetic Natural Convection Flow of a Heat Absorbing Fluid Within a Rotating Vertical Channel in Porous Medium with Hall effects, *Journal of Applied Fluid Mechanics*, **2015**, 8 (4), 767-779.

[9] SA Shehzad, T Hayat and A Alsaedi, Three-dimensional MHD flow of Casson Fluid in Porous Medium with Heat Generation, *Journal of Applied Fluid Mechanics*, **2016**, 9 (1), 215-223.

[10] MA Samad, MD Hossain and MM Alam, MHD Free Convection and Mass Transfer Flow through a Vertical Oscillatory Porous Plate with Hall, Ion-Slip Currents and Heat Source in a Rotating System, *Procedia Engineering* (*Elsevier*), **2015**, 105, 56–63.