# High Speed 3D-DCT/IDCT CORDIC Algorithm for DSP Application 

Bharati Y Masram ${ }^{1}$ and PT Karule ${ }^{2}$<br>${ }^{1}$ Department of Electronics and Telecommunications Engineering, YCCE, Nagpur, India<br>${ }^{2}$ Department of Electronics Engineering, YCCE, Nagpur, India<br>bharatimasram@gmail.com


#### Abstract

This paper elaborates the high speed $8 x 8 x 83 D-D C T / I D C T$ processor based on CORDIC algorithm for various DSP applications such as image and video processing coding with reduced size and cost of hardware. In this paper, CORDIC based 3D DCT/IDCT i.e. both forward and inverse computations required for transform has been presented. By using this algorithm image and video can be processed through the DCT/IDCT for the compression/decompression data. The number of CORDIC Computational requirement for $N=8, N=163 D$ DCT/IDCT for $8 * 8 * 8$ is $(N 3+16) / 12$ has been compared with 2D-DCT algorithm. Based on this algorithm a new processor 3DDCT/IDCT architecture is designed and simulated in Xilinx ISE 14.7 and FPGA implementation has been done in Altera Quartus cyclone II (DE2 board).


Keywords: 3D-Discrete Cosine Transform/Inverse Discrete Cosine Transform, CORDIC Algorithm, XILINX 14.7

## INTRODUCTION

Now a day, various digital VLSI signal processing is based on the application such as various transform DCT, IDCT, DFT, FFT etc [1-3] VLSI based signal processing using HDL and input as image or video is processed through various transform is simulated using HDL, where compression and decompression is done for signal processing of video standard format such as MPEG, JPEG, H.265, HDTV etc. Far year ago research was done on 1DDCT computation for various compression techniques using various methods such as row column Frame (RCF) [45], Distributed Arithmetic [6] and CORDIC algorithm [7] which includes the less complexity in the computation of transform.

High performance architecture based on computation algorithms, efficiency, power consumption for the 2D-DCT and IDCT were discussed in [8]. Moreover, conventional the double size Fast Fourier transform (FFT) algorithm which gives computation time $\mathrm{O}(\log 2 \mathrm{~N}+1)$ where as several computations for the DCT is $\mathrm{O}(\mathrm{N}-\log 2 \mathrm{~N})$ [9-11]. Coding decoding techniques with throughput for the 2D DCT /IDCT as $118.2 \mathrm{MHz}, 150 \mathrm{MHz}, 300 \mathrm{MHz}$, with data input rate [12-14]. Owing to the rapid growth of the three dimensional applications in the field of video processing based on the VLSI technology is now very much popular.

Now to achieve near-optimal compression of individual frames widely used standard frames formats are JPEG, MPEG etc used with the spatial and temporal dimensions is described with the three dimensional DCT. Various standard format is compared for the compression based output in [15]. The compression technique has been applied to 3D-DCT data based on $4 \times 4 \times 4$ cubes resulting 6:1 compression ratio. The technique of "natural "extension of 2DDCT with two stages of 1D-DCT and similarly 3D-DCT with 2D transform followed with 1D-DCT (or vice-versa) has been used [16].

In this paper we have proposed an algorithm for 3D-DCT/IDCT using CORDIC (Co-ordinate Rotation Digital Computer). The Fig. 1 shows the block diagram of 3D-DCT process with compressed output. In this we have given video input vector in the hexadecimal form as this video vector coefficient has been converted into hexadecimal number using MATLAB and then it is processed through 3D DCT to get the compressed output. The description of Basic CORDIC Algorithm is given in further section. The Algorithm is written for the 3D DCT with the help of basic CORDIC Algorithm which is explained in next Section.


Fig. 1 Block diagram of 3D-DCT Compressed output using CORDIC Algorithm


Fig. 2 Block diagram of 3D-IDCT Decompressed output using CORDIC Algorithm
Fig. 2 shows the block diagram of 3D-IDCT Decompressed output using CORDIC algorithm which has been implemented in Altera Cyclone-II which has been briefly described in next Section. Further Section has calculated hardware Computation required in the 3D DCT /IDCT comparing with the 2D-DCT processor [18]. Further continues with the structure using CORDIC algorithm. Synthesis result has been presented in tabular Section. Finally, paper ends with the conclusion of the 3D DCT/IDCT Algorithm application.

## BASIC CORDIC ALGORITHM

The Conventional CORDIC Algorithm was first developed by VOLDER in 1959 [4] for the computation of the various elementary functions, hyperbolic and trigonometric functions such as $\sin , \cos$, tan. Let us consider Fig. 3 Vector Rotation example of CORDIC algorithm and X and Y are the vector component similarly $\mathrm{X}^{\prime}$ and $\mathrm{Y}^{\prime}$ are another vector component after the increment of angle.


Fig. 3 Vector Rotation example of CORDIC algorithm

$$
\begin{align*}
& X=P \cos (\rho)  \tag{1}\\
& Y=P \sin (\rho)  \tag{2}\\
& X^{\prime}=P \cos (\varphi+\rho)  \tag{3}\\
& Y^{\prime}=P \sin (\varphi+\rho)  \tag{4}\\
& X^{\prime}=X \cos (\varphi)-Y \sin (\varphi)  \tag{5}\\
& Y^{\prime}=Y \cos (\varphi)+X \sin (\varphi)  \tag{6}\\
& X_{i+1}=\cos \left(\tan ^{-1}\left( \pm 2^{-i}\right)\right) X_{i}-Y_{i} d_{i} 2^{-i}  \tag{7}\\
& Y_{i+1}=\cos \left(\tan ^{-1}\left( \pm 2^{-i}\right)\right) Y_{i}+X_{i} d_{i} 2^{-i}  \tag{8}\\
& Z_{i+1}=z_{i}-\operatorname{ditan}^{-1}\left(2^{-i}\right) \tag{9}
\end{align*}
$$

$$
\left[\begin{array}{l}
X^{\prime}  \tag{10}\\
Y^{\prime}
\end{array}\right]=\underbrace{\left[\begin{array}{ll}
\cos \phi & \sin \phi \\
-\sin \phi & \cos \phi
\end{array}\right]}_{R(\phi)}\left[\begin{array}{l}
X \\
Y
\end{array}\right]
$$

Equation (10) can be Rearrange as:

$$
\left[\begin{array}{l}
X^{\prime}  \tag{11}\\
Y^{\prime}
\end{array}\right]=[\cos \phi]\left[\begin{array}{lc}
1 & -\tan \phi \\
\tan \phi & 1
\end{array}\right]\left[\begin{array}{l}
X \\
Y
\end{array}\right]
$$

Allowing only iterative rotations so that

$$
\begin{equation*}
\tan \beta= \pm 2^{-i} \tag{12}
\end{equation*}
$$

In this way number of iteration can be calculated by rotation direction up to equation of the basic CORDIC algorithm of the ith iteration are as follows:

$$
\begin{align*}
& X_{i+1}=\cos \left(\tan ^{-1}\left( \pm 2^{-\mathrm{i}}\right)\right)\left[\mathrm{X}_{i}-\mathrm{Y}_{i} d_{i} 2^{-i}\right]  \tag{13}\\
& Y_{i+1}=\cos \left(\tan ^{-1}\left( \pm 2^{-\mathrm{i}}\right)\right)\left[\mathrm{Y}_{i}+\mathrm{X}_{i} d_{i} 2^{-i}\right]  \tag{14}\\
& \cos \left(\tan ^{-1}\left(2^{-\mathrm{i}}\right)\right)=\cos \left(\tan ^{-1}\left(-2^{-\mathrm{i}}\right)\right) \\
& \cos \left(\tan ^{-1}\left(2^{-\mathrm{i}}\right)\right) \text { term is the gain Ki of an iteration }
\end{align*}
$$

As

$$
K i=\cos \left(\arctan ^{-1}\left(2^{-i}\right)\right)=\frac{1}{\sqrt{1+2^{-2 i}}}
$$

We can compute K offline for all n iterations:

$$
K=\prod_{n} K i
$$

The gain approaches 0.6037 , if n goes to infinity for a single CORDIC micro-rotation the resulting the following equations

$$
\begin{aligned}
& \mathrm{X}_{i+1}=\mathrm{X}_{i}-\mathrm{Y}_{i} \mathrm{~d}_{i} 2^{-i} \\
& Y_{i+1}=\mathrm{Y}_{i}+\mathrm{X}_{i} \mathrm{~d}_{i} 2^{-i} \\
& \mathrm{Z}_{i+1}=\mathrm{Z}_{i}-d_{i} \tan ^{-1}\left(2^{-i}\right)
\end{aligned}
$$

## 3D-DCT USING CORDIC ALGORITHM

The expression for 3D DCT [1] for an $N * N * N$ is given by the expression

$$
F(u, v, w)=\alpha_{3 D}(u, v, w) \sum_{x=0}^{N_{1}-1} \sum_{y=0}^{N_{2}-1} \sum_{z=0}^{N_{3}-1} f_{3 D}(x, y, z)\left\{\cos \left[l_{1}\right] \cos \left[l_{2}\right] \cos \left[l_{3}\right]\right\}
$$

where

$$
\begin{aligned}
& u \in\left\{0,1, \ldots \ldots . N_{1}-1\right\}, v \in\left\{0,1, \ldots \ldots \ldots N_{2}-1\right\}, w \in\left\{0,1, \ldots \ldots \ldots . N_{3}-1\right\} \\
& \text { and } \alpha_{3 D}(u, v, w) \text { is defined as }
\end{aligned}
$$

$$
\begin{align*}
& \alpha_{3 D}(u, v, w)=\left\{\begin{array}{l}
\sqrt{\frac{1}{N_{k}}}, \text { if } u=v=w=0 \\
\sqrt{\frac{2}{N_{k}}}, \text { otherwise } u \neq v \neq w \neq 0
\end{array}\right.  \tag{18}\\
& l 1=\frac{\pi(2 x+1) u}{2 N_{1}}, l 2=\frac{\pi(2 y+1) v}{2 N_{2}}, l 3=\frac{\pi(2 z+1) w}{2 N_{3}}
\end{align*}
$$

To rewrite the discrete cosine transform we need to observe the trigonometric identities.
$\cos \frac{2 j+1}{2 N} u \pi$ and $\sin \frac{2 j+1}{2 N} u \pi, \cos \frac{2 j+1}{2 N} v \pi$ and $\sin \frac{2 j+1}{2 N} v \pi \cos \frac{2 j+1}{2 N} w \pi$ and $\sin \frac{2 j+1}{2 N} w \pi$

We have,

$$
\begin{align*}
& \cos \frac{2 j+1}{2 N}\left(N-u^{\prime}\right) \pi=(-1)^{i} \sin \frac{2 j+1}{2 N} u^{\prime} \pi  \tag{19}\\
& \sin \frac{2 j+1}{2 N}\left(N-u^{\prime}\right) \pi=(-1)^{i} \cos \frac{2 j+1}{2 N} u^{\prime} \pi  \tag{20}\\
& \text { if } j=\frac{N}{K}-1-j^{\prime} K=1,2,4,8,16 \ldots \ldots . \\
& \text { when } u=0, k, 2 k, 3 k \ldots . .
\end{align*}
$$

Similarly, for the $\mathrm{v} \& \mathrm{w}$ above terms can be written

$$
\cos \frac{2\left(\frac{N}{K}-1-j^{\prime}\right)+1}{2 N} u \pi=(-1)^{\frac{u}{K}} \cos \frac{2 j^{\prime}+1}{2 N} u^{\prime} \pi
$$

And when $=\mathrm{K} / 2,3 \mathrm{~K} / 2,5 \mathrm{~K} / 2, \ldots \ldots . \mathrm{K} 2$ we have

$$
\begin{align*}
& \cos \frac{2\left(\frac{K}{N}-1-j^{\prime}\right)+1}{2 N} u \pi=(-1)^{\frac{u-K / 2}{K}} \sin \frac{2 j^{\prime}+1}{2 N} u \pi  \tag{22}\\
& \sin \frac{2\left(\frac{K}{N}-1-j^{\prime}\right)+1}{2 N} u \pi=(-1)^{\frac{u-K / 2}{K}} \cos \frac{2 j^{\prime}+1}{2 N} u \pi \tag{23}
\end{align*}
$$

Now the analysis of Equation (18) and (37) namely Forward 3D-Discrete Cosine Transform and Inverse 3DDiscrete Cosine Transform can be performed as follows.

$$
\begin{aligned}
& \alpha_{3 D}(u, v, w=0)=1 / \sqrt{2} \\
& \alpha_{3 D}(u, v, w)=1, u, v, w=1,2, \ldots \ldots \ldots . N-1 \\
& \text { Assume } N=2^{n} \text { and } \sqrt{2 / N}=\text { const } \\
& \text { for } u \neq v \neq w \neq 0 \\
& \alpha_{3 D}(u, v, w)=\sqrt{2 / N} \text { can be removed }
\end{aligned}
$$

The Equation (22) can be split into two parts

$$
\begin{align*}
& F(u)=\sum_{j=0}^{N / 2-1} \alpha_{3 D}(u, v, w) \cos \left(\frac{2 j+1}{2 N} u \pi\right) \cos \left(\frac{2 j+1}{2 N} v \pi\right) \\
& \cos \left(\frac{2 j+1}{2 N} w \pi\right) f(j)+\sum_{j=N / 2}^{N-1} \alpha_{3 D}(u, v, w) \cos \left(\frac{2 j+1}{2 N} u \pi\right) \\
& \cos \left(\frac{2 j+1}{2 N} v \pi\right) \cos \left(\frac{2 j+1}{2 N} w \pi\right) f(j)  \tag{24}\\
& =\sum_{j=0}^{N / 2-1} \alpha_{3 D}(u, v, w) \cos \left(\frac{2 j+1}{2 N} u \pi\right) \cos \left(\frac{2 j+1}{2 N} v \pi\right) \\
& \cos \left(\frac{2 j+1}{2 N} w \pi\right) \times\left[f(j)+(-1)^{u} f(N-1-j)\right]
\end{align*}
$$

For $\mathrm{K}=1$, we can define two sets of functions and to express equation (24) the above function

$$
\begin{equation*}
f_{1}^{+}(j)=f_{1}^{-}(j)=f(j) \quad j=0,1,2 \ldots ., N-1 \tag{25}
\end{equation*}
$$

and for $K=2,4,8,16, \ldots \ldots . N / 2, \quad j=0,1,2, \ldots \ldots ., N-K / 2$,
Let

$$
\begin{equation*}
f_{K}^{-}(j)=f_{K / 2}^{+}(j)-f_{K / 2}^{+}(2 N / K-1-j) \tag{26}
\end{equation*}
$$

$$
\begin{equation*}
f_{K}^{+}(j)=f_{K / 2}^{+}(j)+f_{K / 2}^{+}(2 N / K-1-j) \tag{27}
\end{equation*}
$$

From equation (26) \& (27) may be split into two parts. In general, let $K=2,4,8, \ldots \ldots$. $N / 2$, when $u=0$, $\mathrm{K} / 2, \mathrm{~K}, 3 \mathrm{~K} / 2,2 \mathrm{~K} \ldots \ldots, \mathrm{~N}-\mathrm{K} / 2$, we obtain

$$
\begin{align*}
& F(u)=\sum_{j=0}^{\frac{N}{2 K}-1} \alpha_{3 D}(u, v, w) \cos \left(\frac{2 j+1}{2 N} u \pi\right) \cos \left(\frac{2 j+1}{2 N} v \pi\right) \cos \left(\frac{2 j+1}{2 N} w \pi\right) \times \\
& \left\{\left[f_{K / 2}^{+}(j)+(-1)^{\frac{2 u}{K}} f_{k / 2}^{+}\left(\frac{2 N}{K}-1-j\right)\right]\right\}+\sum_{j=N / 2 K}^{\frac{N}{K}-1} \alpha_{3 D}(u, v, w) \cos \left(\frac{2 j+1}{2 N} u \pi\right)  \tag{28}\\
& \cos \left(\frac{2 j+1}{2 N} v \pi\right) \cos \left(\frac{2 j+1}{2 N} w \pi\right)\left\{\left[f_{K / 2}^{+}(j)+(-1)^{\frac{2 u}{K}} f_{k / 2}^{+}\left(\frac{2 N}{K}-1-j\right)\right]\right.
\end{align*}
$$

observe that the value of the right hand side does not depend on K , but it can only be used to compute the value of $\mathrm{F}(\mathrm{u}, \mathrm{v}, \mathrm{w})$ for a set of values of $\mathrm{u}, \mathrm{v} \& \mathrm{w}$ which depends on the value of K . For $\mathrm{u}, \mathrm{v}, \mathrm{w}=\mathrm{K} / 2,3 \mathrm{~K} / 2,5 \mathrm{~K} / 2, \ldots \ldots$, $\mathrm{N}-$ $\mathrm{K} / 2, \alpha_{3 D}(u, v, w)=1$, using formula (22) and (26), the above expression reduces to

$$
F(u, v, w)=\sum_{j=0}^{\frac{N}{2 K}-1}\left\{\begin{array}{l}
\cos \left(\frac{2 j+1}{2 N} u \pi\right) \cos \left(\frac{2 j+1}{2 N} v \pi\right) \cos \left(\frac{2 j+1}{2 N} w \pi\right) f_{K}^{-}(j)+  \tag{29}\\
(-1)^{\frac{u-K / 2}{K}} \times \sin \left(\frac{2 j+1}{2 N} u \pi\right) \sin \left(\frac{2 j+1}{2 N} v \pi\right) \sin \left(\frac{2 j+1}{2 N} w \pi\right) \\
f_{K}^{-}\left(\frac{N}{K}-1-j\right)
\end{array}\right\}
$$

For $\mathrm{u}=\mathrm{v}=\mathrm{w}=0, \mathrm{~K}, 2 \mathrm{~K}, \ldots, \mathrm{~N}-\mathrm{K}$, using formulas (22) and (27), then (28) becomes

$$
\begin{align*}
& F(u)=\sum_{i=0}^{\frac{N}{2 K}-1} \alpha_{3 D}(u, v, w) \cos \left(\frac{2 j+1}{2 N} u \pi\right) \cos \left(\frac{2 j+1}{2 N} v \pi\right)  \tag{30}\\
& \cos \left(\frac{2 j+1}{2 N} w \pi\right) \times\left\{f_{K}^{+}(i)+(-1)^{\frac{u}{K}}\left[f_{K}^{+}\left(\frac{N}{K}-1-j\right)\right\}\right.
\end{align*}
$$

Which again can be split as in (28) until there is only one term left in the sum (i. e. for $\mathrm{K}=\mathrm{N} / 2$ ), allowing the computation of $\mathrm{F}(0)$ and $\mathrm{F}(\mathrm{N} / 2)$. Note that

$$
\alpha_{3 D}(0)=1 / \sqrt{2}=\cos (\pi / 4)=\sin (\pi / 4)
$$

Therefore,

$$
\begin{equation*}
F(0)=\cos \left(\frac{\pi}{4}\right) f_{N / 2}^{+}(0)+\sin \left(\frac{\pi}{4}\right) f_{N / 2}^{+}(1) \tag{31}
\end{equation*}
$$

For $\mathrm{u}=\mathrm{K}=\mathrm{N} / 2$ we get from (30)

$$
\begin{equation*}
F\left(\frac{N}{2}\right)=-\left\{\cos \left(\frac{\pi}{4}\right) f_{N / 2}^{+}(1)-\sin \left(\frac{\pi}{4}\right) f_{N / 2}^{+}(0)\right\} \tag{32}
\end{equation*}
$$

We can replace u in formula (29) with N - $\mathrm{u}^{\prime}$, using formulae (19) and (20), when $\mathrm{K}=2,4,8, \ldots$, $\mathrm{N} / 2$, so for $\mathrm{u}^{\prime}=$ $\mathrm{K} / 2,3 \mathrm{~K} / 2,5 \mathrm{~K} / 2, \ldots . \mathrm{N}-\mathrm{K} / 2$, we have an alternative expression:

$$
\left\{\begin{array}{l}
\sin \left(\frac{2 j+1}{2 N} u^{\prime} \pi\right) \sin \left(\frac{2 j+1}{2 N} v^{\prime} \pi\right) \sin \left(\frac{2 j+1}{2 N} w^{\prime} \pi\right) f_{K}^{-}(j)  \tag{33}\\
-(-1)^{\frac{u^{\prime}-K / 2}{K}} \cos \left(\frac{2 j+1}{2 N} u^{\prime} \pi\right) \cos \left(\frac{2 j+1}{2 N} v^{\prime} \pi\right) \\
\cos \left(\frac{2 j+1}{2 N} w^{\prime} \pi\right) \times f_{K}^{-}\left(\frac{N}{K}-1-j\right)
\end{array}\right\}
$$

Because $-\sin \theta=\sin (-\theta), \cos \theta=\cos (-\theta)$, the sign term $(-1)(u-K / 2) / K$ in formulas (13) and (17) may also be moved into the sin function. The angles $\left.(-1)^{\frac{u-K / 2}{K}}(2 i+1) u \pi / 2 N\right)$ may be thus expressed as follows:

$$
\begin{aligned}
& (-1)^{\frac{u-K / 2}{K}}(2 j+1) u \pi / 2 N=m(j, u) \pi+\theta(j, u) \\
& m(j, u)=0, \pm 1, \pm 2, \ldots,-\frac{\pi}{2}<\theta(j, u)<\frac{\pi}{2}
\end{aligned}
$$

So we have

$$
\begin{align*}
& (-1)^{\frac{u-K / 2}{K}} \sin \frac{2 j+1}{2 N} u \pi=\sin (-1)^{\frac{u-K / 2}{K}} \frac{2 j+1}{2 N} u \pi=(-1)^{m(j, u)} \sin \theta(j, u) \\
& \cos \frac{2 j+1}{2 N} u \pi=\cos (-1)^{\frac{u-K / 2}{K}} \frac{2 j+1}{2 N} u \pi=(-1)^{m(j, u)} \cos \theta(j, u) \tag{34}
\end{align*}
$$

For a given value of K the two expressions (29) and (33) can compute the same set of values $\mathrm{F}(\mathrm{u})$ when u ranges over $u=K / 2,3 K / 2,5 K / 2, \ldots ., N-K / 2$. So with $u$ from half the range, $u=K / 2,3 K / 2,5 K / 2, \ldots .,(N-K / 2), F(u)$ and $\mathrm{F}(\mathrm{N}-\mathrm{u})$ together then provide the complete set of values:

$$
\begin{align*}
& F(u)=\sum_{j=0}^{\frac{N}{2 K}}(-1)^{m(j, u)}\left\{f_{K}^{-}(i) \cos \theta(j, u)+f_{K}^{-}\left(\frac{N}{K}-1-i\right) \sin \theta(j, u)\right\}  \tag{35}\\
& F(N-u)=\sum_{j=0}^{\frac{N}{2 K}-1}(-1)^{j+\frac{u-K / 2}{K}+m(j, u)} \times\left\{f_{K}^{-}\left(\frac{N}{K}-1-j\right) \cos \theta(j, u)-f_{K}^{-}(i) \sin \theta(i, j)\right\} \tag{36}
\end{align*}
$$

when $u=K / 2,3 K / 2, \ldots .(\mathrm{N}-\mathrm{K}) / 2$ and $\mathrm{K}=2,4,8, \ldots \mathrm{~N} / 2, \mathrm{~F}(0)$ and $\mathrm{F}(\mathrm{N} / 2)$ are calculated according to the formulas (15), (16). Comparing the above formulae with the CORDIC algorithm, it is found that the $\mathrm{N}=2 \mathrm{u}$ DCT F(u) ( $u=0,1,2,3, \ldots, \mathrm{~N}-1$ ) can be computed using the CORDIC algorithm. Therefore

$$
\begin{align*}
& F(u)=\sqrt{\frac{2}{N}} \frac{1}{K_{c}} \sum_{j=0}^{N / 2 K-1} Y(j, u)  \tag{37}\\
& F(N-u)=\sqrt{\frac{2}{N}} \frac{1}{K_{c}} \sum_{j=0}^{N / 2 K-1} X(j, u) \tag{38}
\end{align*}
$$

for $\mathrm{u}=\mathrm{K} / 2,3 \mathrm{~K} / 2,5 \mathrm{~K} / 2, \ldots,(\mathrm{~N}-\mathrm{K}) / 2, \mathrm{~K}=2,4,8, \ldots, \mathrm{~N} / 2$, and
re the values

$$
\begin{array}{lr}
F(0)=\sqrt{\frac{2}{N}} \frac{1}{K_{c}} Y(0,0) F\left(\frac{N}{2}\right)=\sqrt{\frac{2}{N}} \frac{1}{K_{c}} Y(0,0) \\
X(j, u)=(-1)^{j+\frac{u-K / 2}{K}+m(j, u)} X q & X(0,0)=-X q \\
Y(j, u)=(-1)^{m(j, u)} Y q, & Y(0,0)=Y q
\end{array}
$$

are computed from results of CORDIC computations yielding the values Xq and Yq after q steps, using suitable initial values (for $\mathrm{K}=0, \mathrm{X} 0=f_{N / 2}^{+}(1), \mathrm{Y} 0=f_{N / 2}^{+}(0)$ and for $\mathrm{K}=2,4,8, \ldots ., \mathrm{N} / 2, \mathrm{X} 0=f_{K}^{-}(N / K-1-j)$, $\mathrm{Y} 0=f_{K}^{-}(i)$ and rotation through angles $\theta(\mathrm{i}, \mathrm{u})$.

## 3D-IDCT USING CORDIC ALGORITHM

The one dimensional even inverse discrete cosine transform (IDCT) [4] is defined by

## Inverse 3D-DCT

$$
f(x, y, z)=\sum_{u=0}^{N_{1}-1} \sum_{v=0}^{N_{2}-1} \sum_{w=0}^{N_{3}-1}\left\{\alpha_{3 D}(u, v, w) F(u, v, w) \cos \left(l_{1}\right) \cos \left(l_{2}\right) \cos \left(l_{3}\right)\right\}
$$

where

$$
\begin{equation*}
l_{1}=\frac{\pi(2 x+1) u}{2 N_{1}}, l_{2}=\frac{\pi(2 y+1) v}{2 N_{2}}, l_{3}=\frac{\pi(2 z+1) w}{2 N_{3}} \tag{39}
\end{equation*}
$$

$$
\begin{aligned}
& \mathrm{j}=0,1,2,3, \ldots \ldots, \mathrm{~N}-1 \\
& \alpha_{3 D}(0)=1 / \sqrt{2}, \alpha_{3 D}(u, v, w)=1 \\
& u=v=w=1,2, \ldots ., N-1
\end{aligned}
$$

We will initially not consider the constant for simplicity. The above formula is also divided into two parts, then replacing u with $\mathrm{N}-\mathrm{u}^{\prime}$ for the second term, using formula (20), f (i) becomes

$$
\begin{align*}
& f(x, y, z)=\sum_{u=0}^{N / 2-1} \sum_{v=0}^{N / 2-1} \sum_{w=0}^{N / 2-1} \alpha_{3 D}(u, v, w) \cos \left(l_{1}\right) \cos \left(l_{2}\right) \cos \left(l_{3}\right) \\
& F(u, v, w)+\sum_{u=N / 2}^{N-1} \sum_{v=N / 2}^{N-1} \sum_{w=N / 2}^{N-1} \alpha_{3 D}(u, v, w) \cos \left(l_{1}\right) \cos \left(l_{2}\right) \cos \left(l_{3}\right) F(u, v, w) \\
& =\frac{F(0)}{\sqrt{2}}+(-1)^{\left[\frac{j+1}{2}\right]} \frac{F(N / 2)}{\sqrt{2}}+\sum_{u=1}^{N / 2-1}\left\{\begin{array}{l}
\cos (l 1) \cos (l 2) \cos (l 3) F(u, v, w) \\
+(-1)^{j} \sin (l 1) \sin (l 2) \sin (l 3) \\
F(N-u) F(N-v) F(N-w)
\end{array}\right\} \\
& =\sum_{u=0}^{N / 2-1} \sum_{v=0}^{N / 2-1} \sum_{w=0}^{N / 2-1} g(j, u) g(j, v) g(j, w) \tag{40}
\end{align*}
$$

Where for $\mathrm{u}, \mathrm{v}, \mathrm{w}=1,2, \ldots, \mathrm{~N} / 2$ and all j

$$
\begin{aligned}
& g(j, 0)=\cos \left(\frac{\pi}{4}\right) F(0)+(-1)^{\left[\frac{j+1}{2}\right]} \sin \left(\frac{\pi}{4}\right) F\left(\frac{N}{2}\right) \\
& g(j, u)=\cos \left(\frac{2 j+1}{2 N} u \pi\right) F(u)+(-1)^{j} \sin \left(\frac{2 j+1}{2 N} u \pi\right) F(N-u)
\end{aligned}
$$

Similarly for $g(j, v)$ and $g(j, w)$ can be write
Similar to the DCT, let $\mathrm{K}=1,2,4,8 \ldots$, $\mathrm{N} / 2$ using formula (22), (23), when $\mathrm{u}=\mathrm{K}, 2 \mathrm{~K}, 3 \mathrm{~K}, \ldots \ldots$, we get

$$
\begin{align*}
& g\left(\frac{N}{K}-1-j, u\right)=(-1)^{u / K}\left\{\cos \left(\frac{2 j+1}{2 N} u \pi\right) F(u)+(-1)^{j} \sin \left(\frac{2 j+1}{2 N} u \pi\right) F(N-u)\right\}=(-1)^{u / K} g(j, u) \\
& \quad=\mathrm{K} / 2,3 \mathrm{~K} / 2,5 \mathrm{~K} / 2, \ldots ., \text { let } \mathrm{K}=2,4,8, \ldots, \mathrm{~N} / 2 \text {, we get: }  \tag{41}\\
& g\left(\frac{N}{K}-1-j, u\right)=(-1)^{\frac{u-K / 2}{K}}\left\{\sin \left(\frac{2 j+1}{2 N} u \pi\right) F(u)+(-1)^{j} \cos \left(\frac{2 j+1}{2 N} u \pi\right) F(N-u)\right\}=g^{\prime}(j, u) \tag{42}
\end{align*}
$$

Where we define $\mathrm{g}^{\prime}(\mathrm{i}, \mathrm{u})$ as follows (note that there is a relation between u and K )

$$
\begin{aligned}
& g^{\prime}(0,0)=\sin \left(\frac{\pi}{4}\right) F(0)-\cos \left(\frac{\pi}{4}\right) F\left(\frac{N}{2}\right)=-g(1,0) \\
& g^{\prime}(j, u)=(-1)^{(u-K / 2) / K}\left\{\sin \left(\frac{2 j+1}{2 N} u \pi\right) F(u)-(-1)^{j} \cos \left(\frac{2 j+1}{2 N} u \pi\right) F(N-u)\right\}
\end{aligned}
$$

Exploiting the symmetries a complete set of values $g(j, u), g(j, v) \& g(j, w)$ needed for the computation of ( $f(x, y, z)$ according to (19), using (20) and (21), can be calculated from (N3 +16)/12 (g(j, u), $g^{\prime}(\mathrm{j}, \mathrm{u})$ ) , $\left(g(j, v), g^{\prime}(j, v)\right), g(j, w), g^{\prime}(j, w)$ pairs of values. Results of all $g(j, u) g(j, v), g(j, w)$ for $N=16, u=0,1,2, \ldots, 7, j=0,1,2, \ldots$, 15 is calculated. Similar to calculating the DCT using CORDIC, we move the sign $(-1) j$ to the sin function. The angle $(-1) j(2 j+1) / u / 2 N$ can be expressed by

$$
\begin{align*}
& (1) \mathrm{j}(2 \mathrm{j}+1) \mathrm{u} / 2 \mathrm{~N}=\mathrm{h}(\mathrm{j}, \mathrm{u})+(\mathrm{j}, \mathrm{u}), \\
& \mathrm{h}(\mathrm{j}, \mathrm{u})=0,1,2, \ldots . \mathrm{j} \quad-/ 2<(\mathrm{j}, \mathrm{u})</ 2  \tag{43}\\
& g^{\prime}(j, u)=(-1)^{K(j, u)}=*\{\cos \beta(j, u) F(u)+\sin \beta(j, u) F(N-u)\} \\
& g^{\prime}(j, u)=(-1)^{j+\frac{u-K / 2}{K}}+h(j, u)=*\{\cos \beta(j, u) F(u)+\sin \beta(j, u) F(u)\} \\
& g(0,0)=\cos (\pi / 4) F(0)+\sin (\pi / 4) F(N / 2) \\
& g^{\prime}(0,0)=-\{\cos (\pi / 4) F(N / 2)-\sin (\pi / 4) F(0)\}
\end{align*}
$$

So we have
$g(j, u), g^{\prime}(j, u), g(j, v), g^{\prime}(j, v)$ and $g(j, w), g^{\prime}(j, w)$ and $\left(g(0,0), g^{\prime}(0,0)\right)$ can be computed using $q$ steps of the CORDIC algorithm.

$$
\begin{array}{ll}
g^{\prime}(j, u)=(-1)^{j+\frac{u-K / 2}{K}+h(j, u)} X_{q^{\prime}}^{\prime} & g^{\prime}(0,0)=-X_{q^{\prime}}^{;} \\
g^{\prime}(j, u)=(-1)^{u(j, u)} Y_{q^{\prime}}^{\prime} & g^{\prime}(0,0)=Y_{q^{\prime}}^{\prime} \\
g^{\prime}(j, v)=(-1)^{j+\frac{u-K / 2}{K}+h(j, u)} X_{q^{\prime}}^{\prime} & g^{\prime}(0,0)=-X_{q^{\prime}}^{;} \\
g^{\prime}(j, v)=(-1)^{u(j, u)} Y_{q^{\prime}}^{\prime} & g^{\prime}(0,0)=Y_{q^{\prime}}^{\prime} \\
g^{\prime}(j, w)=(-1)^{j+\frac{u-K / 2}{K}+h(j, u)} X_{q^{\prime}}^{\prime} & g^{\prime}(0,0)=-X_{q^{\prime}}^{;} \\
g(j, w)=(-1)^{u(j, u)} Y_{q^{\prime}}^{\prime} & g^{\prime}(0,0)=Y_{q^{\prime}}^{\prime} \\
X_{l+1}^{\prime}=X_{l}^{\prime}-\delta_{1} Y_{1}^{\prime} x 2^{-1} \\
Y_{l+1}^{\prime}=Y_{l}^{\prime}-\delta_{1} Y_{1}^{\prime} x 2^{-1} & \\
\beta_{l+1}=\beta_{1}+\delta_{1} \alpha_{1} & l=0,1, \ldots, q-1,
\end{array}
$$

Where $\delta_{1}$ is chosen such that $\beta_{1}$ converge to the angle $\beta(\mathbf{i}, \mathbf{u})$, using initial values $\mathrm{X}^{\prime}=\mathrm{F}(\mathrm{N}-\mathrm{u}), \mathrm{Y}_{0}^{\prime}=\mathrm{F}(\mathrm{u})$, for $\mathrm{u}=0$ $\mathrm{X}^{\prime}=\mathrm{F}(\mathrm{N} / 2), \mathrm{Y}^{\prime}=\mathrm{F}(0)$.

## CORDIC COMPUTATIONS OF 3D-DCT/IDCT ALGORITHM

In this section, we will see how many CORDIC computations are needed in this algorithm [18] for DCT/IDCT. Consider the values of K and $\mathrm{u}=\mathrm{v}=\mathrm{w}=\mathrm{K} / 2,3 \mathrm{~K} / 2,5 \mathrm{~K} / 2, \ldots \ldots(\mathrm{~N}-\mathrm{K}) / 2 \mathrm{j}$ will change from 0 to $\mathrm{N} / 2 \mathrm{~K}-1$, hence there are $\mathrm{N} * \mathrm{~N} * \mathrm{~N} /(2 \mathrm{~K} * 2 \mathrm{~K})$ CORDIC Computations .Now including computations required for $\mathrm{F}(0)$ and $\mathrm{F}(\mathrm{N} / 2)$ for 3D DCT is ( $\mathrm{N} 3+8$ ) /12. Table -1 shows number of CORDIC computations required for the 3D DCT and can be calculated. We can have observed that depends upon the iterations required the CORDIC computations goes on increasing from 44 to 342 for $\mathrm{N}=8$ and $\mathrm{N}=16$, that may lead to the increase Of course area size but obviously reduces the complexity in the computations of the DCT/IDCT transform that will lead to the less power consumptions in the digital signal processing application.

Table -1 CORDIC Computations

| Computations Formula | Algorithms | $\mathrm{N}=8$ | $\mathrm{~N}=16$ |
| :---: | :---: | :---: | :---: |
| $(\mathrm{~N} 2+8) / 12$. | 2D-DCT | 6 | 22 |
| $(\mathrm{~N} 3+16) / 12$. | 3D-DCT | 44 | 342 |

## 3D-DCT/IDCT STRUCTURE USING CORDIC ALGORITHM

From the above Table -1 using CORDIC computation, the structure for the 3D DCT/IDCT would be used for $\mathrm{N}=8$ Computations required 44 and for $\mathrm{N}=16$ Computations required is 342 . The parallel computations is needed for calculation of latency, area and speed. The block diagram of 3d-DCT/IDCT structure can be drawn from the computations as shown in Fig.4, the Image Inputs vectors in the form of coefficients are given to the Selector, which will select the binary ' 0 ' or ' 1 ' generated by the select line. When the selector will Select ' 0 ' then the processor will work as DCT i.e Discrete Cosine Transform And when '1' then processor will work as Inverse discrete transform .The RAM buffer is used to store the output vector from the selector and proceed to the Processor .The complex computation required in the DCT/IDCT is computed by the CORDIC algorithm. This will not only lead to the area minimization but also speed boosting from the following synthesis result.


Fig. 4 Block diagram of 3D-DCT/IDCT structure

## SYNTHESIS RESULT AND DISCUSSION

Based on this algorithm a new processor 3D-DCT/IDCT architecture is design and synthesis results with the help of Xilinx ISE 14.7 tools and FPGA implementation has been done with Altera Quartus cyclone-II (DE2 board). The results have been compared with $[19,20]$ 2D-DCT /IDCT structure. we can observe the improvement in working frequency in proposed work hence area wise decrement as shown in the given Table -2.

Table -2 Comparative Analysis and Synthesis Results

| Parameters | Proposed 3D-DCT/IDCT structure | 2D-DCT/IDCT structure[13,19] |
| :---: | :---: | :---: |
| Number of Slices | 447 | 727 |
| Occupied LUT's | 1412 | 1181 |
| RAM | 32 | -- |
| Working Frequency | 310 MHz | 111.08 MHz |
| Dynamic Power | 10 mW | 9 mW |
| Total ON chip power | 458 mW | 1.198 W |

## CONCLUSION

This paper has presented the algorithms of 3D-DCT and IDCT using CORDIC which leads to reduce the complex computation required in the Discrete Cosine Transform and Inverse Discrete Cosine transform. Using CORDIC algorithm frequency has been increased. Faster CMOS based Architecture is very much useful in the today's era of technology. Comparing 2D-DCT [21,22] and 3D-DCT[proposed] we found that CORDIC computation has been incremented by $38 \%$ for $\mathrm{N}=8$, whereas for $\mathrm{N}=16$, large number of computations are required for the 3D-DCT algorithm.

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