



Mathematical Formulations of the Phasor Measurement Units (PMU) component for Fault Location in Power System Networks

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ABSTRACT

This research proposes a strategy suitable for the placement of Phasor Measurement Units (PMUs) for fault location in power networks. In this paper, the well-designed method utilizes the development of an Evolutionary Computing (EC) approach which is based on a Modified Sparsity Genetic Algorithm (GA) optimizer (MS-GAO) and is presented and validated with Mathematical Formulations of the PMU components. MATLAB Simulink program (2022a) and GNU Octave program (8.4.0) are the materials used, The data were sourced and collected from open-source data and IEEE benchmarks – IEEE 6 bus and 14 bus power system networks. The Fault Index was simulated in MATLAB using the approximate Positive Admittance Sequence (aPSA). Also, several IEEE Benchmarks were validated using the MATLAB program. Fault localization is done using an evolutionary computing technique based on the Genetic Algorithms and a modified version – the Modified Sparsity Genetic Algorithm (GA) optimizer (MS-GAO) as described. The conclusions that may be derived from this survey are two-fold: first, there is the need for highly accurate optimizers, and second the need for fast optimizers. These requirements coupled with the need for less complex AI optimizers have very important implications for power system operators and maintenance engineers alike

Keywords: IEEE 6 bus and 14 bus, Octave program, Positive Admittance Sequence, Mathematical Formulations, MATLAB Simulink

INTRODUCTION

The demand for electrical power is generally in the increase at a fast rate in economically developing countries like Nigeria. So, the power distribution networks are becoming highly loaded; so, the issue of protection scheme has become a great concern in most of the injection substation power distribution networks. A number of works have been carried out in the area of electric power distribution protection and solutions have been proposed to improve the protection of the distribution network (Ijeoma and Amadi, 2024).

The developed nations rode on reliable electricity to attain and sustain their present status. Today, they can boast of years of uninterrupted power supply which have helped their industries, utilities, hospitals, schools, etc to make life comfortable for their citizens. This means that, we can comfortably say that no nation can move from an underdeveloped status to becoming a developed nation without reliable electricity; no reliable electricity, no development. Sustainable developments have continued to elude Nigeria as a nation due to lack of reliable electricity (Ibinabo and Ijeoma, 2019).

Currently, there is a growing need to develop and implement better heuristics to solve the PMU Optimal Placement Problem (PMU-OPP) for the localization of faults in power system networks. In this regard, a large number of Artificial intelligence (AI) based PMU-OPP solution techniques have been researched and developed. However, it is a known fact that due to the No Free Lunch Theorem (NFLT), no technique is inherently better than another considering a wide variety of tasks. It is on this basis that this research seeks to make a case for unique solutions to the PMU-OPP for fault localization. This section details an overview of past research on PMU fault location strategies and in addition, discusses the current trends in the state-of-the-art as it pertains to the location of faults along power network Transmission Line (TL) using PMUs with a specific emphasis on the PMU-OPP. The PSO approach represents a widely used and very popular metaheuristic strategy used in many power systems domains

and even in other fields. Thus, it deserves special attention here considering the research works on the OPP problem.

Hachimenum et al. (2025) explains that a power distribution unit is a device fitted with multiple outputs designed to distribute electric power, especially to racks of computers and networking equipment located within a data centre. Data centres face challenges in power protection and management solutions. This is why many data centres rely on Protocol Data Unit or Power Distribution Unit (PDU) monitoring to improve efficiency, uptime, and growth. PDUs vary from simple and inexpensive rack-mounted power strips to larger floor-mounted PDUs with multiple functions including power filtering to improve power quality, intelligent load balancing, and remote monitoring and control by LAN or Simple Network Management Protocol (SNMP). This kind of PDU placement offers intelligent capabilities such as power metering at the inlet, outlet, and PDU branch circuit level and support for environment sensors. The newer generation of intelligent PDUs allows for IP consolidation, which means many PDUs can be linked in an array under a single IP address. Next-generation models also offer integration with electronic locks, providing the ability to network and manage PDUs and locks through the same appliance.

In Saleh and Wadoud (2017), the Binary Particle Swarm Optimization (BPSO) technique is used to solve the OPP problem considering the objective of minimum number of PMUs and the constraints of a numerical Complete Network Observability (CNO).

In Rather et al. (2015), the BPSO technique is used to solve the OPP problem considering the objective of minimum realistic cost in addition to the constraints of a CNO and $N \geq 1$ contingency.

In Mishra et al. (2016), BPSO technique is used to solve the OPP problem considering the objective of minimum number of substations in addition to the CNO constraint and additional constraints requiring the prioritization of the sub-station critical elements.

In Maji and Acharjee (2017), BPSO technique is used to solve the OPP problem considering the objective of minimum number of PMUs and the constraints of a CNO, $N-1$ contingency as well as multistage installation scheme constraints.

In Wang et al (2012), the OPP problem was solved using a hybridized approach combining the SA with the Particle Swarm Optimizer (PSO). Furthermore, they exploited crossover and genetic mutation operations to ensure diversity in the particles used for the OPP solution. In another study, Rather et al (2013) introduced a constriction factor BPSO to overcome the local optimality issue and local trapping problem during the search for the solution space for the OPP.

Mazhari et. al. (2013) proposed a binary-based Particle Swarm Optimizer (PSO) including support for zero-injection bus for minimizing the number of PMUs used in a power network. However, no support for optimizing the fault location function was provided.

Ren et. al. (2014) presented dual step strategy where candidate locations are found by iterating line segments in the first step using synchrophasors placed at one terminal end and in the second step where the actual location is found by comparing voltage phasors from junction nodes calculated by synchrophasors placed at both ends. The results on the impact of resistive uncertainties of line parameters on estimated distance were reported.

A Genetic Algorithm (GA) technique that is constrained in terms of its topology (Constrain-GA) was proposed in Zhao et al., (2015) for solving the OPP problem. Their proposed approach included topology reconfiguration rules and rules for PMU reconfiguration as well. This involved removal of low-priority buses in the solution space to reduce the amount of non-feasible solutions.

Zhao et al (2021) proposed the use of Singular Value Decomposition (SVD). Accuracies were reported in terms of the Normalized Mean Squared Errors (NMSE), the Mean Absolute Errors (MAE) and compression ratios (CR). Their results showed that their proposed improved SVD solution gave higher CRs compared to traditional methods and their MAE is competitive with the existing ones.

Sheta et al (2021) proposed an Iterative Support Detection (ISD) and PMUs for fault location in distribution networks. Their proposed approach obviates the need for fault-type pre-determination and was validated on an 11-bus and a 104-bus distribution system considering performance measures such as fault type, fault location, fault resistance, loss of PMU data, and measurement noises. Their proposed approach was found to give better error response, lower bound limits in the case of missing PMU, and faster run times.

Hyacinth and Gomathi (2021) proposed a 3-stage heuristic approach for minimizing the number of PMUs used in a power network. Their approach did not include support for zero-injection bus; also, the approach did not include channel limitation constraints and $N1$ -contingency constraints and no optimization of fault location was considered in this study.

Integer Linear Programming (ILP) approach based on the MATLAB optimal tool was proposed by Elimam et al (2021) for minimizing the number of PMUs used in a power network. Their approach included support for zero-injection bus; however, the approach did not include channel limitation constraints and no optimization of fault location was considered in this study.

In the early 1960s, the only solution algorithm available for power system optimization was the Gauss-seidel type method. In 1968, Dommel and Tinney introduced a reduced gradient steepest-descent algorithm with a simple

exterior penalty function, which until recently, was the only successful algorithm in use. Two drawbacks were associated with the algorithm: slow convergence with the steepest descent direction, and ill-conditioning resulting from the penalty function associated with the inequality constraints. In an attempt to solve OPF problems during the last decade, numerous optimization algorithms have been employed with varied successes.

Among the frequently used techniques include: the linear programming (LP) method, the Newton-based (NB) method, the quadratic programming (QP) method, the simplex algorithm (SA) method, and the interior point (IP) method.

The linear programming (LP) algorithm is based on the linear/piece-wise linear approximation of objectives and constraints to locate the optimal feasible solution by using an iterative approach to satisfy the non-linear constraints, where iteration is based on the linear approximation with the optimal search direction.

The Newton-based (NB) method employs an iterative scheme based on an approximation of the Hessian matrix, which is calculated in the iteration.

The Quadratic programming (QP) technique uses the Kuhn-Tucker conditions and the Newton search direction at iteration, where the non-linear constraints are linearized and the sub-problems are solved.

Interior point methods are usually classified into three main categories: Projective methods, affine-scaling methods, and primal-dual methods.

The projective method which includes Karmarkar's original algorithm, is responsible for the great interest set to the area.

Soon after 1984, affine-scaling methods were obtained as a simplification of the projective methods. Affine-scaling methods does not share all the good qualities of projective method, but its reduced computational complexity and simplicity made it become very popular at the time. In practice, it was among the most effective.

One of the drawbacks of (IP) methods was their difficulty in detecting infeasibility. Fortunately, new homogeneous and self-dual models for (IP) can easily detect infeasibility while their computational complexity is quite similar to the complexity of the most efficient primal-dual methods. Several optimization packages already have this method as an option.

The computational effort of an (IP) algorithm is dominated by the solution of large, sparse linear systems. Therefore, the performance of any (IP) code is dependent highly on the linear algebra kernel.

Networks arise in numerous settings and in a variety of styles such as in transportation, electrical, biological, social, and communication networks parade our daily lives. Network representations also are widely used for problems in such diverse areas as production, distribution, project planning, facilities location, resource management, and so on. A network representation provides the component systems that are used in virtually every field of scientific, social, and economic endeavors.

One of the most exciting developments in operations research (OR) in recent years has been the usually rapid advance in both the methodology and the application of network optimization models. Several algorithmic breakthroughs have had a major impact, as have the ideas from computer science concerning data structures and efficient data manipulation.

Many network optimization models actually are special types of linear programming problems. For example, special types of linear programming problems include the transportation problem and the assignment problem. They are sometimes called network optimization problems and can be solved with network representations.

Networks are necessary for the movement of people, transportation of goods, communication information, and control of the flow of matter and energy. Network application is quite vast with such phenomena that are represented and analyzed as roads, railways, cables, and pipelines. In networking, the cost, time, and complex nature of the network increases in different kinds of network-based systems, e.g. Television cable networks, Telephone networks, Electricity supply networks, Gas pipe networks, and water supply systems. Therefore, the cost, time, and complexity of a network are considered greatly in solving networking problems.

Graph theory based on Minimum Solution Technique (MST) approach has been discussed in various papers with varied focuses. An algorithm for finding the shortest path for power routing between two nodes in an electrical network used in the airlines presents MST for network topological observability analysis. In 1994, the application of the MST for finding the connectivity in the VLSI circuits was conducted.

In past studies, the minimization of energy losses in distribution systems by applying a general search method to a Brazil power network has been presented. Here outages were not considered as an important factor to address. The use of a Floyd – Warshall's based MST to find the time scheduling in the data flow graph of a Digital Signal Processor (DSP) in addition to the learning classifier system for loss minimization in a power system have been equally investigated.

The present work deals with a methodology proposed for fault localization in power systems based on Phasor Measurement Units (PMUs) for system restoration in power networks following the tenets of an MST as a dynamic optimization problem in the time-domain, model is an efficient route that can be used in planning power network fault locators. This network model also finds applications in telecommunications and transportation planning.

Mathematical optimization (algorithmic) methods used over the years for many power systems planning, operation, and control problems are derived under certain assumptions, and even with these assumptions; the solution of large-scale power systems is not simple. Most recently deregulation of power utilities has introduced new issues to the existing problems. The solution to power system problems should be optimum globally, but the solution searched by mathematical optimization normally produces optimum locally. These facts make it difficult to deal effectively with many power system problems through strict mathematical formulation alone.

Therefore, artificial intelligence (AI) techniques which promise a global optimum or nearly so, have emerged in recent years in power systems as a complementary tool to mathematical approaches. These include expert systems (ES), artificial neural network (ANN), genetic algorithm (GA), and fuzzy logic.

Despite the successes of the algorithmic approaches described in the previous section, there remains a large class of problems that elude complete solution in a conventional setting.

These problems require:

- Use of knowledge bases to store human knowledge.
- Operator judgment particularly in practical solutions.
- Experience gained over some time.
- Characterization by network uncertainty, load variations, etc.

In the following sub-sections the techniques based on Genetic Algorithm and Evolutionary computing is presented and discussed. In addition, the technique of local search based on Hill-Climbing (HC) heuristic is equally discussed. The research borders on fault localization using PMUs. It concerns the optimal placement of PMUs in power system networks considering the use of dynamic programming strategies based on Artificial Intelligence and applied to IEEE power networks.

In particular, the study findings demonstrates potential applications in the field of evolutionary inspired Artificial Intelligence (AI) based power systems network planning and design in the context of efficient fault protection systems based on the localization of PMUs.

MATERIALS AND METHOD

In this paper, the well-designed method utilizes the development of an Evolutionary Computing (EC) approach which is based on a Modified Sparsity Genetic Algorithm (GA) optimizer (MS-GAO) is presented and validated.

Materials:

The following materials were used.

1. MATLAB Simulink program (2022a)
2. GNU Octave program (8.4.0)

The data were sourced and collected from open-source data and IEEE benchmarks – IEEE 6 bus and 14 bus power system networks.

Mathematical Formulations of the PMU Component Selection Objective

Despite the high reliability of PMUs, they are exposed to failures as all other measurement devices. When one of the PMUs fails, the bus that was observed via the PMU will become unobservable if the bus is not monitored by other PMUs. Hence, this problem is addressed by ensuring that each bus is observed with two PMUs (except the radial bus). This will ensure that in case one of the PMUs fails, the observability of the power system will remain assured.

Observability of Bus 6,

$$F1 = 1 - (x1 + x2) \quad (1)$$

$$F2 = 1 - (x1 + x2 + x3 + x4) \quad (2)$$

$$F3 = 1 - (x2 + x3 + x4 + x5 + x6) \quad (3)$$

$$F4 = 1 - (x2 + x3 + x4 + x5) \quad (4)$$

$$F5 = 1 - (x3 + x4 + x5 + x6) \quad (5)$$

$$F6 = 1 - (x3 + x5 + x6) \quad (6)$$

A minimum of two PMUs must be installed at nodes 1 and 5 or 2 and 6 to guarantee maximum network observability from the Bus 6 network.

For example, for IEEE 14-bus system to observe bus 1 and satisfy the first constraint (f1) a PMU at least should be installed on one of the buses of 1, 2 and 5, and to fully observability of the IEEE 14-bus at least 4 PMUs are required to fulfil the observability on buses 2, 6, 7 and 9. However this is not the optimal solution, see Ademola et al (2024).

Observability of Bus 14 from the Fig. 3.2,

$$Node1 = x1 + x2 + x5 \geq 1 \quad (7)$$

$$Node 2 = x1 + x2 + x3 + x4 + x5 \geq 1 \quad (8)$$

$$\text{Node 3} = x_2 + x_3 + x_4 \geq 1 \quad (9)$$

$$\text{Node 4} = x_2 + x_3 + x_4 + x_5 + x_7 + x_9 \geq 1 \quad (10)$$

$$\text{Node 5} = x_1 + x_2 + x_4 + x_5 + x_6 \geq 1 \quad (11)$$

$$\text{Node 6} = x_5 + x_6 + x_{11} + x_{12} + x_{13} \geq 1 \quad (12)$$

$$\text{Node 7} = x_4 + x_7 + x_8 + x_9 \geq 1 \quad (13)$$

$$\text{Node 8} = x_7 + x_8 \geq 1 \quad (14)$$

$$\text{Node 9} = x_4 + x_7 + x_9 + x_{10} + x_{14} \geq 1 \quad (15)$$

$$\text{Node 10} = x_9 + x_{10} + x_{11} \geq 1 \quad (16)$$

$$\text{Node 11} = x_6 + x_{10} + x_{11} \geq 1 \quad (17)$$

$$\text{Node 12} = x_6 + x_{12} + x_{13} \geq 1 \quad (18)$$

$$\text{Node 13} = x_6 + x_{12} + x_{13} + x_{14} \geq 1 \quad (19)$$

$$\text{Node 14} = x_9 + x_{13} + x_{14} \geq 1 \quad (20)$$

According to figure 2 node 7 is a Zero Injection Node (ZIN) because it does not have a load or generation source attached to it, while node 8 is radial, since node 8 is a radial node, it is chosen to be integrated with the ZIN. Therefore, a minimum of three PMUs must be installed at node 2, 6, and 9 to guarantee maximum network observability from these newly created constraints of IEEE 14 Bus network, Thus, the problem can be generalized and reduced to a minimizing one, see eqn 21:

$$\text{Min. } f_n : \sum_{i=1}^n (f_1 \cdots f_n) \quad (21)$$

Where,

$n = 14$, in this case.

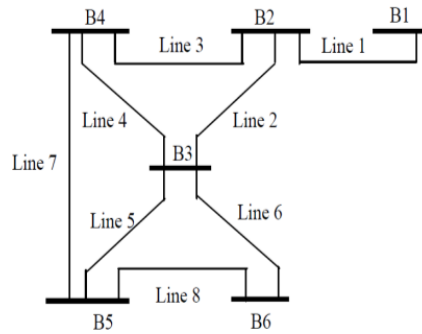


Figure 1: Hypothetical Six Bus Network; Source: Pokharel and Brahma, (2009)

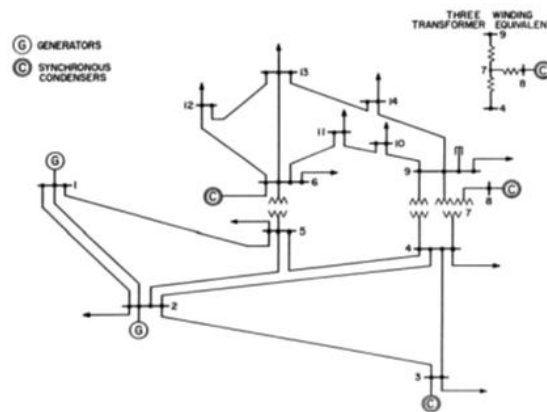


Figure 2: IEEE 14-Bus Network; Source: Pokharel and Brahma, (2009).

RESULTS AND DISCUSSION

This research upshot gotten from the MATLAB, and GNU Octave software simulations were discussed. The Fault Index was simulated in MATLAB using the approximate Positive Admittance Sequence (aPSA). Also several IEEE Benchmarks were validated using MATLAB program. Fault localization is done using evolutionary computing technique based on the Genetic Algorithms and a modified version – the Modified Sparsity Genetic Algorithm (GA) optimizer (MS-GAO) as described.

Simulations considering a Hypothetical 6-bus Network

Simulations to the 6-bus network as presented in this section. The solution of problem (21) is obtained using program routine, ga_pmu_6bus.m as defined in the Main Program (6-Bus Network); this program is based on standard GA used in most power system researches. The fitness response are as shown in Figure 3 while numerical results showing 10 different trial runs and correspondingly the candidate PMU locations for fault localizations are as provided in Table 1.

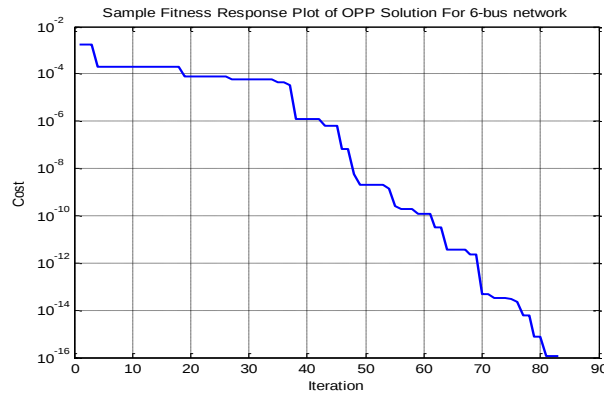


Figure 3: Fitness response of 6bus network – Trial 1 (6-Bus Network)

Table 1: PMU Placement States for 10 different trials: 1 (PMU) 0 (No PMU)

SN: trials	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5	Bus 6
1	1	0	1	0	1	0
2	1	1	0	1	0	1
3	0	0	1	0	0	0
4	1	1	0	0	1	1
5	1	0	1	0	0	1
6	0	1	1	0	0	0
7	1	1	0	0	1	1
8	1	1	0	0	1	0
9	1	1	1	0	0	0
10	1	1	1	0	0	0

As seen in Table 1, the PMU placements on the ON (1) states are more predominant for Bus numbers 1 to 3, and Bus number 6 than in Bus number 4 implying placing PMU at Bus number 4 will not be an efficient operation, Cost Function (6-Bus Network).

Discussions considering 6-bus power network

As shown in Figure 3, the fitness response of the GAO improved as the iterations progresses until around the 80th iteration where convergence begins to set. From the 80th iteration, the solution to the optimal placement of PMU's can be found. Table 1 shows the solution states for the placement of PMU's where a 0 indicates that no PMU should be placed while a 1 indicates the placement of pmu is allowed. It is important to emphasize here that depending on the trial run, the placement positions and geometry might differ which is attributed to the stochastic behavior of GA's. However, this also gives it a variety of possible solutions making it more of a global optimizer.

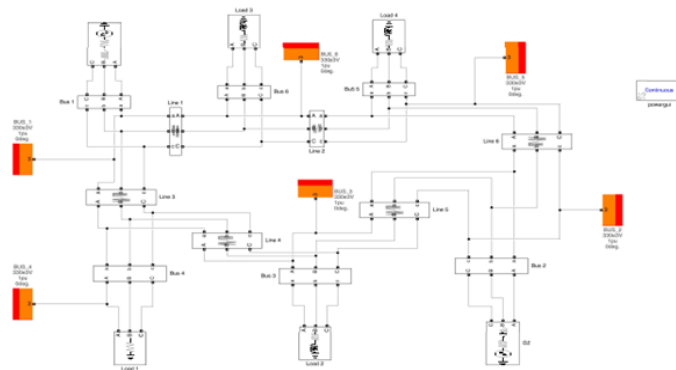


Figure 4: 6 Bus IEEE Network

Load flow analysis of the IEEE 6 bus network simulated in MATLAB Simulink tool.
Creating line outage fault on the network to ascertain the fault level and the impact of PMU.

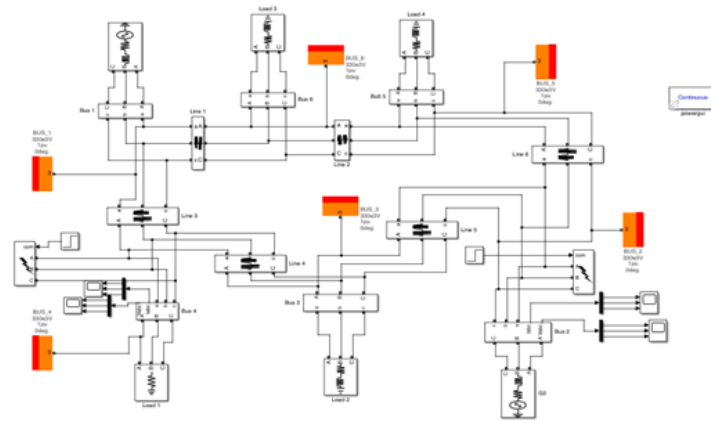


Figure 5: Circuit of 6 bus IEEE Network with Fault Application in MATLAB SIMULINK

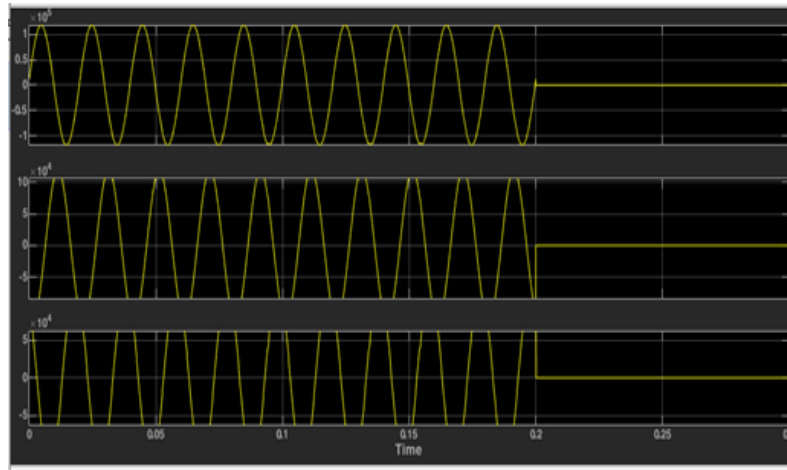


Figure 6: Voltage Profile of bus 1 when a three-phase short circuit occurred at 0.2 seconds

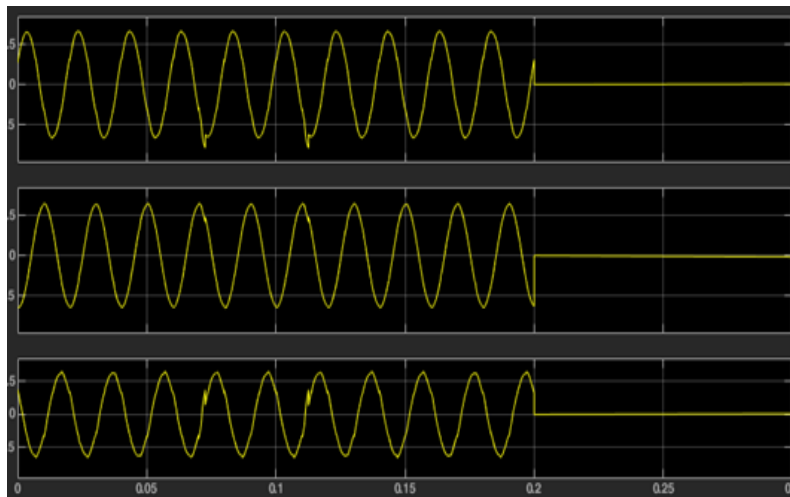


Figure 7: Voltage Profile of bus 2 when three phase short circuit at 0.2 seconds

Three phase symmetrical fault was applied on bus two at 0.2 seconds to ascertain the voltage profile of the network. As shown in Figure 7, the voltage collapsed to zero when a three-phase fault was applied to the system.

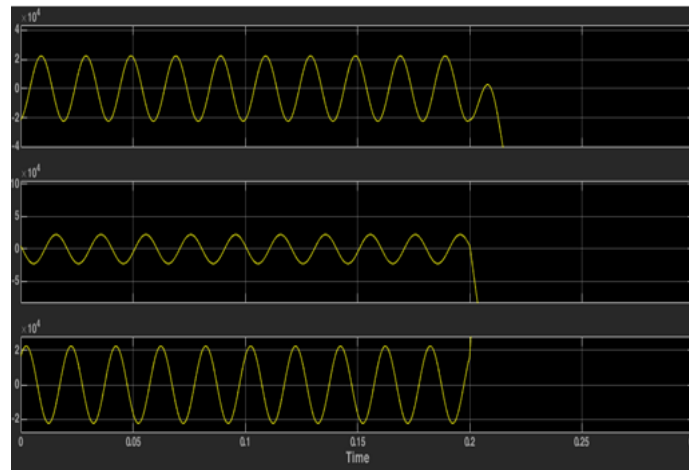


Figure 8: Current Profile of bus 2 when three phase short circuit fault occurred at 0.2 seconds

There is an upward shoot of current when a symmetrical fault occurred at bus 2

CONCLUSION

Currently, there is a growing need to develop and implement better heuristics to solve the PMU Optimal Placement Problem (PMU-OPP) for the localization of faults in power system networks. In this regard, a large number of Artificial Intelligent (AI) based PMU-OPP solution techniques have been researched upon and developed. However, it is a known fact that due to the No Free Lunch Theorem (NFLT), no technique is inherently better than another considering a wide variety of tasks. It is on this basis that this research seeks to make a case for unique solutions to the PMU-OPP for fault localization. This section details an overview of past researches on PMU fault location strategies and in addition discusses the current trends in the state-of-the-art as it pertains to the location of faults along power network Transmission Line (TL) using PMUs and with a specific emphasis on the PMU-OPP. The PSO approach represents a widely used and very popular metaheuristic strategy used in many power systems domains and even in other field. Thus, it deserves special attention here considering the research works on the OPP problem. Mathematical Formulations of the PMU component selection, despite the high reliability of PMUs, they are exposed to failures as all other measurement devices. When one of the PMUs fails, the bus that was observed via the PMU will become unobservable if the bus is not monitored by other PMUs. Hence, this problem is addressed by ensuring that each bus is observed with two PMUs (except the radial bus). This will ensure that in case one of the PMUs fails, the observability of the power system will remain assured. Simulations to the 6-bus network as presented in this section. The solution of problem (21) is obtained using program routine, `ga_pmu_6bus.m` as defined in the Main Program (6-Bus Network); this program is based on standard GA used in most power system researches. The fitness response are as shown in Figure 3 while numerical results showing 10 different trial runs and correspondingly the candidate PMU locations for fault localizations are as provided in Table 1.

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