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Influence of Stokes waves on horizontal and vertical movements of water particles in a wave tank with a linear slope bottom

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ABSTRACT

Ocean circulation accounts for the movement of currents and water particles in the oceans, both at the surface and at depth. This circulation plays an important role in the description of surface waves. We will in our work consider a numerical waves tank for an amplitude A=0.5 a wavelength λ=0.25 and an average height He=10 and a Froude number fixed at 1.105. Numerical wave tank analysis is used to reproduce the natural phenomenon of wave propagation in an experimental model. In this study, the evolution of horizontal and vertical velocities can be determined by considering a linear stokes wave as the driving force of the motion in the tank. To track the evolution of the horizontal and vertical particle components velocities as well as their intensities, we will use a fixed boundary condition at the tank outlet to modify the wave profile over time. The nonlinear stokes theory and the finite difference method calculation will be studied in this case in order to provide hydrodynamic solutions through the Navier- stokes equations. We will use in this work the iterative method of relaxation line by line of Gauss-Siedel by using the Successive over relaxation (S.O.R.) to resolve numerically the nonlinear equations.

Keywords: Ocean circulation, wave tank, horizontal velocities, vertical velocities, nonlinear stokes theory

INTRODUCTION: THE NEW FRONTIER

Ocean circulation explains the movement of currents in the oceans, both at the sur-face and at depth. It is caused by a variety of complex natural phenomena that have been understood through experiments and theoretical modelling, such as differences in salinity and temperature, which generate convective currents; the rotation of the moon around the earth, which causes tides; the rotation of the earth; and the effect of wind on the ocean surface. In this work, we are going to use a consequence of the effects of the wind, in this case the swell, to describe the movement of the particles. The propagation of the swell induces a movement of fluid particles from the surface to the bottom [1]. The horizontal and vertical components of the particles as they pass through the swell make the latter an excellent source of renewable energy. It is therefore of the utmost importance to understand the evolution of particle velocity under the influence of the swell. The horizontal component is also very important for sediment transport. It is therefore important to understand the hydrodynamic aspects of wave propagation, such as variations in velocity components, in order to better describe the wave phenomenon. In fact, it can be said that in linear wave theory; horizontal velocity is proportional to stream function and the vertical component of wave velocity is proportional to potential velocity function see Figure 1 [2].

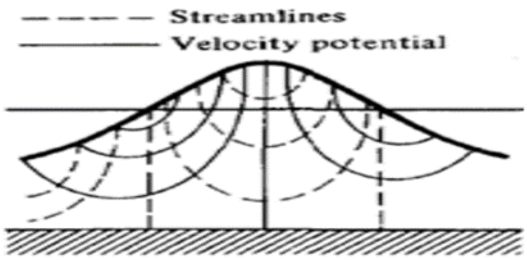


Figure 1: Streamlines and velocity potentials for a progressive wave

To understand swell propagation and its hydrodynamic behaviour, researchers have been experimenting with swell channels to reproduce the real thing, and mathematical and numerical theories have been adopted for some time. [3] present two different numerical methodologies to generate regular gravity waves in a wave tank. We performed numerical simulations of wave generation through the FLUENT® package, using the Volume of Fluid (VOF) multiphase model to reproduce the wave propagation in the tank.[4] numerically solve the second-order Stokes theory using the Crank–Nicholson finite difference method in order to simulate the potential flow and the surface elevation and then to deduct the pressure loads without taking into account the phenomena observed during the flow of the fluid (breaking, reflection, compressibility...) using a 2-D tank.[5] present two different numerical software capabilities to generate regular gravity waves in a wave tank. The wave generation was performed using the FLUENT package and Flow-3D. Both models are based on Navier-Stokes and VOF equations. The results of the mentioned models were compared with theoretical results. Free surface elevation and horizontal component of wave particle velocity were the two parameters which have been considered for comparison.

Our goal is to study variations in the horizontal and vertical movements of particles under the effect of the propagation of a Stokes wave. We will simulate our model from a numerical wave tank with a fixed linear bottom initially actuated by an incident linear stokes wave. The determination of the horizontal and vertical velocity field requires beforehand a detailed and explicit study of the hydrodynamic aspect, namely the evolution of the free surface elevation and the distribution of the velocity potential in the wave tank.

To do this we will first describe the physical model to be studied and describe the calculation through the mathematical formulation. The equations obtained being nonlinear, we will finally approach the numerical formulation to give the results.

SYSTEM DESCRIPTION AND PROBLEM FORMULATION IN THE PHYSICAL DOMAIN

In order to clarify the approach used, we will expose our problem by proposing a numerical wave tank to describe the calculation. During the last decade, research has been done to develop numerical wave tanks [6]. Research has developed different numerical methods to simulate ocean waves. [7] and [8] implemented a source function method to generate ocean waves, based on the Boussinesq model. Based on the 2D form of the Navier-Stokes equations, [9] established a 2D numerical wave reservoir to simulate small amplitude waves and solitary waves. [10] numerically simulated the overtopping of waves against levees in the case of regular waves. We consider a 2D and irrotational flow of a non-viscous and incompressible fluid under the influence of a Stokes wave in a wave tank of length L, wavelength λ and amplitude A see Figure 2. The bottom condition follows a linear evolution given by equation 1.

$$z = h(x) = \frac{h_0}{L}x \quad (1)$$

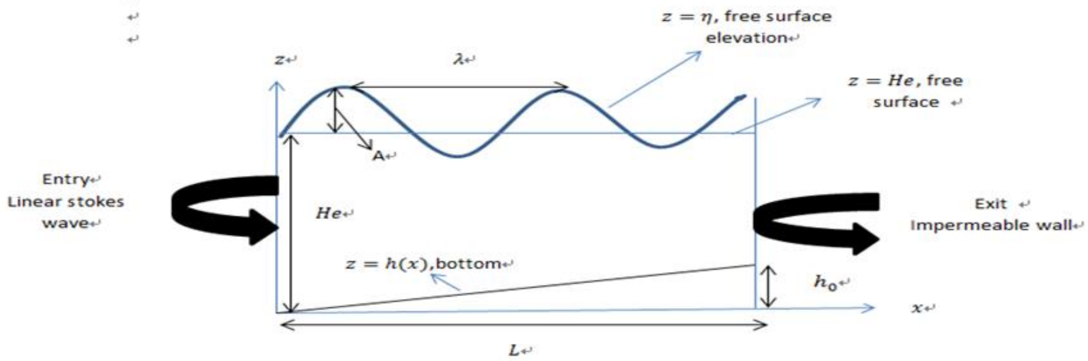


Figure 2: Geometry of the problem

We propose to numerically solve the nonlinear system of equations (2) representing the propagation of a Stokes wave and which are written in terms of the velocity potential and free surface elevation and consisting respectively of

- Laplace equation
- Kinematic free surface condition: water particles cannot cross the free surface. To satisfy the condition of particle velocity must be equal to the normal speed at the free surface
- Dynamic free surface condition: the pressure at the free surface is zero for any position and time. Basically consists of applying the Bernoulli equation to the free surface
- Bottom condition: the bottom can be considered as variable and impermeable

$$\begin{cases} \Delta\phi = 0 & h(x) < z < \eta(x;t); 0 \leq x \leq L \\ \frac{\partial \eta}{\partial t} = -\frac{\partial \phi}{\partial x}\frac{\partial \eta}{\partial x} + \frac{\partial \phi}{\partial z} & z = \eta(x;t)\ ; 0 \leq x \leq L \\ \frac{\partial \emptyset}{\partial t} + \frac{1}{2}\left[\left(\frac{\partial \phi}{\partial x})^2 + \frac{\partial \phi}{\partial z})^2\right)\right] + g\eta = 0 & z = \eta(x;t); 0 \leq x \leq L \\ \frac{\partial \phi}{\partial z} - \frac{\partial \phi}{\partial x}\frac{\partial h}{\partial x} = 0 & z = h(x); 0 \leq x \leq L \end{cases} \quad (2)$$

To remove the difficulties of knowing the treatment of the problem in unsteady regime and the determination of the conditions at the entry and exit boundaries, we will consider an incident linear wave upstream of the tank propagating on a flat bottom of constant depth He of which we know all its hydrodynamics characteristics and which will drive the movement. The hydrodynamic parameters of the incident wave are given by

$$\eta(x;t) = Acos(kx - \omega t) \quad (3)$$

$$\phi(x;z;t) = \frac{Ag}{\omega}\frac{\cosh[k(H_e+z)]}{\sinh(kH_e)}\ sin(kx - \omega t) \quad (4)$$

Thus we can give the initial conditions and complete with the boundary conditions by giving the entry and exit conditions for the free surface elevation η and the velocity potential ϕ

Initial condition

At t=0 the tank is initially disturbed by a linear stokes wave

$$\eta(x;t=0) = H_e + Acos(kx) \quad (5)$$

$$\phi(x\,;z\,;t=0) = \frac{Ag}{\omega}\frac{\cosh[k(z+H_e)]}{\sinh(kH_e)}\ sin(kx) \quad (6)$$

Entry condition: x=0; h(0)<z<η(0;t)

For the entry conditions, we consider that the incident wave upstream of the tank is a linear Stokes wave and that its impact at the entrance creates the wave motion at t>0 .We then have for the entry conditions the free surface elevation and the velocity potential

$$\eta(0;t) = Acos(\omega t) + H_e \quad (7)$$

$$\phi(0;z;t) = \frac{Ag}{\omega}\frac{\cosh[k(z+H_e)]}{\sinh(kH_e)}\ sin(-\omega t) \quad (8)$$

Exit condition: x=L; h(L)<z<η(L;t)

For the exit conditions, we consider a solid impermeable wall. We will then have for the exit conditions of the free surface elevation and of the velocity potential

$$\eta(x=L;t) = Acos(kL - \omega t) + H_e \quad (9)$$

$$\frac{\partial \phi}{\partial x} = 0 \quad (10)$$

The solution of the equations for the velocity potential finally obtained allows us to determine the velocity field in the channel. In fact, the hypothesis of irrotational flow is mathematically translated by

$$\vec{V} = \vec{\nabla}\phi \quad (11)$$

VELOCITY FIELD

On the basis of the irrotational flow hypothesis, we deduce the velocity field from the potential field. Thus, taking V_x (x,z) and V_z (x,z) as the horizontal and vertical components of velocity respectively, we obtain:

$$\begin{cases} v_x(x;z) = \frac{\partial \phi}{\partial x} & pour \quad h(x) < z < \eta(x;t); 0 \leq x \leq L \\ v_z(x;z) = \frac{\partial \phi}{\partial z} & pour \quad h(x) < z < \eta(x;t); 0 \leq x \leq L \end{cases} \quad (12)$$

Boundary conditions

Entry condition : x=0; h(0)<z<η(0;t)

$$\begin{cases} v_x(0;z) = \frac{\partial \phi}{\partial x} = \frac{a.g.k}{\omega}\frac{\cosh[k(H_e+z)]}{\sinh(kH_e)}\ cos(\omega t) \\ v_z(0;z) = \frac{\partial \phi}{\partial z} = -\frac{a.g.k}{\omega}\frac{\sinh[k(H_e+z)]}{\sinh(kH_e)}\ sin(\omega t) \end{cases} \quad (13)$$

With $\phi = \frac{ag}{\omega}\frac{\cosh[k(H_e+z)]}{\sinh(kH_e)}\ sin(kx - \omega t)$

Exit condition : x=L; h(L)<z<η(L;t)

$$\begin{cases} v_x(L;z) = 0 \\ v_z(L;z) = 0 \end{cases} \quad (14)$$

Bottom condition : z=h(x);0≤x≤L

$$\begin{cases} v_x(x;z) = \frac{\partial \phi}{\partial x} \\ v_z(x;z) = 0 \end{cases} \quad (15)$$

Free surface condition : z=η(x;t) ;0≤x≤L

$$\begin{cases} v_x(x;z) = \frac{\partial \phi}{\partial x} \\ v_z(x;z) = \frac{\partial \phi}{\partial z} \end{cases} \quad (16)$$

DIMENSIONLESS STUDY

Considering the multiplicity of the parameters which intervene in the whole of the system of equation of our mathematical model, it would be useful to find a technique to reduce them. To do this we can agglomerate them in the form of dimensionless grouping having a physical significance and which allow

- Obtain information on the solution before solving the problem
- To optimize a possible experimental approach

To each quantity of the equations which govern the flow, one can correspond to a dimensionless quantity from the characteristic quantities. Indeed we have:

$$t=t^{*}\cdot t_0\ ;\ x=x^{*}\cdot L\ ;\ z=z^{*}\cdot h_0;\ \ \eta=\eta^{*}\cdot h_0;\ \ \phi=\phi^{*}\cdot L^2/t_0\ ;\ h^{*}(x^{*})=x^{*};\ [\![A=A]\!]^{*}.h_0\ ;$$
$$[\![H_e=H_e]\!]^{*}.h_0\ ;\ [\![k=k]\!]^{*}/L$$

With t_0 , L and h_0 respectively the time, the characteristic length along the x axis and the characteristic length along the z axis

We thus obtain from the dimensionless system dimensionless numbers, namely the Froude number and a number characteristic of the geometry of the channel which we will note β

The Froude number being the ratio between the forces of inertia and the forces of gravity and its expression is given by

$$1/_{Fr} = \frac{gh_0 t_0^2}{L^2} = \frac{gh_0}{L^2\omega^2}$$

The characteristic number of the geometry of the problem is given by

$$\beta = \frac{L}{h_0}$$

This number tells us about the shape of the slope of the wave tank

This problem as it is proposed, essentially presents the following difficulties: nonlinear, unsteady problem, written on a domain with curved and evolutionary border

DIMENSIONLESS FORMULATION IN CUVILINEAR COORDINATES $(\chi^{*};\zeta^{*})$

In order to facilitate the processing of boundary conditions on these boundaries, an orthogonal curvilinear coordinate system will be used. In this work we use a system of curvilinear coordinates which makes it possible to marry the shape of the free surface at all times and to take account of the irregularity of the bottom. This transformation facilitates the writing of boundary conditions on the irregular and evolving boundaries of the domain. The homotopic transformation T which transforms the physical region (D) into a rectangular domain (Dt) at each dimensionless instant t^{*} is defined by

$$\begin{cases} T: D \to Dt \\ (x^{*}, z^{*}) \mapsto (\chi^{*}, \xi^{*}) \end{cases}$$

$$\chi^{*} = x^{*} \qquad \zeta^{*} = \frac{z^{*} - h^{*}(x^{*})}{\eta^{*}(x^{*};t^{*}) - h^{*}(x^{*})} \qquad t^{*} = t^{*} \tag{17}$$

We then move from the physical domain to a rectangular mathematical domain to describe the numerical calculation as shown in Figure 3

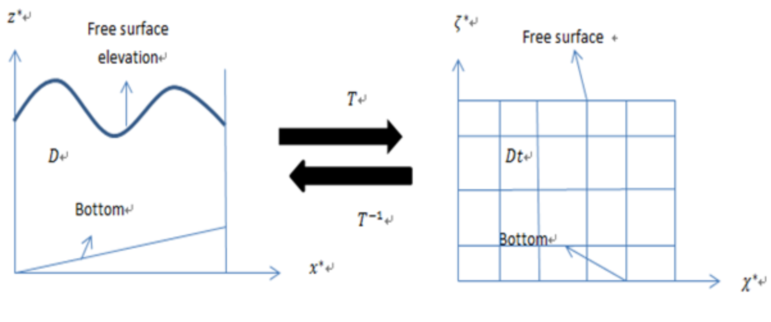


Figure 3: Transformation of the physical domain into rectangular domain

The new system to be solved consisting of the Laplace equation; the conditions at the boundaries of the tank and the initial conditions will be written according to the curvilinear coordinates in the transformed and dimensionless domain.The kinematic condition and the Laplace equation written according to the curvilinear coordinates in the transformed and dimensionless domain give respectively the free surface elevation and the velocity potential in order to get the velocity for the new system.

DIMENSIONLESS VELOCITIES COMPONENTS IN CURVILINEAR COORDINATE SYSTEM

The numerical solution of the coupled and non-linear equations allows us to deduce the velocity field in the wave

$$\begin{cases} v_x^*(\chi^*;\zeta^*) = \frac{\partial\phi^*}{\partial\chi^*} + G^*(\chi^*,\zeta^*)\,\frac{\partial\phi^*}{\partial\zeta^*} & 0<\zeta^*<1; 0\le\chi^*\le 1 \\ v_z^*(\chi^*;\zeta^*) = \frac{1}{\eta^*(\chi^*;t^*)-h^*(\chi^*)}\beta\frac{\partial\phi^*}{\partial\zeta^*} & 0<\zeta^*<1; 0\le\chi^*\le 1 \end{cases} \quad (18)$$

With $G^*(\chi^*;\zeta^*) = \frac{1}{\eta^*(\chi^*;t^*)-h^*(\chi^*)}\left(\zeta^*\left(1-\frac{\partial\eta^*(\chi^*;t)}{\partial\chi^*}\right)-1\right)$

Since the transformation keeps all scalar quantities invariant, the boundary conditions in the dimensionless transformed system can be deduced from those of the dimensionless physical system.

Entry condition x^*=0; 0<ζ^*<1

$$\begin{cases} v_x^*(0;\zeta^*) = \frac{\partial\phi^*}{\partial\chi^*} + G^*(0,\zeta^*)\,\frac{\partial\phi^*}{\partial\zeta^*} \\ v_z^*(0;\zeta^*) = \frac{1}{\eta^*(\chi^*;t^*)-h^*(\chi^*)}\beta\frac{\partial\phi^*}{\partial\zeta^*} \end{cases} \quad (19)$$

With$\phi^*(\chi^*;\zeta^*;t^*) = \frac{1}{Fr}A^*\frac{\cosh\left[\frac{k^*}{\beta}*(\zeta^*(\eta^*(\chi^*;t^*)-\chi^*)+\chi^*+H_e^*)\right]}{\sinh\left(\frac{k^*H_e^*}{\beta}\right)}\,sin(k^*\chi^*-t^*)$

Exit condition x^*=1; 0<ζ^*<1

$$\begin{cases} v_x^*(1;\zeta^*) = 0 \\ v_z^*(1;\zeta^*) = 0 \end{cases} \quad (20)$$

Bottom condition ζ^*=0;0≤χ^*≤1

$$\begin{cases} v_x^*(\chi^*;0) = \frac{\partial\phi^*}{\partial x^*} \\ v_z^*(\chi^*;0) = 0 \end{cases} \quad (21)$$

Free surface condition ζ^*=1;0≤χ^*≤1

$$\begin{cases} v_x^*(\chi^*;1) = \frac{\partial\phi^*}{\partial\chi^*} + G^*(\chi^*,1)\,\frac{\partial\phi^*}{\partial\zeta^*} \\ v_z^*(\chi^*;1) = \frac{1}{\eta^*(\chi^*;t^*)-h^*(\chi^*)}\beta\frac{\partial\phi^*}{\partial\zeta^*} \end{cases} \quad (22)$$

With$G^*(\chi^*;1) = -\frac{1}{\eta^*(\chi^*;t^*)-h^*(\chi^*)}\frac{\partial\eta^*(\chi^*;t)}{\partial\chi^*}$

NUMERICAL PROCEDURE

The discretization of the differential equations makes it possible to transform these differential equations into algebraic equations where the continuous variations of the variables of the flow are represented by values at discrete points. Discrete locations in space are represented by nodal points (or nodes) chosen from a numerical grid (mesh) that subdivides the flow domain as shown in figure 4. The discretization procedure makes approximations to the spatial derivatives of the variables of the flow present in the differential equation, at each node of the grid

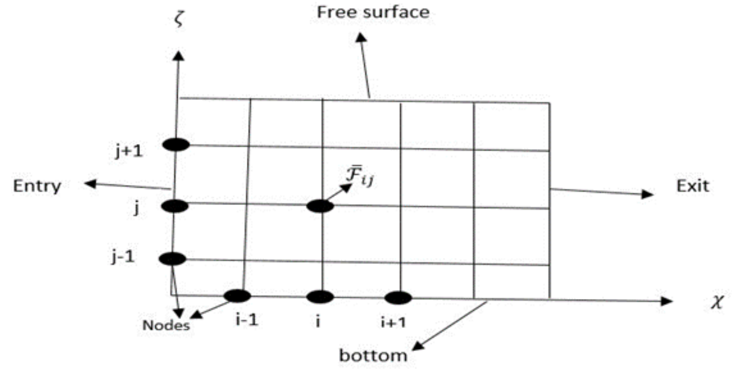


Figure 4: Meshing of the study area

The spatial partial derivatives obtained from the equations of the system will be approximated by classical finite difference schemes [11] all of order 2, namely the scheme decentered downstream or upstream for the parietal conditions and the scheme centered inside of the domain (Laplace condition). The choice of the upstream or downstream off-center diagram will be justified according to the geometry of our problem and will be elucidated in the discretization calculation. The purpose of this choice is to ensure that the movement of the fluid involved is confined within the calculation domain.

To approximate the time derivative, we want to use an implicit two-level scheme of the Euler type with a constant time step δt

NUMERICAL RESOLUTION TECHNIC

To determine the numerical results of the free surface elevation and the potential field inside the domain we will solve the matrix systems respectively from the kinematic condition at the free surface and from the Laplace condition using iterative methods which are based on the repeated application of a simple algorithm leading to eventual convergence after a finite number of repetitions (iterations). We will use in this work the iterative method of relaxation line by line of Gauss-Siedel [12] by using the Successive Over relaxation (S.O.R.) [13]. The principle of iterative methods consists in seeking the solution of the system using a series of successive approximations. By giving an arbitrary vector of components (f_i)^k , we can find (f_i)^(k+1) at the next iteration. The process is stopped when the following convergence criterion is met by considering equation (23)

$$\frac{\sum_{i=1}^{i=i_m}|(f_i)^k-(f_i)^{k+1}|}{\sum_{i=1}^{i=i_m}|(f_i)^{k+1}|} \leq \varepsilon_f \qquad (23)$$

Besides the criterion of convergence of the iterative processes equation (23) it is also necessary to define a criterion for stopping the calculation program. We stop the calculations when, at high times, the variations of our functions between two consecutive times are very small. Under these conditions, we take as stopping criterion by using equations (24) and (25)

$$max\{e_\eta\ ;\ e_\phi\} \leq \ e_t \qquad (24)$$

With$e_f = \frac{\sum_{i=1}^{i=i_m}|(f_i)^{n+1}-(f_i)^n|}{\sum_{i=1}^{i=i_m}|(f_i)^{n+1}|}$ (25)

RESULTATS AND DISCUSSION

In this chapter, results mainly concern the evolution of the free surface and its influence to the distribution of the velocities components over time. For the fixed dimensionless physical parameters of the problem in our modelling, we will in our work consider a numerical waves tank for an amplitude A=0.5 a wavelength λ=0.25 and an average height He=10. The results presented are from simulations performed for a fixed slope β=100 , the error criteria for the convergence of the iterative calculations for the free surface elevation and the velocity potential is set to 1.10^3.After writing the calculation program in FORTRAN language, we will visualize the numerical results using MATLAB software. Our results are validated by considering [3] in the next sub-section.

We present the numerical results obtained from the calculation code that we developed with the FORTRAN 2003 Double Precision software and that simulates the propagation of a non-linear stokes wave in a variable bottom wave tank

VALIDATION

[3] present two different numerical methodologies to generate regular gravity waves in a wave tank. They performed numerical simulations of wave generation through the FLUENT® package, using the Volume of Fluid (VOF) multiphase model to reproduce the wave propagation in the tank. In Figure 6 (a) and (b) respectively, the topology of the velocities in the x direction and y direction are shown. Considering Figure 5, it is observed a good approximation of the characteristics of velocities components along a wave obtained by numerical simulation with the characteristics proposed by the theory, as shown in Figure 7 (a) and (b)

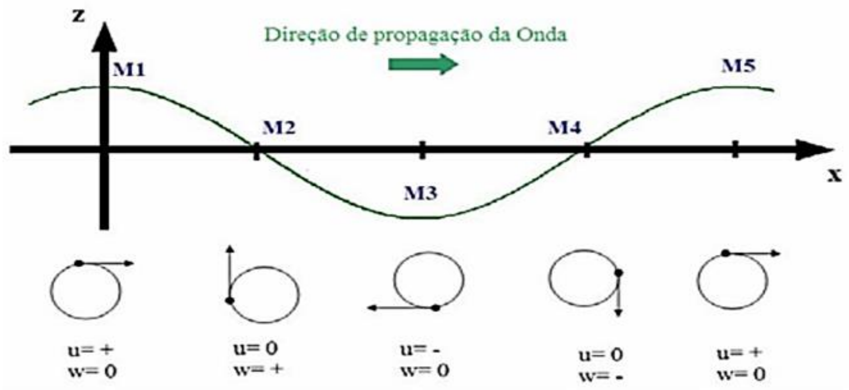


Figure 5: Characteristic of the wave velocity along a wavelength

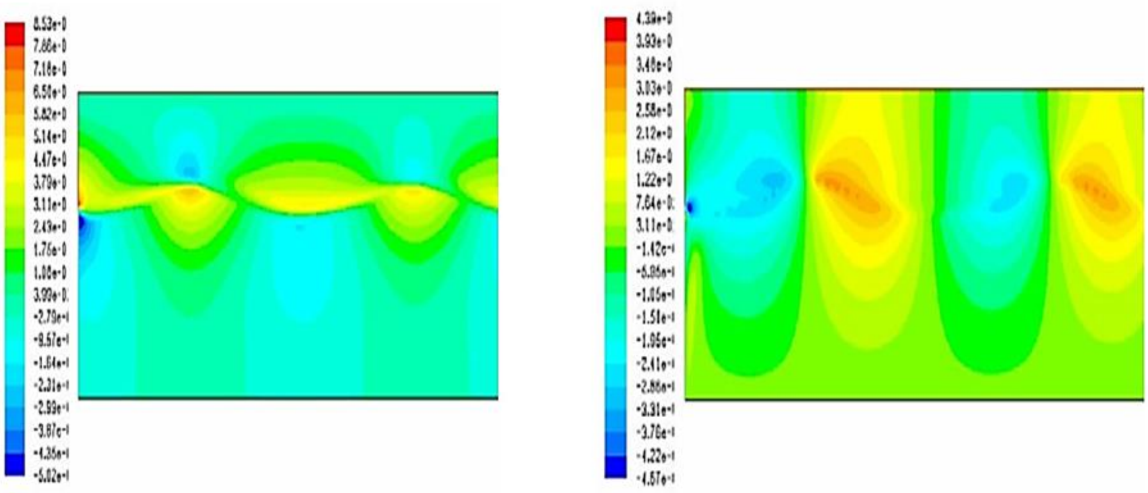


Figure 6: (a) Topology of velocity in x direction; (b) Velocity in the y direction along a wave

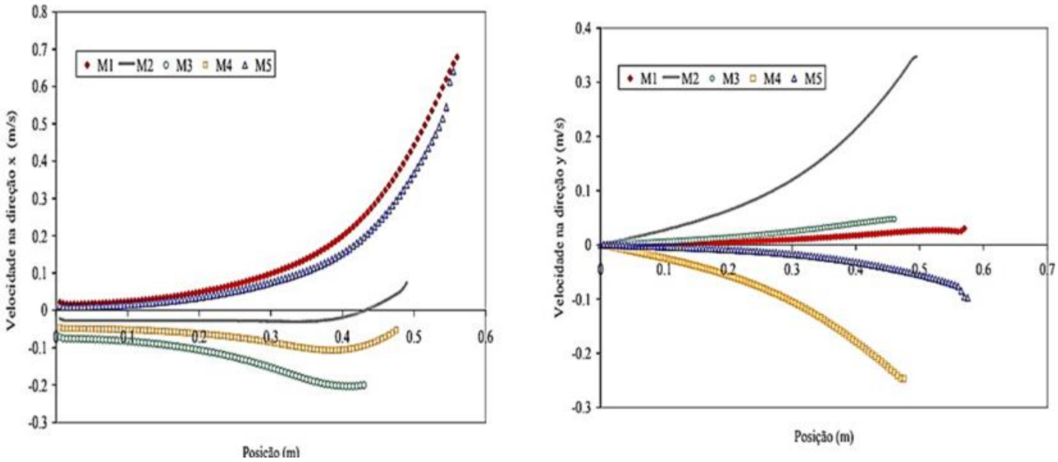
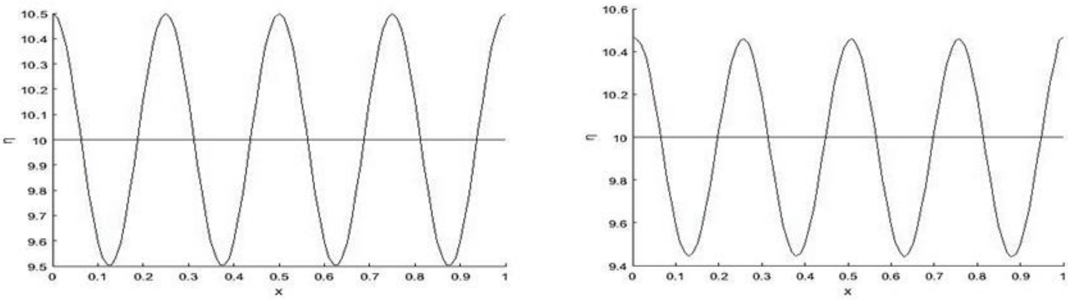


Figure 7: (a) Velocity in x direction along a wave; (b) Velocity in the y direction along a wave

LONGITUDINAL PROFILES OF THE WAVE OVER TIME FOR β=100

Figure 8 shows Influence of the output condition on the wave profile. Indeed, we initially observe a periodic wave with equal peak heights and trough depths. However, over time the profile changes with a decrease in the height of the peaks in favor of an increase in the depth of the troughs [1]

a) b)



c) d)

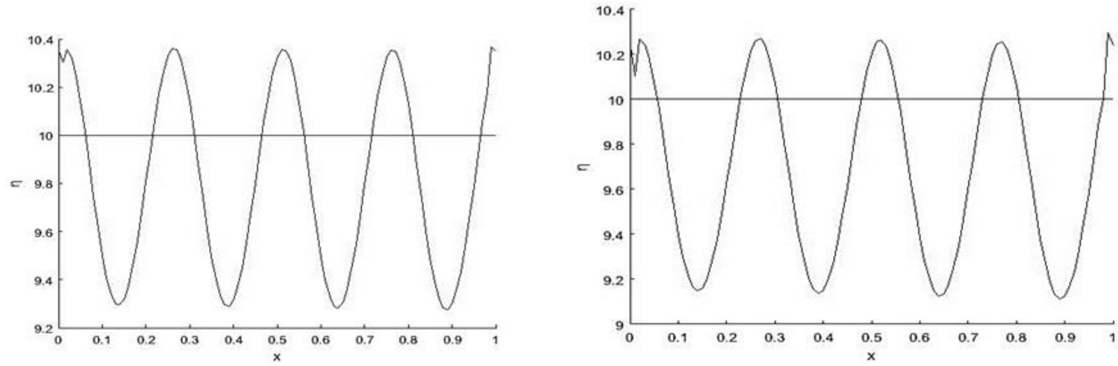
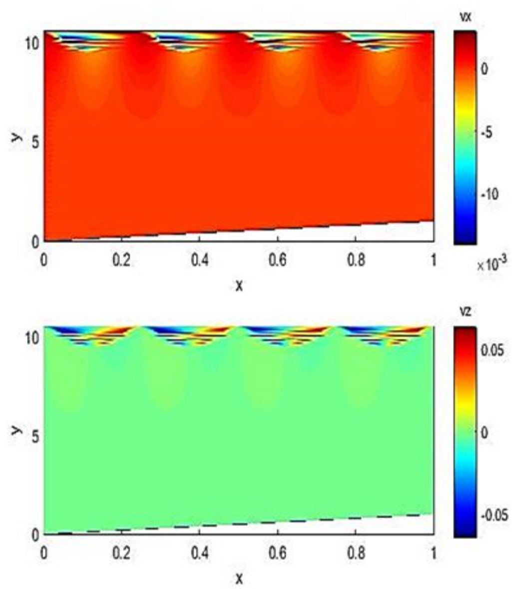


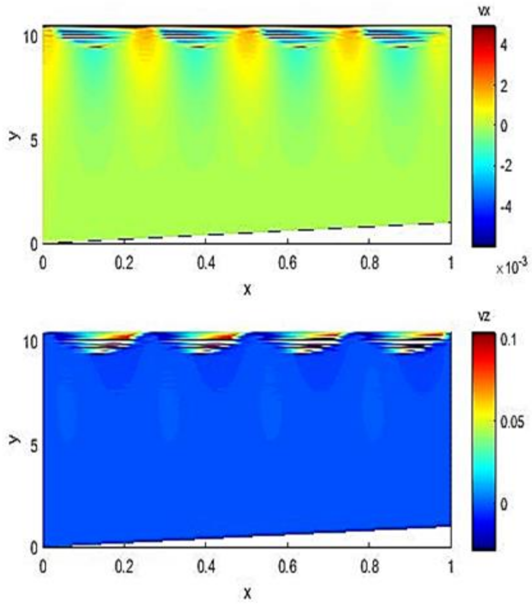
Figure 8: Longitudinal profile of the wave over time: a) t= 2.500000E-03, b) t= 0.36, c) t= 0.8, d) t=1.07

EVOLUTION OF VELOCITIES COMPONENTS OVER TIME FOR β=100

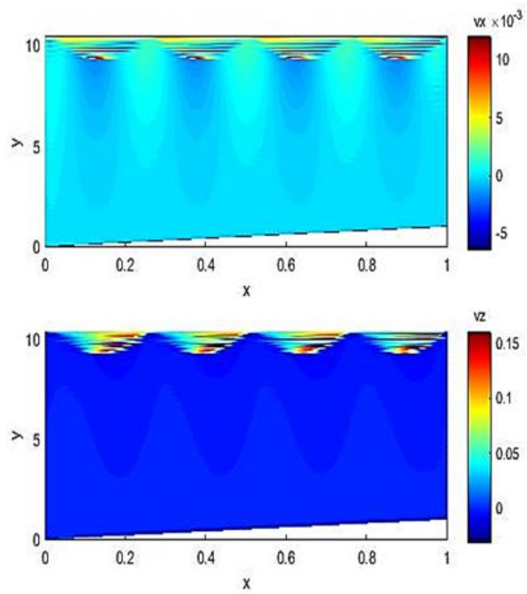
a)



b)



c)



d)

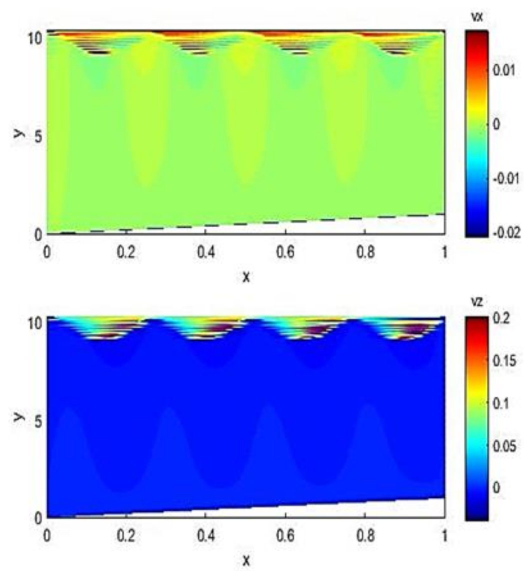


Figure 9: Variation of the horizontal (top) and vertical (bottom) components of speed over time:

a) t= 2.500000E-03, b) t= 0.36, c) t= 0.8, d) t=1.07

Figure 9 shows the evolution of horizontal and vertical velocities under the influence of the wave in the tank. Initially, at t= 2.510-3 and for the horizontal movement it takes place under the crest in the direction of the wave and in the opposite direction under the trough with maximum intensities at the free surface and almost equal under the crests and under the troughs as can also be seen in figure 9. The horizontal movement takes place below the free surface until a certain depth in the channel where it cancels out, which is in line with the study considering [3] See figure 6 (a).

For vertical movement the particle follows the evolution of the wave crest trough downward movement which cancels at the trough, trough crest upward movement which cancels at the crest with equal maximum intensities at the free surface. The vertical movement is only felt on the free surface. We obtain the same topology for vertical movement as in the study by Gomes et al (3), see figure 6 (b) and figure 7 (b).

Over time Horizontal movement take place in a deeper zone as time increases with an identical flow profile for the initial instant with an increase in the intensity of the movement under the trough in favour of a decrease in the intensity under the crest. This is due to the variation of the wave over time (increase in trough depth at the profit of a decrease in crest height) see figure 8. For vertical movement, the intensity of the peak- trough movement is very low compared with that of the trough peak movement; a high intensity is required to bring the particle back to the peak. The intensity of the trough peak movement increases as the depth of the trough increases (see figure 8).

CONCLUSION

A numerical wave tank was studied in this work to observe the effect of the passage of the stokes wave on horizontal and vertical particles' movement For this purpose we observed a simulation using the nonlinear stokes theory to describe the calculation in a hydrodynamic approach using a tank with fixed bottom. The hydrodynamic results are obtained using the finite difference method and the iterative method of relaxation line by line of Gauss-Siedel by using the Successive Over relaxation (S.O.R.).

The numerical simulations show that the particles' movement influenced by the passage of the stokes wave [1] acts differently horizontally and vertically in the tank. For horizontal movement we have particles in the tank acting from the free surface to a depth where we feel a weak intensity of the movement. However, for vertical movement, The particles follow the evolution of the wave and only act on the free surface. These results are in perfect harmony with those in [3].

Finally, we found that the variation of the wave with time due to the exit condition has a considerable influence on the intensity of the horizontal and vertical movement of the water particles. The wave behaviour of the ocean surface induces horizontal and vertical movements of the fluid particles with intensities that vary according to the nature of the wave.

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NOMENCLATURE

Latin letters

A, wave amplitude (m)
e_t, criterion for stopping the temporal iterative process
e_f, relative error of the temporal iterative process
f, free surface elevation or velocity potential
F_r, Froude number
g, gravity intensity (m.s^(-2))
h(x), bottom profile (m)
h_0, slope height at the exit of the tank (m)
H_e, mean height of the tank (m)
J, Jacobian of the transformation (m)
k, wave number (m^(-1))
L, length of the tank (m)
t, time (s)
v_x,v_z, horizontal and vertical coordinates in physical domain (ms^(-1))
x,z, horizontal and vertical coordinates in physical domain (m)

Greek symbols

η, free surface elevation (m)
ϕ, velocity potential (m^2.s^(-1))
ω, wave pulse (rad.s^(-1))

λ, wavelength (m)
β, parameter providing information on the slope
χ^*,ζ^*, dimensionless horizontal and vertical curvilinear coordinates
ε_f, criterion for stopping the itterative process
Superscripts
*, related to dimensionless parameters
n, related to time
k, related to number of iteration

REFERENCES

[1]. A M Ndiaye, F Diop, S Dia and C Mbow, Linear And Non Linear Stokes Waves Theory : Numerical Hydrodynamic and Energy Studies, 2023, 13,61-79.
[2]. R G Dean and R A Dalrymple, Water Wave Mechanics for Engineers and Scientists, illustrée, Prentice-Hall, University of California, 1984
[3]. M N Gomes,C R Olinto,L A O Rocha,J A Souza and L A Isoldi,Computational Modeling of a Regular Wave Tank,Engenharia termica (Thermal Engineering),2009, 44-50
[4]. M A Maatoug,M, M Ayadi,Numerical Simulation of The Second Order Stokes Theory Using Difference Method,Alexandia Engineering Journal, 2016, 55(3), 3005-3013
[5]. M Zabihi, S Mazaheri, A R Mazyak, Wave Generation in a Numerical Wave Tank, International Journal of Coastal and Offshore Engineering, 2017, 33-44
[6]. Du Qingjie, Y C Dennis and Leung, 2D, Numerical simulation of Ocean waves, World Renewable Energy Congress, Marine and Ocean Technology, 2011, Sweden, 8-13
[7]. G Wei, J T Kirby and, A Sinha, Generation of waves in Boussinesq Models using a source function method, Coastal Engineering, 1999, 36(4), 271-299.
[8]. A Chawla and J T Kirby, A source function method for generation of waves on currents in Boussinesq models, Applied Ocean Research, 2000, 22(2), 75-83.
[9]. C M Dong and, C J Huang, Generation and propagation of water waves in a Two-dimensional numerical viscous wave flume, Journal of Waterway, Port, Coastal And Ocean Engineering, 2004, 130(3), 143-153.
[10]. Y j Lu Numerical simulation of two-dimensional overtopping against Seawalls armored with artificial units in regular waves, Journal of Hydrodynamics, 2007, 19(3), 322-329.
[11]. E G Da Silva, Méthodes et Analyse Numériques, Engineering school, Institut Polytechnique, Grenoble, 2007
[12]. J Beuneu, Algorithmes pour le calcul scientifique, Cours de l'Ecole Universitaire d'Ingénieurs, Lille, 1999
[13]. A Bejan, Convection Heat Transfer., John Wiley & Sons, New York, 2013