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Basic resolution of the axisymmetric disk under five different thermoelasticity theories

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ABSTRACT

In this work, the governing equations for the one-dimensional disk thermoelasticity model were formulated, in the context of five different theories of thermoelasticity. The thermoelastic response of the disc under axial thermal shock loading was studied. These governing equations are solved to obtain general solutions using the boundary conditions for stress and heat flow. The Laplace transform method is used to obtain analytical solutions in the transformation field. The inverse of the transformation can be determined numerically for the temperature, thermal stress, and displacement distributions. These solutions are represented graphically and discussed for several cases of the applied heating.

Keywords: Thermal shock; Disk, Laplace transform, Coupled thermoelasticity, Generalized thermoelasticity

INTRODUCTION

Coupled thermoelasticity is a field of study that combines the principles of thermodynamics and elasticity to describe the behavior of materials under the influence of both thermal and mechanical loads. In this coupled phenomenon, the thermal and mechanical effects interact with each other, leading to complex material behavior. Biot [1] developed the classical coupled dynamical theory of thermoelasticity with parabolic field equations. Generalized thermoelasticity is a field that extends the classical theory of thermoelasticity to account for more complex and realistic phenomena. Lord and Shulman [2] formulated the theory of generalized thermoelasticity with one relaxation time. Much research has provided an explanation and application of this theory. Bagri and Eslami [3] investigated a one-dimensional generalized thermoelasticity model of a disk under the Lord–Shulman theory. Yong and other [4] discussed Thermo-mechanical of multi-layered media based on the Lord-Shulman theory. Hadi and other [5] studied dynamical crack propagation under generalized thermal shock based on Lord-Shulman theory. Othman and Mondal [6] investigated memory-dependent derivative effect on two dimensional problem of generalized thermoelastic rotating medium with Lord–Shulman theory. Papers [7-10] provide applications to Lord–Shulman theory. Green and Lindsay theory [11] which is known as the theory of generalized thermoelasticity with two relaxation times. Khader and other [12] used Green and Lindsay theory to discussed Heat transient response in the surfaces of an infinitely long annular cylinder with internal heat source. Ezzat [13] introduced the fundamental solution in thermoelasticity with two relaxation times for cylindrical regions. Othmann [14] investigated the effect of rotation on plane waves in generalized thermo-elasticity with two relaxation times Papers [15-17] provide applications to Green and Lindsay theory. Green and Nagdi [18] developed the theory of thermoelasticity without energy dissipation. These theories use hyperbolic field equations to describe heat as a wave. Abd El-Latief and Khader [19] introduced the Exact Solution of Thermoelastic Problem for a 1 D Bar without Energy Dissipation. Khedr and Khader [20] solved a problem in thermoelasticity with and without energy dissipation. Yadav and other [21] studied the thermo dynamical interactions in a non local initially stressed fiber-reinforced thermoelastic medium with micro temperatures under GN-II theory. Papers [22-25] provide applications to GN-II model. Tzou [26] proposed a universal constitutive equation between the heat flux vector and the temperature gradient to cover the fundamental behaviors of propagation, wave, phonon-electron interactions, and pure phonon scattering. Barak and other [27] investigated the energy analysis at the boundary interface of elastic and piezo thermo elastic half-spaces. Abouelregaland other [28] studied the influence of a non-local Moore–Gibson–Thompson heat transfer model. El-Nabulsi and Anukool [29] discussed fractal non local thermo elasticity of thin elastic nano-beam with apparent negative thermal conductivity. Many researches [30-32] have provided applications to the theory of generalized thermoelasticity.

MATHEMATICAL EQUATIONS

The linear equations of an isotropic and homogeneous one-dimensional disk in the context of five different theories are:

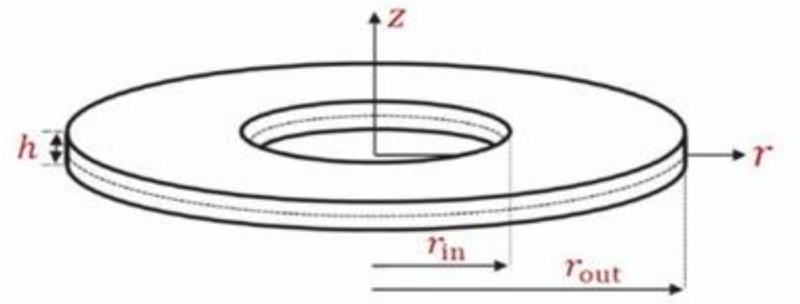

Figure 1: Schematic and geometry of the annular disk

Equation of motion

$$\rho\frac{\partial^2 u_i}{\partial t^2}=(\bar{\lambda}+\mu)u_{j,ij}+\mu u_{i,jj}-\bar{\beta}\left(1+\upsilon_0\frac{\partial}{\partial t}\right)T_{,i} \tag{1}$$

Heat conduction equation

$$K\left(n^*+\tau_1\frac{\partial}{\partial t}\right)T_{,ii}=\rho C_E\left(n^*+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t}+\bar{\beta}T_0\left(n_1+n_0\tau_0\frac{\partial}{\partial t}\right)\frac{\partial u_{i,i}}{\partial t} \tag{2}$$

Stress-displacement relation

$$e_{ij}=\frac{1}{2}\left(u_{i,j}+u_{j,i}\right) \tag{3}$$

Stress-strain relation

$$\sigma_{ij}=\left[\bar{\lambda}e-\bar{\beta}\left(1+\upsilon_0\frac{\partial}{\partial t}\right)T\right]\delta_{ij}+2\mu e_{ij} \tag{4}$$

the plane stress condition

$$\bar{\beta}=\frac{2\mu}{\lambda+2\mu}\beta\ ,\bar{\lambda}=\frac{2\mu}{\lambda+2\mu}\lambda \tag{5}$$

where $\beta=(3\lambda+2\mu)\alpha_t$

Equations (2) and (3), can be expressed by the five different theories as follows:

I - Coupled thermoelastcity

We put $\tau_0=\tau_1=\upsilon_0=0, n_1=n^*=1$, the equations (1, 2, and 4), can be written as

$$\rho\frac{\partial^2 u_i}{\partial t^2}=(\bar{\lambda}+\mu)u_{j,ij}+\mu u_{i,jj}-\bar{\beta}T_{,i}$$

$$KT_{,ii}=\rho C_E\frac{\partial T}{\partial t}+\bar{\beta}T_0\frac{\partial u_{i,i}}{\partial t}$$

$$\sigma_{ij}=\left[\bar{\lambda}e-\bar{\beta}T\right]\delta_{ij}+2\mu e_{ij}$$

II - Lord-Shulman (L-S) theory

We put $\tau_1=\upsilon_0=0, \tau_0>0, n_1=n^*=n_0=1$, the equations (1, 2, and 4), can be written as

$$\rho\frac{\partial^2 u_i}{\partial t^2}=(\bar{\lambda}+\mu)u_{j,ij}+\mu u_{i,jj}-\bar{\beta}T_{,i}$$

$$KT_{,ii}=\rho C_E\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t}+\bar{\beta}T_0\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial u_{i,i}}{\partial t}$$

$$\sigma_{ij}=\left[\bar{\lambda}e-\bar{\beta}T\right]\delta_{ij}+2\mu e_{ij}$$

III - Green-Lindsay (G-L) theory

We put $\tau_1 = 0,\ \upsilon_0 \geq \tau_0 > 0, n_1 = n^* = 1, n_0 = 0$, the equations (1, 2, and 4), can be written as

$$\rho \frac{\partial^2 u_i}{\partial t^2} = (\bar{\lambda} + \mu) u_{j,ij} + \mu u_{i,jj} - \bar{\beta}\left(1 + \upsilon_0 \frac{\partial}{\partial t}\right) T_{,i}$$

$$K T_{,ii} = \rho C_E \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \frac{\partial T}{\partial t} + \bar{\beta} T_0 \frac{\partial u_{i,i}}{\partial t}$$

$$\sigma_{ij} = \left[\bar{\lambda} e - \bar{\beta}\left(1 + \upsilon_0 \frac{\partial}{\partial t}\right) T\right] \delta_{ij} + 2\mu e_{ij}$$

IV - Green-Naghdi type II (G-N-II) theory

We put $\tau_1 = \upsilon_0 = 0,\ \tau_0 = 1, n_1 = 0, n^* > 0, n_0 = 1$, the equations (1, 2, and 4), can be written as

$$\rho \frac{\partial^2 u_i}{\partial t^2} = (\bar{\lambda} + \mu) u_{j,ij} + \mu u_{i,jj} - \bar{\beta} T_{,i}$$

$$n^* K T_{,ii} = \rho C_E \left(n^* + \frac{\partial}{\partial t}\right) \frac{\partial T}{\partial t} + \bar{\beta} T_0 \frac{\partial^2 u_{i,i}}{\partial t^2}$$

$$\sigma_{ij} = \left[\bar{\lambda} e - \bar{\beta} T\right] \delta_{ij} + 2\mu e_{ij}$$

V - Chandrasekharaiah and Tzou (C-T) theory

We put $\tau_1 > 0, \upsilon_0 = 0,\ \tau_0 > 0, n_1 = n^* = n_0 = 1$, the equations (1, 2, and 4), can be written as

$$\rho \frac{\partial^2 u_i}{\partial t^2} = (\bar{\lambda} + \mu) u_{j,ij} + \mu u_{i,jj} - \bar{\beta} T_{,i}$$

$$K\left(1 + \tau_1 \frac{\partial}{\partial t}\right) T_{,ii} = \rho C_E \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \frac{\partial T}{\partial t} + \bar{\beta} T_0 \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \frac{\partial u_{i,i}}{\partial t}$$

$$\sigma_{ij} = \left[\bar{\lambda} e - \bar{\beta} T\right] \delta_{ij} + 2\mu e_{ij}$$

METHOD AND SOLUTION

Consider a hollow disk under axisymmetric thermal shock load applied into its inner or outer radii. The inner radii occupy the region $0 < r \leq a$. The outer radii occupy the region $0 < r \leq b$.

The mechanical and thermal boundary conditions, takes the form

First case

$$\begin{cases} q_r = h(f(t) - T),\ u_r = 0 \ \ at\, r = a \\ \sigma_{rr} = 0 \ ,\ T = 0 \ \ at\, r = b \end{cases} \tag{6a}$$

Second case

$$\begin{cases} T = H(t),\ u_r = 0 \ \ at\, r = a \\ \sigma_{rr} = 0 \ ,\ T = 0 \ \ at\, r = b \end{cases} \tag{6b}$$

The general equations can be written as

$$\mu \nabla^2 u_r + (\bar{\lambda} + \mu)\, grad\, div\, u - \bar{\beta}\left(1 + \upsilon_0 \frac{\partial}{\partial t}\right) grad\, T = \rho \frac{\partial^2 u_r}{\partial t^2} \tag{7}$$

$$K\left(n^* + \tau_1 \frac{\partial}{\partial t}\right) \nabla^2 T = \rho C_E \left(n^* + \tau_0 \frac{\partial}{\partial t}\right) \frac{\partial T}{\partial t} + \bar{\beta} T_0 \left(n_1 + n_0 \tau_0 \frac{\partial}{\partial t}\right) \frac{\partial^2 u_r}{\partial t \partial r} \tag{8}$$

$$\sigma_{rr} = 2\mu \frac{\partial u_r}{\partial r} + \bar{\lambda} e - \bar{\beta}\left(T - T_0 + \upsilon_0 \frac{\partial T}{\partial t}\right) \tag{9}$$

$$\sigma_{\phi\phi} = 2\mu \frac{u_r}{r} + \bar{\lambda} e - \bar{\beta}\left(T - T_0 + \upsilon_0 \frac{\partial T}{\partial t}\right) \tag{10}$$

Now we introduce the non-dimension variables

$$r^* = \frac{r}{\eta}, u^* = \frac{(\bar{\lambda} + 2\mu)}{\eta \bar{\beta} T_0} u, \theta = \frac{T}{T_0}, \sigma_{ij}^* = \frac{\sigma_{ij}}{\bar{\beta} T_0}, \{t, \tau_1, \tau_0, \upsilon_0\}^* = \frac{C_0}{\eta}\{t, \tau_1, \tau_0, \upsilon_0\}, q^* = \frac{q\eta}{K T_0}$$

$$C_0^2 = \frac{(\bar{\lambda} + 2\mu)}{\rho}, \eta = \frac{K}{\rho C_0 C_E}$$

The equations (7) – (10), in non-dimensional form become

$$\nabla^2 u_r + (w^2 - 1)\, grad\, div\, u_r - w^2 \left(1 + \upsilon_0 \frac{\partial}{\partial t}\right) grad\, T = w^2 \frac{\partial^2 u_r}{\partial t^2} \tag{11}$$

$$\left(n^* + \tau_1 \frac{\partial}{\partial t}\right)\nabla^2 T = \left(n^* + \tau_0 \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t} + \varepsilon \left(n_1 + n_0 \tau_0 \frac{\partial}{\partial t}\right)\frac{\partial^2 u_r}{\partial t \partial r} \tag{12}$$

$$\sigma_{rr} = \frac{2}{w^2}\frac{\partial u_r}{\partial r} + \left(1 - \frac{2}{w^2}\right) e - \left(1 + \upsilon_0 \frac{\partial}{\partial t}\right)\theta \tag{13}$$

$$\sigma_{\phi\phi} = \frac{2}{w^2}\frac{u_r}{r} + \left(1 - \frac{2}{w^2}\right) e - \left(1 + \upsilon_0 \frac{\partial}{\partial t}\right)\theta \tag{14}$$

where $w^2 = \dfrac{\overline{\lambda} + 2\mu}{\mu}$, $\varepsilon = \dfrac{\eta \overline{\beta}^2 T_0 C_0}{k(\overline{\lambda} + 2\mu)}$

We have use the potential function $u_r = \dfrac{\partial \psi}{\partial r}, e = \nabla^2 \psi$, equations (11)-(14), takes the form

$$\left[\nabla^2 - \frac{\partial}{\partial t^2}\right]\psi = \left(1 + \upsilon_0 \frac{\partial}{\partial t}\right)\theta \tag{15}$$

$$\left[\left(n^* + \tau_1 \frac{\partial}{\partial t}\right)\nabla^2 - \left(n^* + \tau_0 \frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\right]\theta = \varepsilon \left(n_1 + n_0 \tau_0 \frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\nabla^2 \psi \tag{16}$$

$$\sigma_{rr} = \frac{2}{w^2}\frac{\partial^2 \psi}{\partial r^2} + \left(1 - \frac{2}{w^2}\right)\nabla^2 \psi - \left(1 + \upsilon_0 \frac{\partial}{\partial t}\right)\theta \tag{17}$$

$$\sigma_{\phi\phi} = \frac{2}{w^2}\frac{1}{r}\frac{\partial \psi}{\partial r} + \left(1 - \frac{2}{w^2}\right)\nabla^2 \psi - \left(1 + \upsilon_0 \frac{\partial}{\partial t}\right)\theta \tag{18}$$

SOLUTION PROBLEM IN THE LAPLACE TRANSFORM DOMAIN LAPLCE TRANSFORM

Applying the Laplace transform with parameter *s*, to both sides of equations (15)-(18), we obtain

$$\left[\nabla^2 - s^2\right]\overline{\psi} = (1 + \upsilon_0 s)\overline{\theta} \tag{19}$$

$$\left[(n^* + \tau_1 s)\nabla^2 - s(n^* + \tau_0 s)\right]\overline{\theta} = \varepsilon\, s(n_1 + n_0 \tau_0 s)\nabla^2 \overline{\psi} \tag{20}$$

$$\overline{\sigma}_{rr} = \frac{2}{w^2}\frac{\partial^2 \overline{\psi}}{\partial r^2} + \left(1 - \frac{2}{w^2}\right)\nabla^2 \overline{\psi} - \left(1 + \upsilon_0 \frac{\partial}{\partial t}\right)\overline{\theta} \tag{21}$$

$$\overline{\sigma}_{\phi\phi} = \frac{2}{w^2}\frac{1}{r}\frac{\partial \overline{\psi}}{\partial r} + \left(1 - \frac{2}{w^2}\right)\nabla^2 \overline{\psi} - \left(1 + \upsilon_0 \frac{\partial}{\partial t}\right)\overline{\theta} \tag{22}$$

Eliminating $\overline{\psi}$ from Eq. (19) and (20), we get

$$\left\{(n^* + \tau_1 s)\nabla^4 - \nabla^2\left[s^2(n^* + \tau_1 s) + s(n^* + \tau_0 s) + \varepsilon\, s(1 + \upsilon_0 s)(n_1 + n_0 \tau_0 s)\right] + s^3(n^* + \tau_0 s)\right\}\overline{\theta} = 0 \tag{23}$$

As the same eliminating $\overline{\theta}$ from Eq. (19) and (20), we get

$$\left\{(n^* + \tau_1 s)\nabla^4 - \nabla^2\left[s^2(n^* + \tau_1 s) + s(n^* + \tau_0 s) + \varepsilon\, s(1 + \upsilon_0 s)(n_1 + n_0 \tau_0 s)\right] + s^3(n^* + \tau_0 s)\right\}\overline{\psi} = 0 \tag{24}$$

The solution of Eq. (23) and (24)

$$\overline{\theta} = \sum_{i=1}^{2}\left[A_i \left(k_i^2 - s^2\right) I_0(k_i r) + B_i \left(k_i^2 - s^2\right) K_0(k_i r)\right] \tag{25}$$

$$\overline{\psi} = (1 + \upsilon_0 s)\sum_{i=1}^{2}\left[A_i I_0(k_i r) + B_i K_0(k_i r)\right] \tag{26}$$

where k_1^2 and k_2^2 are the roots of the characteristic equation

$$(n^* + \tau_1 s)\nabla^4 - \nabla^2\left[s^2(n^* + \tau_1 s) + s(n^* + \tau_0 s) + \varepsilon\, s(1 + \upsilon_0 s)(n_1 + n_0 \tau_0 s)\right] + s^3(n^* + \tau_0 s) = 0$$

From Eq. (26), we obtain

$$\overline{u_r} = (1 + \upsilon_0 s)\sum_{i=1}^{2}\left[A_i\, k_i I_1(k_i r) - B_i\, k_i K_1(k_i r)\right] \tag{27}$$

Substituting equations (25) and (26) into equations (21) and (22), we get

$$\overline{\sigma}_{rr} = (1 + \upsilon_0 s)\sum_{i=1}^{2}\left\{A_i\left[s^2 I_0(k_i r) - \frac{2k_i}{w^2 r} I_1(k_i r)\right] + B_i\left[s^2 K_0(k_i r) + \frac{2k_i}{w^2 r} K_1(k_i r)\right]\right\} \tag{28}$$

$$\bar{\sigma}_{\phi\phi} = (1+\upsilon_0 s)\sum_{i=1}^{2}\left\{A_i\left[\left(s^2 - \frac{2}{w^2}k_i^2\right)I_0(k_i r) + \frac{2k_i}{w^2 r}I_1(k_i r)\right] + B_i\left[\left(s^2 - \frac{2}{w^2}k_i^2\right)K_0(k_i r) - \frac{2k_i}{w^2 r}K_1(k_i r)\right]\right\} \tag{29}$$

Applying the Laplace transform, to both sides of equations (6)

$$\begin{cases} \bar{q}_r = h(\bar{f}(s) - \bar{\theta}) \ , \bar{u}_r = 0 \ \ at\, r = a \\ \bar{\sigma}_{rr} = 0 \ \ , \ \bar{\theta} = 0 \ \ at\, r = b \end{cases} \tag{30}$$

$$\begin{cases} \bar{\theta} = \dfrac{\theta_0}{s} \ , \bar{u}_r = 0 \ \ at\, r = a \\ \bar{\sigma}_{rr} = 0 \ \ , \ \bar{\theta} = 0 \ \ at\, r = b \end{cases} \tag{31}$$

Substituting equations (25), (27), and (28) into equations (30, and 31), we obtain the linear system of equations with unknown four constant

Case 1

$$\sum_{i=1}^{2}\left[A_i\, k_i\left(k_i^2 - s^2\right)I_1(k_i a) - B_i k_i\left(k_i^2 - s^2\right)K_1(k_i a)\right] - h\sum_{i=1}^{2}\left[A_i\left(k_i^2 - s^2\right)I_0(k_i a) + B_i\left(k_i^2 - s^2\right)K_0(k_i a)\right] = h\,\bar{f}(s) \tag{32}$$

$$\sum_{i=1}^{2}\left[A_i\, k_i I_1(k_i a) - B_i\, k_i K_1(k_i a)\right] = 0 \tag{33}$$

$$\sum_{i=1}^{2}\left\{A_i\left[s^2 I_0(k_i b) - \frac{2k_i}{w^2 b}I_1(k_i b)\right] + B_i\left[s^2 K_0(k_i b) + \frac{2k_i}{w^2 b}K_1(k_i b)\right]\right\} = 0 \tag{34}$$

$$\sum_{i=1}^{2}\left[A_i\left(k_i^2 - s^2\right)I_0(k_i b) + B_i\left(k_i^2 - s^2\right)K_0(k_i b)\right] = 0 \tag{35}$$

Case 2

$$\sum_{i=1}^{2}\left[A_i\left(k_i^2 - s^2\right)I_0(k_i a) + B_i\left(k_i^2 - s^2\right)K_0(k_i a)\right] = \frac{\theta_0}{s} \tag{36}$$

$$\sum_{i=1}^{2}\left[A_i\, k_i I_1(k_i a) - B_i\, k_i K_1(k_i a)\right] = 0 \tag{37}$$

$$\sum_{i=1}^{2}\left\{A_i\left[s^2 I_0(k_i b) - \frac{2k_i}{w^2 b}I_1(k_i b)\right] + B_i\left[s^2 K_0(k_i b) + \frac{2k_i}{w^2 b}K_1(k_i b)\right]\right\} = 0 \tag{38}$$

$$\sum_{i=1}^{2}\left[A_i\left(k_i^2 - s^2\right)I_0(k_i b) + B_i\left(k_i^2 - s^2\right)K_0(k_i b)\right] = 0 \tag{39}$$

Solve this system of linear equations to find $A_1, A_2, B_1, and\ B_2$

NOMENCLATURE

n^* parameter in Green-Nagdhi theory
k thermal conductivity
n_1, n_0 parameters
$H(t)$ Heaviside unit step function
μ_0 magnetic permeability
K_0 ,I_0 modified Bessel function
τ_0, υ_0 the relaxation times
α_t coefficient of thermal expansion
e_{ij} components of the linear strain tensor
c_E specific heat at constant strain
u displacement vector
λ, μ Lamé's modulii
T absolute temperature
P density
T_0 reference temperature

INVERSION THE LAPLACE TRANSFORMS

To invert the Laplace transforms, we used the method as in [12] and [33]

NUMERICAL RESULTS AND DISCUSSION

For the purposes of numerical evaluations, the material of the disk is assumed to be aluminum [31] $T_0 = 239\,\text{K}$, $\rho = 1707\ \text{kg/m}^3$, $c_E = 896\ \text{J/(kg K)}$, $\alpha_t = 8.418\times10^{-5}\,\text{K}^{-1}$,

$k = 204\ \text{W/(m K)}$, $\lambda = 4.7\times10^{10}\ \text{kg/(ms}^2)$, $\mu = 2.68\times10^{10}\ \text{kg/(ms}^2)$

We have taken the inner radii $a = 1$ and the outer radii $b = 2$, $f(t) = 32 + 12\,Cos(13t)$, and the Heaviside unit step function that is $H(t) = 1$ for $t > 0$ and $H(t) = 0$ for $t \le 0$.

The figure (2-4), represents the temperature, displacement, and stresses distribution for the first case.

The figure (5-7), represents the temperature, displacement, and stresses distribution for the second case.

In figure (2), the non-dimension temperature is start with the maximum value and decrease. The wave front of thermal wave is $r = 1.06\,(C.T), r = 1.65\,(L.S), r = 1.65\,(G.L), r = 1.46\,(G.N), r = 1.16\,(Tzou)$.

In figure (3), the non-dimension displacement is start from zero and increasing until to reach the maximum value, then it decreasing. In figure (4), the non-dimension stresses in the beginning namely constant and then decreasing rapidly. From the figure (2, 3, 4), we show that the beaver of the temperature, displacement, and stresses are the same for (C.T and Tzou), and same for (L.s, G.l, and G.N)

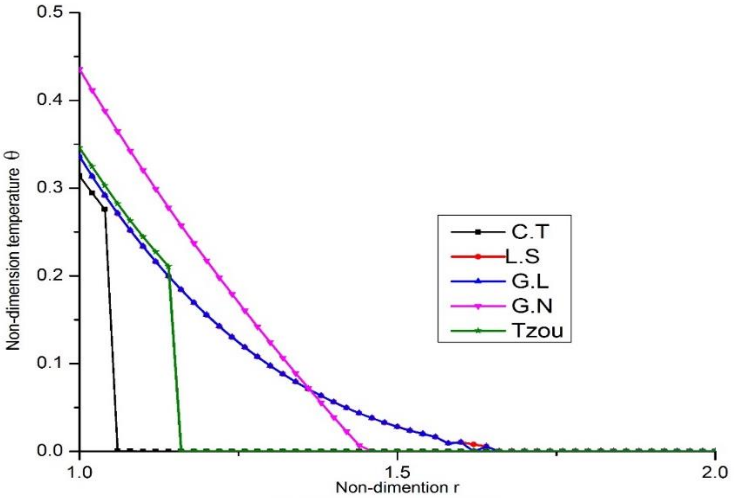

Fig. 2 Temperature Distribution
case 1 for t = 0.1

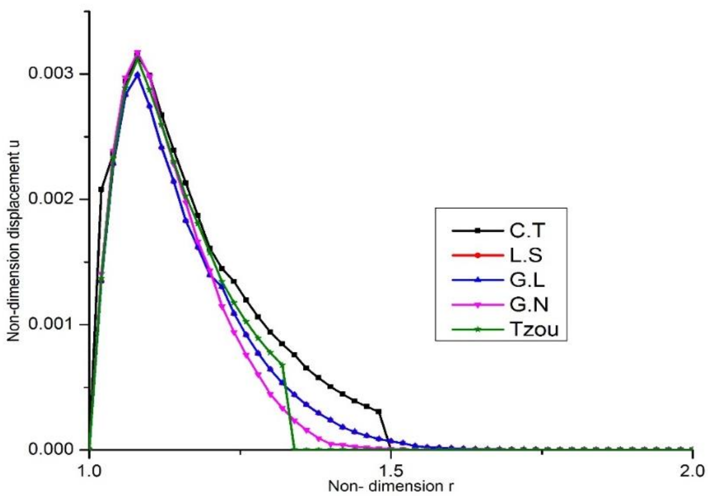

Fig. 3 Displacement Distrubution
Case 1 for t = 0.1

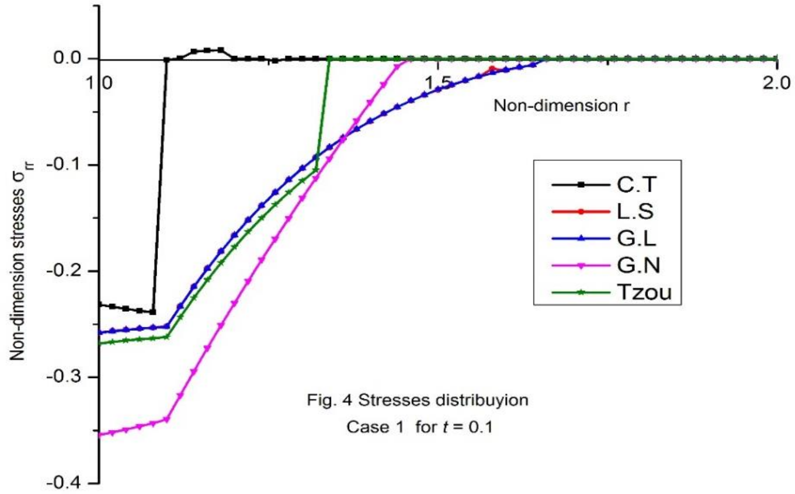

Fig. 4 Stresses distribuyion
Case 1 for t = 0.1

In figure (5), the non-dimension temperature is start with value one, then decreasing, the thermal wave from are ($r = 1.02\,(C.T), r = 1.64\,(L.S), r = 1.64\,(G.L), r = 1.76\,(G.N), r = 1.32\,(Tzou)$. in figure (6), the non-dimension displacement is started from maximum value, ten decreasing sharply until to reach the certain value to change the direction. In figure (7), the non-dimension stresses is start from zero and increasing until to reach the cretin value, then change the direction (discontinuity)

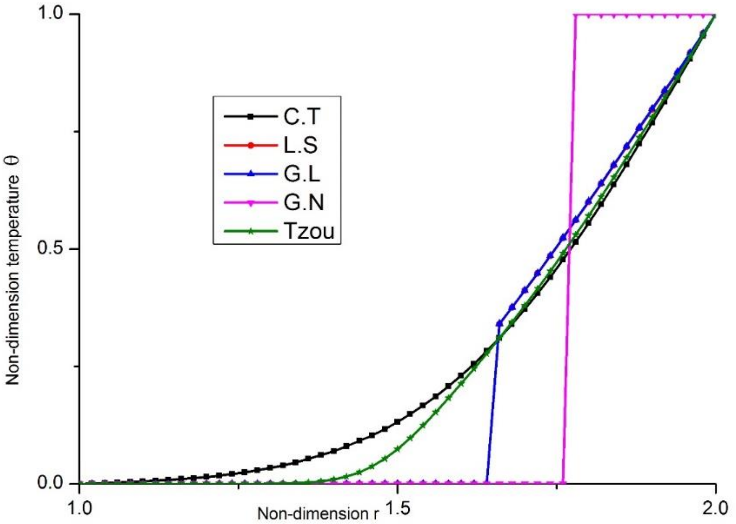

Fig. 5 Temperature Distribution
case 2 for t = 0.1

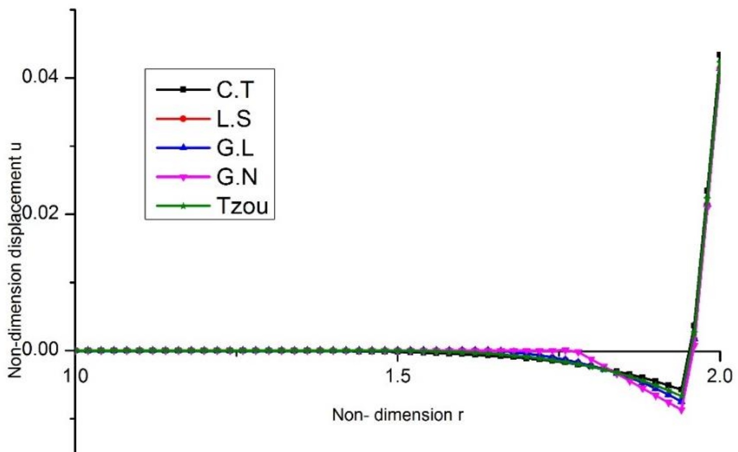

Fig. 6 Displacement Distrubution
Case 2 for t = 0.1

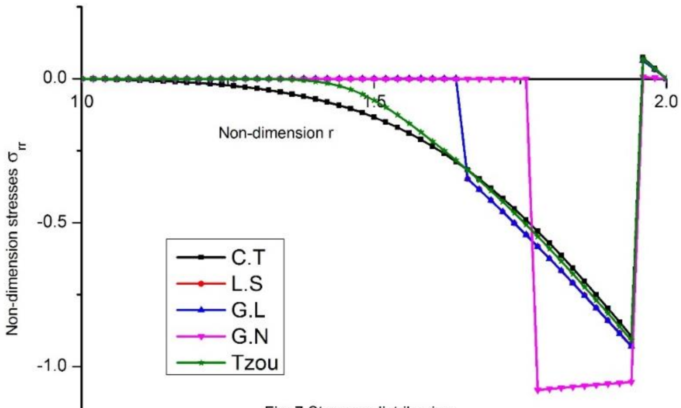

Fig. 7 Stresses distribuyion
Case 2 for t = 0.1

CONCLUSION

The five different theories of generalized thermoelasticity of a hollow is studied in this paper for two different cases. Distributions of temperature, displacement, and radial stresses at certain time and along the radius of the disk are obtained and shown in the figures. In the case of L.S, G.L, and G.N II theory, the solution to all functions is limited to a finite region of space and does not extend to infinity. But in the case of C.T and Tzou theory, where the solution extends to infinity instantaneously, which indicates an infinite speed of wave propagation. The difference between the predictions of the theories of LS, G.L and GN is most apparent in the graphs of the temperature distribution (5). In the LS and G.L theories the temperature decreases monotonically signifying continuous dissipation of heat energy. This is not the case for GN theory.

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