



Ion-acoustic compressive soliton in dusty plasmas having superthermal electrons

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ABSTRACT

Study the characteristics of the ion-acoustic soliton in unmagnetized dusty plasma having ions and superthermal electrons. The Korteweg de Vries (KdV) equation is derived by employing the reductive perturbation method (RPM). The effect of superthermal electron and charge dust grain on the characteristics of the soliton are discussed in details. It is found that increase the superthermal parameter (spectral index), the amplitude (width) of the soliton decreases (increases). Phase velocity of soliton increases with increase superthermal parameter however decreases with increase in charge dust grain. The result of this investigation may be useful to solve the nonlinear structure in space dusty plasma where ions and superthermal electrons are present.

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INTRODUCTION

The behavior of superthermal electrons observed in space environment deviates from Maxwellian distribution and found to obey kappa distribution was introduced by Vasyliunas [1]. Hellberg et al. [2] examined that the super-thermal/ k -distribution function should be greater than $(3/2)$ for superthermal electrons and reduces to maxwellian distribution as the value of $k \rightarrow \infty$. Boubakour et al. [3] studied the ion-acoustic solitary waves in an unmagnetized plasma and found that the effect of k -distribution and positrons on amplitude of the solitary wave. El-Tantawy et al. [4] examined the ion-acoustic wave in unmagnetized dusty plasmas with k -distribution and found that the effect of k -distribution on the characteristics (amplitude and width) of nonlinear structures. El-shamy [5] examined the obliquely propagation of IASWs in magnetized plasma and he found that amplitude and width increases with increase in the spectral index k . Saini et al. [6] derived the ZK equation for ion-acoustic solitary waves and found that the effect of spectral index k on amplitude of the solitary waves in a magnetized plasma. Bains et al. [7] examined the ion acoustic waves in magnetized plasma consisting of spectral index k and found that the spectral index k play important role in the properties of solitary waves. Singhadiya et al. [8] derived the KdV equation for ion-acoustic solitary waves in unmagnetised plasmas and found that the effect of spectral index k on amplitude and width of the ion-acoustic solitary waves.

Rao et al. [9] studied the dust acoustic waves in unmagnetized dusty plasmas and reported that non-linear characteristics of the waves. Shukla and Silin [10] examined dust ion-acoustic wave in unmagnetized collisionless dusty plasma and found that the existence of a new low-frequency electrostatic wave in plasma model. Bharutram and Shukla [11] studied the compressive and rarefactive ion-acoustic solitons in unmagnetized plasma and investigated that the effect of negatively charged dust grains on amplitude of the soliton. Barkan et al. [12] observed the dust -acoustic wave in plasmas. Verheest et al. [13] studied the dust-acoustic solitons in unmagnetized plasmas with Boltzmann distributions and found that negatively charged dust on amplitude of the soliton. Tiwari and Mishra [14] examined ion- acoustic compressive and rarefactive solitons in dusty plasma and found that the effect of positive and negative charged dust on characteristics of the soliton. Tiwari et al. [15] examined the ion-acoustic compressive and rarefactive soliton in unmagnetized plasma with

consisting of massive positively charged dust grains and investigated that amplitude of the soliton increases with decrease of positively charged dust grains. Jain et al. [16] studied the ion-acoustic wave in unmagnetized plasma with Maxwellian distribution and found that the effects of dust concentration on the amplitude the ion-acoustic wave. Sikta et al. [17] examined the dust-acoustic waves in unmagnetized dusty plasma and found that the amplitude of dust-acoustic waves increases with increase negative dust mass.

The layout of this paper is study of the ion-acoustic soliton, in dusty plasma with superthermal electron. The Korteweg de Vries (KdV) equation is derived by employing the reductive perturbation method (RPM). The effect of superthermal electron and charge dust grain on the characteristics of the soliton are discussed in details. In Section 2, the basic governing equations are presented. Section 3 derive the KdV equation and soliton solution of the KdV equation in Section 4. Result and discussion in Section 5 and conclusion in last section.

BASIC EQUATIONS

We consider a dusty plasma consisting of ions and superthermal electrons. The nonlinear dynamics behavior of ion acoustic waves is governed by the following normalized usual equations:

$$\partial_t(N_i) + \partial_x(N_i V_i) = 0 \quad (1)$$

$$\partial_t(V_i) + (V_i)\partial_x(V_i) = -\partial_x\phi$$

$$(2) \quad \partial_t(N_D) + \partial_x(N_D V_D) = 0 \quad (3)$$

$$\partial_t(V_D) + (V_D)\partial_x(V_D) = -\mu\partial_x\phi \quad (4)$$

$$\partial_x^2\phi = [1 + n_1\phi + n_2\phi^2 + n_3\phi^3 + \dots] - \gamma N_D - (1 - \gamma)N_i \quad (5)$$

Where N_D , V_D and N_i , V_i are normalized density and fluid velocity of the plasma dust grains and ions respectively. ϕ , $\mu = Z_D / M_D$, $M_D = m_D / m_i$, $\alpha = N_{D0} / N_{i0}$, and $\lambda_D = (\epsilon_0 T_e / N_0 e^2)^{1/2}$ are the electric potential, ratio of specific charges on dust grains and ions and assuming plasma ions to be singly charged, mass ratio of dust particles and ions, the equilibrium density ratio of dust particles with plasma ions and Debye length speed respectively.

$$\text{Here } \gamma = Z_D \alpha, \quad n_1 = (2k - 1)(2k - 3)^{-1}, \quad n_2 = 2(2k + 1)(2k - 1)(2k - 3)^{-2}, \quad \text{and} \\ n_3 = (2k + 1)(2k - 1)(2k + 3)(2k - 3)^{-3} / 6.$$

Derive the KdV equation with the help of standard RPM. The independent variables are stretched (ξ) and (τ) as: $\xi = \epsilon^{1/2}(x - St)$, $\tau = \epsilon^{3/2}t$ where ϵ is a small parameter and S is the phase velocity of the wave.

The dependent variables are

$$N_{i,D} = 1 + \epsilon N_{i,D}^{(1)} + \epsilon^2 N_{i,D}^{(2)} + \dots$$

$$V_{i,D} = \epsilon V_{i,D}^{(1)} + \epsilon^2 V_{i,D}^{(2)} + \dots$$

$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \dots \quad (6)$$

ABC

DERIVATION OF THE KdV EQUATION

Substituting stretched coordinates and Eq. (6) into (1) - (5) and we obtain the first order quantities as

$$N_D^{(1)} = \frac{\mu}{S^2} \phi^{(1)}, \quad N_i^{(1)} = \frac{1}{S^2} \phi^{(1)}, \quad V_D^{(1)} = \frac{\mu}{S} \phi^{(1)},$$

$$V_i^{(1)} = \frac{1}{S} \phi^{(1)} \quad (7)$$

Solving the above equation, we obtaining the phase velocity (S)

$$S^2 = \frac{\gamma\mu + (1 - \gamma)}{n_1} \quad (8)$$

and obtain the following KdV equation with using first and second order equation

$$\partial_{\tau}\phi + PQ\phi\partial_{\xi}\phi + \frac{1}{2}P\partial_{\xi}^3\phi = 0 \quad (9)$$

Where

$$P = \left[\frac{\mu\gamma + (1-\gamma)}{S^3} \right]^{-1}$$

$$Q = \frac{\{3\mu^2\gamma + 3(1-\gamma) - 2n_2\beta^2 S^4\}}{2S^4} \quad (10)$$

We use $\phi = \phi^{(1)}$, in Eq. (9).

SOLITON WAVE SOLUTIONS OF THE KdV EQUATION

To determine the soliton wave solutions of the KdV equation (9), we consider

$$\eta = \xi - u_1\tau \quad (11)$$

Where u_1 is a constant velocity.

Integrating of Eq. (9) twice w.r.t. η , we obtain

$$\frac{1}{2}(d_{\eta}\phi)^2 + \psi(\phi) = 0 \quad (12)$$

where $\psi(\phi)$ is the Sagdeev potential

$$\psi(\phi) = \frac{2}{P} \left[\frac{1}{6} PQ\phi^3 - \frac{1}{2} u_1\phi^2 \right] \quad (13)$$

In the Eqn. (12) we have used the following boundary conditions as $\eta \rightarrow \pm\infty$, ϕ , $d\phi/d\eta$, and $d^2\phi/d\eta^2 \rightarrow 0$. However, the following boundary conditions on the Sagdeev potential should be satisfied

$$\psi(\phi) = 0 \text{ at } \phi = 0 \text{ and } \phi = \phi_m, \quad \psi'(\phi) = 0 \text{ at}$$

$$\phi = 0,$$

$$\psi'(\phi) > 0 \text{ at } \phi = \phi_m \text{ for compressive soliton,}$$

$$\psi'(\phi) < 0 \text{ at } \phi = \phi_m \text{ for rarefactive soliton.} \quad (14)$$

The soliton wave solution of Eqn. (13) is given by

$$\phi = \phi_m \sec h^2(\eta/W), \quad (15)$$

where the amplitude $\phi_m = 3u_1/PQ$ and width $W = (2P/u_1)^{1/2}$.

RESULT AND DISCUSSION

To investigate the existence regions and nature of the ion - acoustic soliton in dusty plasma, we have done numerical calculations for different set of plasma parameters (k and Z_d).

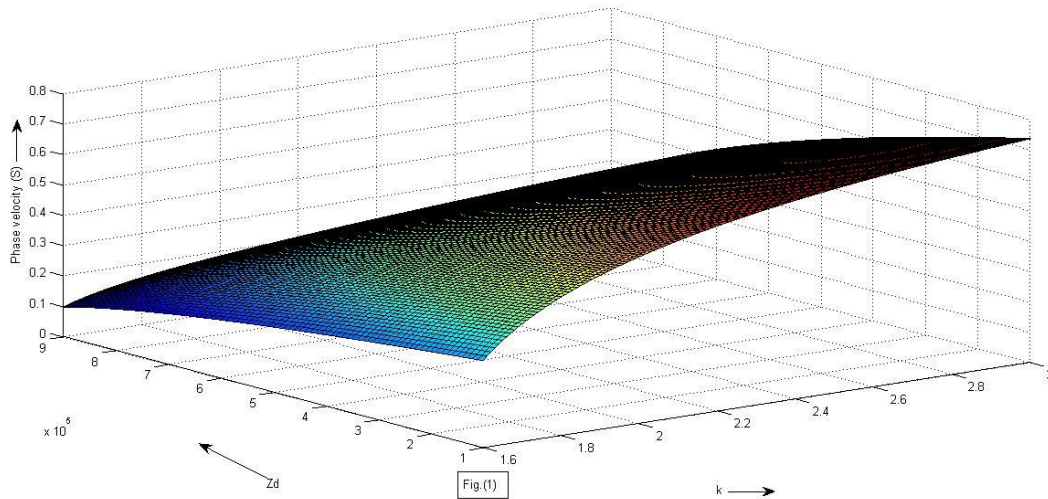


Fig. 1 Variation of S with respect to potential k and Z_D at $\alpha = 1 \times 10^{-6}$, $M_d = 1 \times 10^{16}$.

From 3D curve of figure (1), it may be noted for a given set of parameter, phase velocity S (equation (8)) of the ion acoustic soliton increases with increase in spectral index k but it increases with increase in positive charge dust grain.

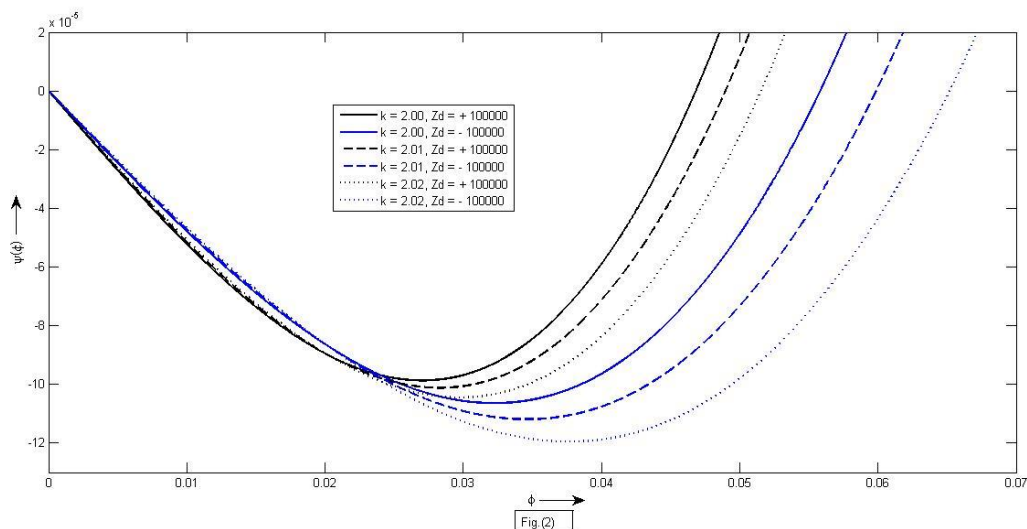


Figure 2. Variation of $\psi(\phi)$ with respect to potential ϕ for different value of $k = 2.00$ (solid line), 2.01 (dashed line) and 2.02 (dotted line), $Z_D = + 1 \times 10^5$ (black color), $Z_D = - 1 \times 10^5$ (blue color), $\alpha = 1 \times 10^{-6}$, $u_1 = 0.001$, $M_d = 1 \times 10^{16}$.

In figure (2), the variation of the $\psi(\phi)$ (equation (13)) with potential (ϕ) for compressive soliton at different value of spectral index k for positive (black color) and negative (blue color) charge dust grain, with fixed values of other parameters. It is seen that as spectral index increases, the amplitude of the soliton increases.

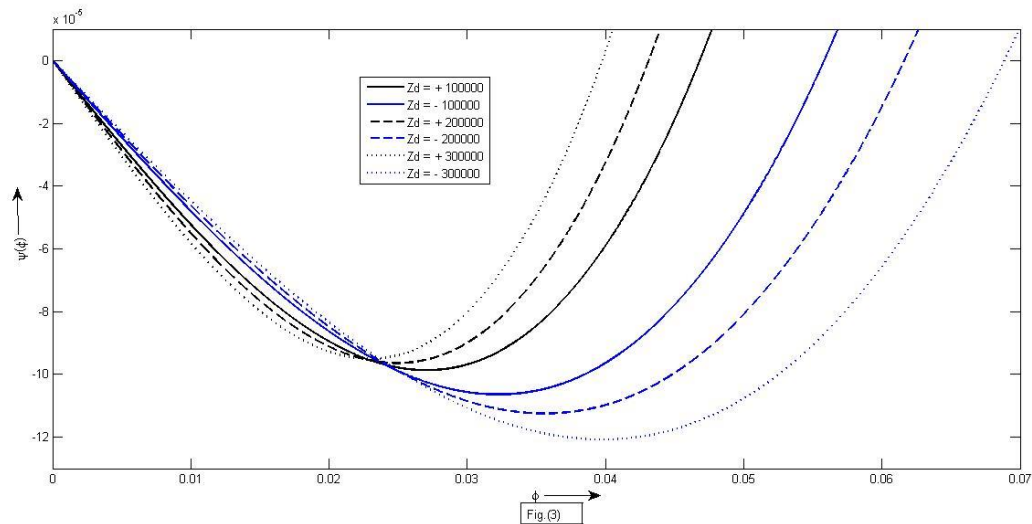


Figure 3. Variation of $\psi(\phi)$ with respect to potential ϕ for different value of $Z_D = + 1 \times 10^5$ (black solid line color), $- 1 \times 10^5$ (blue solid line color), $+ 2 \times 10^5$ (black dashed line color), $- 2 \times 10^5$ (blue dashed line color), $+ 3 \times 10^5$ (black dotted line color), $- 3 \times 10^5$ (blue dotted line color), $k = 2.00$, $\alpha = 1 \times 10^{-6}$, $k = 2.11$, $u_1 = 0.001$, $M_d = 1 \times 10^{16}$.

Figure (3), shows the variation of $\psi(\phi)$ with respect to potential ϕ for compressive soliton at different value of positive and negative charge dust grain, with fixed values of other parameters. It is seen that as charge dust grain increases, the amplitude of the soliton increases.

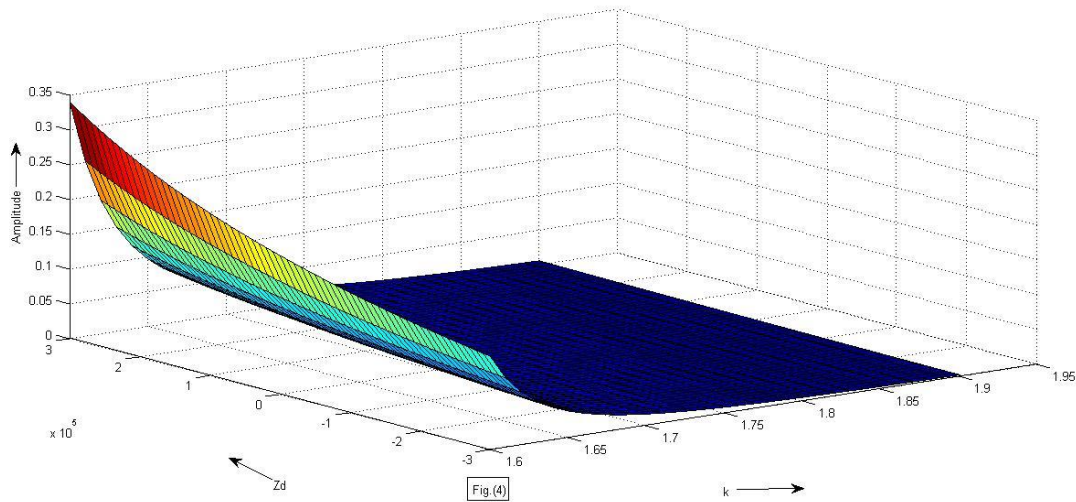


Fig. 4 Variation of Amplitude with respect to potential k and Z_D at, $u_1 = 0.00001$, $M_d = 1 \times 10^{16}$.

Figure (4), depicts the 3D view of the graph, variations of amplitude of the solitons with spectral index k and charge dust grain.

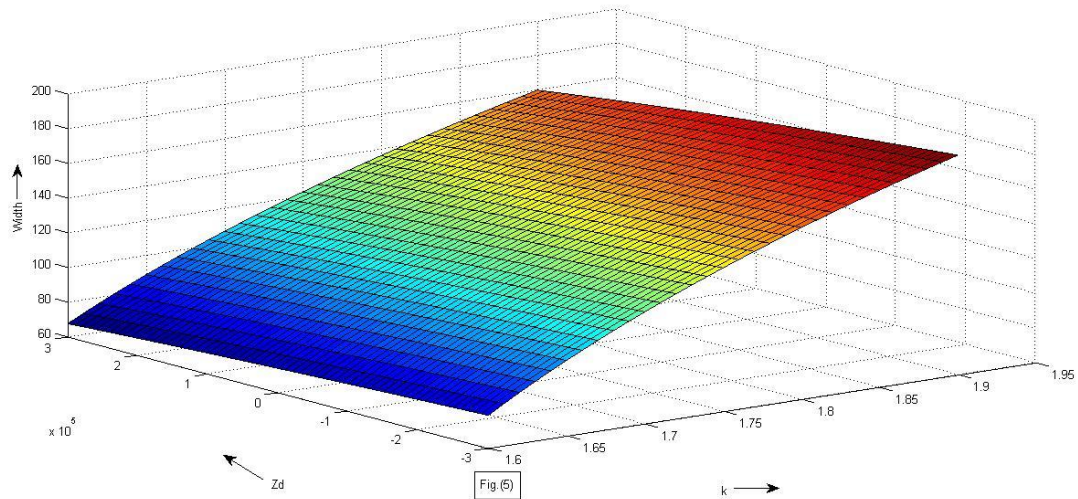


Fig. 5 Variation of Width with respect to potential k and Z_D at $u_1 = 0.00001$, $M_d = 1 \times 10^{16}$.

In figure (5), 3D curve shows that as the parameters spectral index increases, the width of the soliton increases and the parameters charge dust grain increases, the width of the soliton increases.

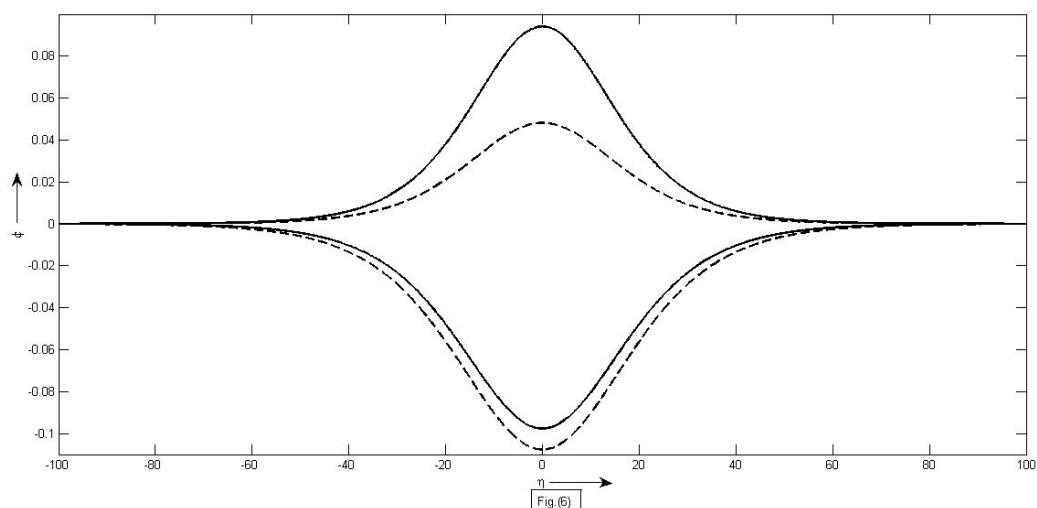


Fig. 6 Variation of η with respect to potential ϕ for different value of $k = 2.02$ and 2.02 , $Z_D = +1 \times 10^5$ (solid line), $Z_D = -1 \times 10^5$ (dashed line), $\alpha = 1 \times 10^{-6}$, $u_1 = 0.001$, $M_d = 1 \times 10^{16}$.

In figure (6), the variation of the potential (ϕ) vs η (equation (15)) for different values of spectral index for positive and negative charge dust grain with fixed values of other parameters.

CONCLUSIONS

In summary, we have addressed the problem of the soliton in unmagnetized dusty plasma with superthermal electrons and ions.

- (1) For a given set of plasma parameters, on increasing k , the amplitude of compressive soliton decreases but width of compressive soliton increases.
- (2) On increasing Z_D , the amplitude of compressive soliton increases but width of compressive soliton decreases for a given set of plasma parameters.
- (3) Phase velocity of soliton increases as k increases and Z_D decreases.

The results obtained in this study may be useful to explain soliton associated with ion-acoustic waves in the astrophysical environment where dust, ions and superthermal electrons are present.

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